Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.4-a+b-x-^m-c+d-x-^n-e+f-x-^p-g+h-x-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [159]. This is test number [15].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.37 (158)	% 0.63 (1)
Mathematica	% 97.48 (155)	% 2.52 (4)
Maple	% 80.5 (128)	% 19.5 (31)
Maxima	% 13.21 (21)	% 86.79 (138)
Fricas	% 29.56 (47)	% 70.44 (112)
Sympy	% 23.9 (38)	% 76.1 (121)
Giac	% 24.53 (39)	% 75.47 (120)

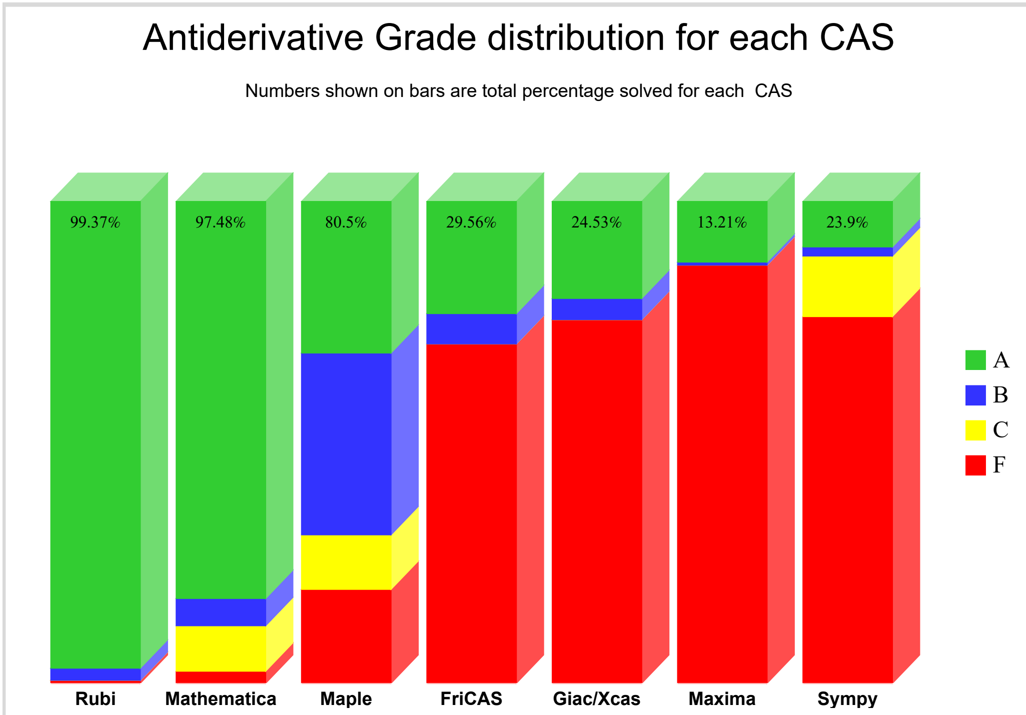
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

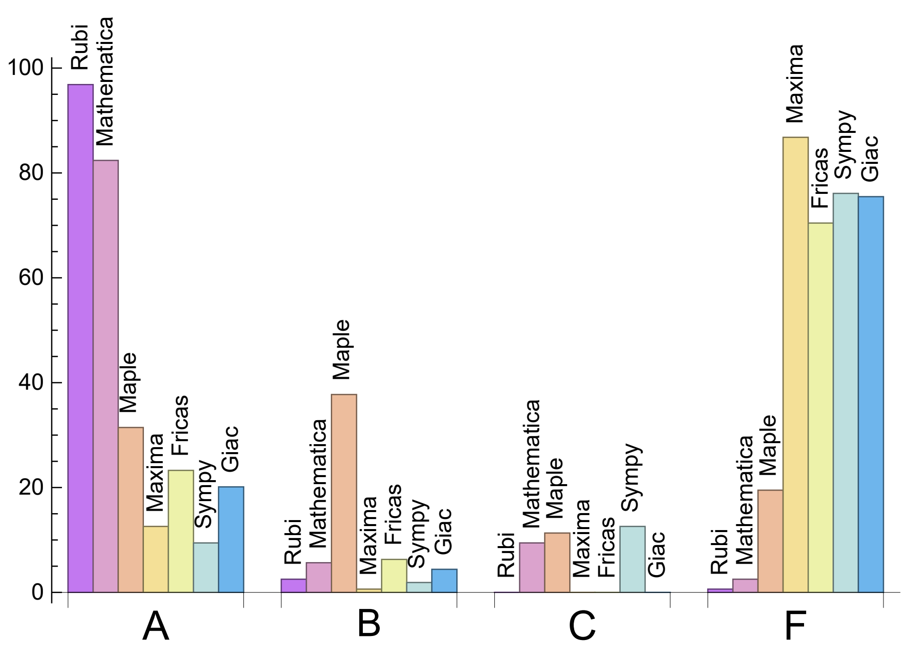
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	96.86	2.52	0.	0.63
Mathematica	82.39	5.66	9.43	2.52
Maple	31.45	37.74	11.32	19.5
Maxima	12.58	0.63	0.	86.79
Fricas	23.27	6.29	0.	70.44
Sympy	9.43	1.89	12.58	76.1
Giac	20.13	4.4	0.	75.47

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.24	223.98	1.05	191.5	1.
Mathematica	1.76	437.15	1.34	153.	0.9
Maple	0.03	1062.64	2.85	196.	1.81
Maxima	2.1	116.67	1.48	105.	1.67
Fricas	1.5	921.19	5.7	203.	3.69
Sympy	21.26	468.95	4.25	218.5	2.71
Giac	2.09	219.62	1.99	142.	1.81

1.4 list of integrals that has no closed form antiderivative

{143}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {136}

Mathematica {72, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 93, 94, 95, 96, 99, 101, 102, 107, 111, 132, 136, 146, 154, 155, 156, 157}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

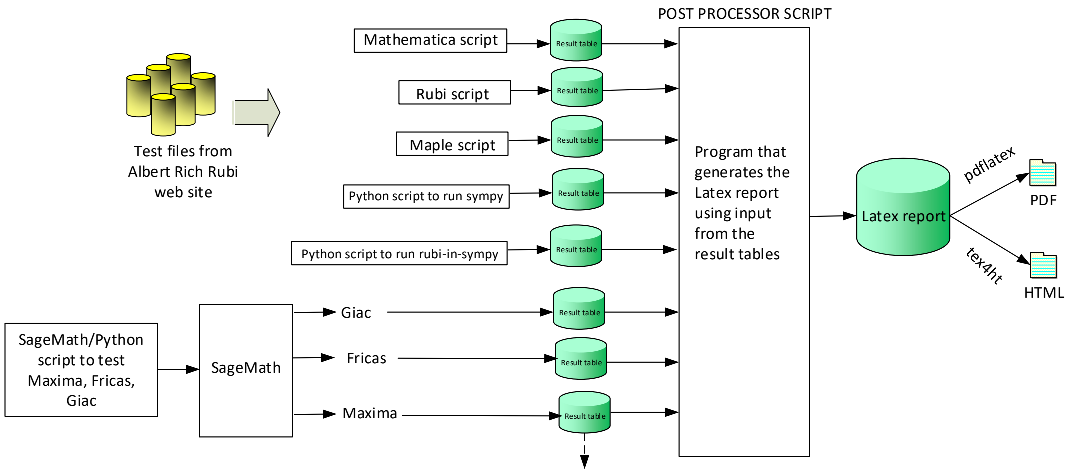
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 158, 159 }

B grade: { 97, 155, 156, 157 }

C grade: { }

F grade: { 111}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 100, 101, 102, 103, 104, 105, 106, 109, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159 }

B grade: { 54, 97, 99, 107, 108, 110, 111, 155, 156 }

C grade: { 33, 34, 43, 58, 59, 68, 69, 70, 71, 72, 73, 74, 75, 76, 132 }

F grade: { 139, 140, 141, 144}

2.1.3 Maple

A grade: { 1, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 51, 52, 53, 55, 58, 60, 61, 62, 63, 65, 70, 88, 95, 102, 104, 109, 143 }

B grade: { 2, 3, 12, 14, 33, 34, 40, 41, 42, 43, 49, 50, 56, 57, 59, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 110, 111, 119, 130, 131, 134, 135 }

C grade: { 22, 23, 24, 25, 26, 54, 64, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

F grade: { 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 31, 32, 143, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159

B grade: { 155 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 143, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

B grade: { 13, 14, 20, 21, 26, 119, 130, 131, 134, 135 }

C grade: { }

F grade: { 4, 5, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.6 Sympy

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 119 }

B grade: { 3, 13, 20 }

C grade: { 22, 23, 24, 25, 26, 27, 28, 29, 30, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

F grade: { 4, 5, 14, 21, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148 }

2.1.7 Giac

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 31, 32, 143, 149, 150, 154, 155, 156, 157 }

B grade: { 27, 28, 29, 30, 119, 158, 159 }

C grade: { }

F grade: { 3, 4, 5, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 151, 152, 153 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	109	146	379	148	203
normalized size	1	1.	1.	0.97	1.3	3.38	1.32	1.81
time (sec)	N/A	0.158	0.049	0.002	1.144	1.08	0.074	1.41

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	123	246	219	359	141	281
normalized size	1	1.	0.98	1.95	1.74	2.85	1.12	2.23
time (sec)	N/A	0.211	0.089	0.004	1.15	1.212	0.972	1.304

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	85	196	140	242	507	0
normalized size	1	1.	1.01	2.33	1.67	2.88	6.04	0.
time (sec)	N/A	0.086	0.071	0.008	1.375	1.484	10.499	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	102	179	181	0	0	0
normalized size	1	1.	0.94	1.66	1.68	0.	0.	0.
time (sec)	N/A	0.11	0.086	0.007	1.647	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	164	164	419	0	0	0
normalized size	1	1.	1.01	1.01	2.57	0.	0.	0.
time (sec)	N/A	0.212	0.247	0.007	1.282	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	66	20	30
normalized size	1	1.	1.	0.87	1.13	2.87	0.87	1.3
time (sec)	N/A	0.011	0.006	0.006	2.159	1.685	0.115	2.225

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	34	46	170	32	42
normalized size	1	1.	0.77	0.79	1.07	3.95	0.74	0.98
time (sec)	N/A	0.037	0.02	0.007	1.232	1.675	0.142	2.168

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	205	301	0	1419	274	456
normalized size	1	1.	0.9	1.33	0.	6.25	1.21	2.01
time (sec)	N/A	0.257	0.264	0.008	0.	1.83	26.773	2.38

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	145	176	0	914	167	271
normalized size	1	1.	0.99	1.21	0.	6.26	1.14	1.86
time (sec)	N/A	0.098	0.176	0.008	0.	1.838	18.801	1.734

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	81	89	0	517	92	142
normalized size	1	1.	1.05	1.16	0.	6.71	1.19	1.84
time (sec)	N/A	0.024	0.15	0.006	0.	1.349	18.418	1.477

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	55	46	0	279	54	77
normalized size	1	1.	1.02	0.85	0.	5.17	1.	1.43
time (sec)	N/A	0.017	0.046	0.008	0.	1.324	3.872	1.531

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	196	0	1013	97	151
normalized size	1	1.	1.	1.94	0.	10.03	0.96	1.5
time (sec)	N/A	0.114	0.115	0.011	0.	1.693	17.092	1.436

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	124	192	0	2114	1204	192
normalized size	1	1.	0.98	1.51	0.	16.65	9.48	1.51
time (sec)	N/A	0.109	0.468	0.013	0.	1.91	52.541	1.673

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	260	424	0	4552	0	405
normalized size	1	1.	1.25	2.04	0.	21.88	0.	1.95
time (sec)	N/A	0.274	0.636	0.016	0.	4.495	0.	1.745

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	204	301	0	1419	274	456
normalized size	1	1.	0.9	1.33	0.	6.28	1.21	2.02
time (sec)	N/A	0.252	0.273	0.008	0.	1.427	26.157	2.56

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	146	176	0	914	167	271
normalized size	1	1.	1.01	1.21	0.	6.3	1.15	1.87
time (sec)	N/A	0.094	0.163	0.007	0.	1.364	18.678	1.754

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	87	89	0	517	92	142
normalized size	1	1.	1.13	1.16	0.	6.71	1.19	1.84
time (sec)	N/A	0.024	0.123	0.007	0.	1.226	18.186	2.756

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	55	46	0	279	54	77
normalized size	1	1.	1.02	0.85	0.	5.17	1.	1.43
time (sec)	N/A	0.016	0.046	0.006	0.	1.293	3.901	2.643

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	100	103	0	1013	100	151
normalized size	1	1.	0.99	1.02	0.	10.03	0.99	1.5
time (sec)	N/A	0.119	0.218	0.013	0.	1.647	15.252	2.624

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	122	137	0	2118	1149	192
normalized size	1	1.	0.95	1.07	0.	16.55	8.98	1.5
time (sec)	N/A	0.116	0.191	0.016	0.	1.959	54.016	2.685

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	259	221	0	4551	0	406
normalized size	1	1.	1.26	1.08	0.	22.2	0.	1.98
time (sec)	N/A	0.279	0.55	0.014	0.	4.975	0.	1.829

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	132	0	165	484	85
normalized size	1	1.	0.8	1.19	0.	1.49	4.36	0.77
time (sec)	N/A	0.045	0.085	0.03	0.	1.263	25.737	1.779

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	111	0	146	393	72
normalized size	1	1.	0.93	1.28	0.	1.68	4.52	0.83
time (sec)	N/A	0.034	0.031	0.013	0.	1.379	18.821	2.59

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	73	90	0	124	269	54
normalized size	1	1.	1.16	1.43	0.	1.97	4.27	0.86
time (sec)	N/A	0.023	0.027	0.011	0.	1.319	14.384	2.491

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	61	70	0	100	133	38
normalized size	1	1.	1.65	1.89	0.	2.7	3.59	1.03
time (sec)	N/A	0.013	0.024	0.015	0.	1.303	7.767	2.546

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	69	0	111	71	59
normalized size	1	1.	1.83	2.38	0.	3.83	2.45	2.03
time (sec)	N/A	0.016	0.024	0.016	0.	1.314	10.317	2.979

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	29	25	0	69	107	119
normalized size	1	1.	0.64	0.56	0.	1.53	2.38	2.64
time (sec)	N/A	0.01	0.012	0.003	0.	1.452	8.864	1.572

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	37	33	0	85	189	176
normalized size	1	1.	0.51	0.45	0.	1.16	2.59	2.41
time (sec)	N/A	0.019	0.015	0.004	0.	1.499	12.744	1.869

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	45	41	0	111	274	236
normalized size	1	1.	0.46	0.42	0.	1.14	2.82	2.43
time (sec)	N/A	0.027	0.017	0.003	0.	1.444	18.872	2.893

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	53	49	0	131	359	293
normalized size	1	1.	0.44	0.4	0.	1.08	2.97	2.42
time (sec)	N/A	0.038	0.019	0.005	0.	1.401	29.943	2.93

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	48	44	28	104	0	58
normalized size	1	1.	1.23	1.13	0.72	2.67	0.	1.49
time (sec)	N/A	0.007	0.015	0.015	3.19	1.324	0.	1.865

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	48	44	28	104	0	58
normalized size	1	1.	1.23	1.13	0.72	2.67	0.	1.49
time (sec)	N/A	0.011	0.006	0.004	1.766	1.328	0.	2.334

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	309	624	0	0	0	0
normalized size	1	1.	2.13	4.3	0.	0.	0.	0.
time (sec)	N/A	0.108	1.447	0.047	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	312	940	0	0	0	0
normalized size	1	1.	1.41	4.25	0.	0.	0.	0.
time (sec)	N/A	0.199	2.175	0.039	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	135	160	0	0	0	0
normalized size	1	1.	0.48	0.57	0.	0.	0.	0.
time (sec)	N/A	0.389	0.416	0.047	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	130	155	0	0	0	0
normalized size	1	1.	0.53	0.64	0.	0.	0.	0.
time (sec)	N/A	0.3	0.268	0.011	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	125	150	0	0	0	0
normalized size	1	1.	0.65	0.78	0.	0.	0.	0.
time (sec)	N/A	0.077	0.232	0.012	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	120	145	0	0	0	0
normalized size	1	1.	0.74	0.9	0.	0.	0.	0.
time (sec)	N/A	0.063	0.17	0.008	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	141	183	0	0	0	0
normalized size	1	1.	0.77	1.01	0.	0.	0.	0.
time (sec)	N/A	0.213	0.766	0.02	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	132	320	0	0	0	0
normalized size	1	1.	0.7	1.69	0.	0.	0.	0.
time (sec)	N/A	0.211	0.752	0.025	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	136	461	0	0	0	0
normalized size	1	1.	0.6	2.03	0.	0.	0.	0.
time (sec)	N/A	0.309	0.696	0.025	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	141	602	0	0	0	0
normalized size	1	1.	0.54	2.29	0.	0.	0.	0.
time (sec)	N/A	0.397	0.754	0.026	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	570	1250	3678	0	0	0	0
normalized size	1	1.	2.19	6.45	0.	0.	0.	0.
time (sec)	N/A	1.286	13.841	0.059	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	130	155	0	0	0	0
normalized size	1	1.	0.53	0.64	0.	0.	0.	0.
time (sec)	N/A	0.297	0.368	0.02	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	125	150	0	0	0	0
normalized size	1	1.	0.61	0.73	0.	0.	0.	0.
time (sec)	N/A	0.213	0.3	0.012	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	120	145	0	0	0	0
normalized size	1	1.	0.74	0.9	0.	0.	0.	0.
time (sec)	N/A	0.062	0.193	0.01	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	115	140	0	0	0	0
normalized size	1	1.	0.88	1.07	0.	0.	0.	0.
time (sec)	N/A	0.051	0.187	0.007	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	76	0	0	0	0
normalized size	1	1.	0.64	0.5	0.	0.	0.	0.
time (sec)	N/A	0.121	0.404	0.013	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	132	320	0	0	0	0
normalized size	1	1.	0.7	1.69	0.	0.	0.	0.
time (sec)	N/A	0.214	0.623	0.016	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	137	461	0	0	0	0
normalized size	1	1.	0.61	2.05	0.	0.	0.	0.
time (sec)	N/A	0.301	0.562	0.016	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	125	150	0	0	0	0
normalized size	1	1.	0.61	0.73	0.	0.	0.	0.
time (sec)	N/A	0.212	0.35	0.023	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	120	145	0	0	0	0
normalized size	1	1.	0.72	0.87	0.	0.	0.	0.
time (sec)	N/A	0.145	0.281	0.014	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	115	140	0	0	0	0
normalized size	1	1.	0.88	1.07	0.	0.	0.	0.
time (sec)	N/A	0.051	0.183	0.012	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	111	55	0	0	0	0
normalized size	1	1.	2.36	1.17	0.	0.	0.	0.
time (sec)	N/A	0.015	0.34	0.011	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	56	0	0	0	0
normalized size	1	1.	0.68	0.54	0.	0.	0.	0.
time (sec)	N/A	0.096	0.401	0.015	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	132	320	0	0	0	0
normalized size	1	1.	0.7	1.69	0.	0.	0.	0.
time (sec)	N/A	0.21	0.653	0.016	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	144	461	0	0	0	0
normalized size	1	1.	0.64	2.05	0.	0.	0.	0.
time (sec)	N/A	0.31	0.421	0.018	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	202	382	0	0	0	0
normalized size	1	1.	0.69	1.3	0.	0.	0.	0.
time (sec)	N/A	0.504	1.713	0.043	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	1195	968	0	0	0	0
normalized size	1	1.	2.66	2.16	0.	0.	0.	0.
time (sec)	N/A	0.672	8.35	0.024	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	125	150	0	0	0	0
normalized size	1	1.	0.62	0.74	0.	0.	0.	0.
time (sec)	N/A	0.216	0.497	0.024	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	120	145	0	0	0	0
normalized size	1	1.	0.73	0.88	0.	0.	0.	0.
time (sec)	N/A	0.153	0.347	0.017	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	115	140	0	0	0	0
normalized size	1	1.	0.89	1.09	0.	0.	0.	0.
time (sec)	N/A	0.063	0.239	0.017	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	187	57	0	0	0	0
normalized size	1	1.	1.91	0.58	0.	0.	0.	0.
time (sec)	N/A	0.037	0.452	0.015	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	79	36	0	0	0	0
normalized size	1	1.	1.65	0.75	0.	0.	0.	0.
time (sec)	N/A	0.014	0.109	0.013	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	99	37	0	0	0	0
normalized size	1	1.	1.94	0.73	0.	0.	0.	0.
time (sec)	N/A	0.07	0.468	0.016	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	132	320	0	0	0	0
normalized size	1	1.	0.7	1.69	0.	0.	0.	0.
time (sec)	N/A	0.22	0.634	0.019	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	147	461	0	0	0	0
normalized size	1	1.	0.65	2.05	0.	0.	0.	0.
time (sec)	N/A	0.31	0.439	0.02	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	180	552	0	0	0	0
normalized size	1	1.	1.31	4.03	0.	0.	0.	0.
time (sec)	N/A	0.061	0.586	0.022	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	319	559	0	0	0	0
normalized size	1	1.	1.12	1.97	0.	0.	0.	0.
time (sec)	N/A	0.169	1.911	0.033	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	225	223	0	0	0	0
normalized size	1	1.	1.36	1.35	0.	0.	0.	0.
time (sec)	N/A	0.374	1.365	0.026	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	321	2842	0	0	0	0
normalized size	1	1.	0.82	7.23	0.	0.	0.	0.
time (sec)	N/A	0.624	4.384	0.08	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	875	875	12191	17330	0	0	0	0
normalized size	1	1.	13.93	19.81	0.	0.	0.	0.
time (sec)	N/A	1.339	17.494	0.257	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	203	184	0	0	0	0
normalized size	1	1.	2.74	2.49	0.	0.	0.	0.
time (sec)	N/A	0.16	0.868	0.097	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	203	181	0	0	0	0
normalized size	1	1.	2.74	2.45	0.	0.	0.	0.
time (sec)	N/A	0.176	0.103	0.025	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	218	212	0	0	0	0
normalized size	1	1.	2.53	2.47	0.	0.	0.	0.
time (sec)	N/A	0.16	0.813	0.096	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	218	205	0	0	0	0
normalized size	1	1.	2.53	2.38	0.	0.	0.	0.
time (sec)	N/A	0.175	0.107	0.024	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	471	471	350	895	0	0	0	0
normalized size	1	1.	0.74	1.9	0.	0.	0.	0.
time (sec)	N/A	0.665	4.381	0.074	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	345	890	0	0	0	0
normalized size	1	1.	0.8	2.07	0.	0.	0.	0.
time (sec)	N/A	0.542	4.156	0.018	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	340	885	0	0	0	0
normalized size	1	1.	0.87	2.26	0.	0.	0.	0.
time (sec)	N/A	0.429	4.066	0.02	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	347	880	0	0	0	0
normalized size	1	1.	0.99	2.51	0.	0.	0.	0.
time (sec)	N/A	0.324	2.933	0.03	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	330	875	0	0	0	0
normalized size	1	1.	0.95	2.51	0.	0.	0.	0.
time (sec)	N/A	0.319	2.793	0.041	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	366	1149	0	0	0	0
normalized size	1	1.	0.94	2.94	0.	0.	0.	0.
time (sec)	N/A	0.429	2.974	0.04	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	251	973	0	0	0	0
normalized size	1	1.	0.76	2.95	0.	0.	0.	0.
time (sec)	N/A	0.395	2.258	0.042	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	259	1160	0	0	0	0
normalized size	1	1.	0.7	3.14	0.	0.	0.	0.
time (sec)	N/A	0.515	2.595	0.044	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	345	890	0	0	0	0
normalized size	1	1.	0.8	2.07	0.	0.	0.	0.
time (sec)	N/A	0.526	3.661	0.031	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	340	885	0	0	0	0
normalized size	1	1.	0.87	2.26	0.	0.	0.	0.
time (sec)	N/A	0.412	3.596	0.02	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	347	880	0	0	0	0
normalized size	1	1.	0.99	2.51	0.	0.	0.	0.
time (sec)	N/A	0.316	3.875	0.019	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	318	875	0	0	0	0
normalized size	1	1.	0.87	2.4	0.	0.	0.	0.
time (sec)	N/A	0.216	1.23	0.021	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	326	870	0	0	0	0
normalized size	1	1.	1.17	3.12	0.	0.	0.	0.
time (sec)	N/A	0.195	2.297	0.026	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	246	786	0	0	0	0
normalized size	1	1.	0.85	2.71	0.	0.	0.	0.
time (sec)	N/A	0.32	1.819	0.028	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	251	973	0	0	0	0
normalized size	1	1.	0.76	2.95	0.	0.	0.	0.
time (sec)	N/A	0.398	1.789	0.031	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	258	1160	0	0	0	0
normalized size	1	1.	0.7	3.14	0.	0.	0.	0.
time (sec)	N/A	0.521	2.196	0.031	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	340	885	0	0	0	0
normalized size	1	1.	0.87	2.26	0.	0.	0.	0.
time (sec)	N/A	0.421	2.445	0.036	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	349	880	0	0	0	0
normalized size	1	1.	0.99	2.51	0.	0.	0.	0.
time (sec)	N/A	0.322	3.148	0.024	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	347	875	0	0	0	0
normalized size	1	1.	0.95	2.4	0.	0.	0.	0.
time (sec)	N/A	0.202	1.369	0.023	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	170	172	0	0	0	0
normalized size	1	1.	1.68	1.7	0.	0.	0.	0.
time (sec)	N/A	0.037	0.546	0.025	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	195	237	330	0	0	0	0
normalized size	1	3.25	3.95	5.5	0.	0.	0.	0.
time (sec)	N/A	0.131	1.729	0.025	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	248	786	0	0	0	0
normalized size	1	1.	0.86	2.71	0.	0.	0.	0.
time (sec)	N/A	0.296	1.805	0.029	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	721	721	6667	18077	0	0	0	0
normalized size	1	1.	9.25	25.07	0.	0.	0.	0.
time (sec)	N/A	0.675	15.126	0.151	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	208	206	4590	0	0	0	0
normalized size	1	1.29	1.28	28.51	0.	0.	0.	0.
time (sec)	N/A	0.093	4.707	0.118	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	347	880	0	0	0	0
normalized size	1	1.	0.99	2.51	0.	0.	0.	0.
time (sec)	N/A	0.319	2.431	0.037	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	347	875	0	0	0	0
normalized size	1	1.	0.74	1.87	0.	0.	0.	0.
time (sec)	N/A	0.28	1.223	0.023	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	170	0	0	0	0
normalized size	1	1.	0.95	1.7	0.	0.	0.	0.
time (sec)	N/A	0.038	0.158	0.021	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	90	134	0	0	0	0
normalized size	1	1.	1.27	1.89	0.	0.	0.	0.
time (sec)	N/A	0.043	0.139	0.022	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	270	237	599	0	0	0	0
normalized size	1	1.38	1.22	3.07	0.	0.	0.	0.
time (sec)	N/A	0.183	1.575	0.03	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	246	786	0	0	0	0
normalized size	1	1.	0.85	2.73	0.	0.	0.	0.
time (sec)	N/A	0.303	1.719	0.031	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	968	968	6638	16526	0	0	0	0
normalized size	1	1.	6.86	17.07	0.	0.	0.	0.
time (sec)	N/A	0.882	14.274	0.087	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	584	2465	0	0	0	0
normalized size	1	1.	2.56	10.81	0.	0.	0.	0.
time (sec)	N/A	0.15	5.571	0.053	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	198	227	270	0	0	0	0
normalized size	1	1.23	1.41	1.68	0.	0.	0.	0.
time (sec)	N/A	0.082	1.289	0.059	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	3247	4660	0	0	0	0
normalized size	1	1.	7.57	10.86	0.	0.	0.	0.
time (sec)	N/A	0.256	14.337	0.095	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	B	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	786	0	7061	21094	0	0	0	0
normalized size	1	0.	8.98	26.84	0.	0.	0.	0.
time (sec)	N/A	0.01	17.458	0.224	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	285	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.279	1.322	0.089	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.496	0.077	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	153	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.074	0.069	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.026	0.068	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.03	0.063	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	170	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.184	0.052	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	177	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.267	0.082	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	149	726	0	1847	8218	2248
normalized size	1	1.	0.89	4.35	0.	11.06	49.21	13.46
time (sec)	N/A	0.131	0.233	0.007	0.	1.607	7.096	2.658

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	120	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.19	0.053	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	115	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.094	0.066	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	193	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	0.395	0.101	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	104	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.217	0.066	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	195	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	0.236	0.065	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	195	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	0.261	0.061	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	189	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.206	0.061	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	221	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.186	0.056	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	205	198	0	0	0	0	0
normalized size	1	1.01	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.249	0.054	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	237	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.331	0.052	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	220	894	0	3309	0	0
normalized size	1	1.	0.61	2.47	0.	9.14	0.	0.
time (sec)	N/A	0.362	0.55	0.008	0.	1.46	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	279	2343	0	6969	0	0
normalized size	1	1.	0.55	4.62	0.	13.75	0.	0.
time (sec)	N/A	0.586	0.769	0.01	0.	1.971	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	815	803	3579	0	0	0	0	0
normalized size	1	0.99	4.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.435	48.597	0.063	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	572	566	422	0	0	0	0	0
normalized size	1	0.99	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.662	1.816	0.059	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	360	227	906	0	3272	0	0
normalized size	1	0.99	0.63	2.5	0.	9.01	0.	0.
time (sec)	N/A	0.397	0.573	0.008	0.	1.604	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	186	182	509	0	1805	0	0
normalized size	1	0.99	0.97	2.71	0.	9.6	0.	0.
time (sec)	N/A	0.098	0.119	0.008	0.	1.505	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	177	190	199	0	0	0	0	0
normalized size	1	1.07	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.399	0.071	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	220	174	0	0	0	0	0
normalized size	1	0.94	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.137	0.068	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	184	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	0.208	0.059	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	530	530	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.179	4.524	0.25	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	393	393	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	1.486	0.173	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	0.905	0.148	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.134	0.002	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.254	0.157	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.637	0.151	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	208	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.32	0.145	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	215	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.443	0.143	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	261	223	0	0	0	0	0
normalized size	1	0.99	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	0.315	0.151	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	508	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.981	1.872	0.152	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	139	134	189	313	123
normalized size	1	1.	0.72	1.76	1.7	2.39	3.96	1.56
time (sec)	N/A	0.142	0.062	0.038	3.068	1.047	46.125	2.052

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	117	105	167	282	97
normalized size	1	1.	0.71	1.86	1.67	2.65	4.48	1.54
time (sec)	N/A	0.061	0.031	0.015	4.261	1.043	21.009	2.571

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	96	89	196	245	0
normalized size	1	1.	1.	2.	1.85	4.08	5.1	0.
time (sec)	N/A	0.184	0.05	0.024	3.423	1.142	27.867	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	97	89	201	221	0
normalized size	1	1.	1.	2.02	1.85	4.19	4.6	0.
time (sec)	N/A	0.183	0.055	0.02	3.045	1.149	27.566	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	56	108	132	154	218	0
normalized size	1	1.	0.79	1.52	1.86	2.17	3.07	0.
time (sec)	N/A	0.188	0.047	0.019	2.734	1.049	34.59	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	151	149	137	147	176	308	130
normalized size	1	1.74	1.71	1.57	1.69	2.02	3.54	1.49
time (sec)	N/A	0.146	0.331	0.023	1.352	1.052	44.788	2.124

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	C	B	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	52	135	126	120	142	150	277	104
normalized size	1	2.6	2.42	2.31	2.73	2.88	5.33	2.
time (sec)	N/A	0.071	0.21	0.016	1.36	1.102	20.739	2.571

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	C	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	135	128	95	86	184	240	96
normalized size	1	2.45	2.33	1.73	1.56	3.35	4.36	1.75
time (sec)	N/A	0.184	0.399	0.019	3.912	1.086	26.248	1.986

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	135	89	96	86	203	216	112
normalized size	1	2.45	1.62	1.75	1.56	3.69	3.93	2.04
time (sec)	N/A	0.182	0.167	0.017	2.016	1.143	27.925	2.665

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	129	82	103	88	173	212	196
normalized size	1	1.55	0.99	1.24	1.06	2.08	2.55	2.36
time (sec)	N/A	0.189	0.121	0.018	2.21	1.057	34.178	1.469

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	171	94	123	119	216	219	266
normalized size	1	1.47	0.81	1.06	1.03	1.86	1.89	2.29
time (sec)	N/A	0.221	0.116	0.019	1.794	1.032	56.645	1.69

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [72] had the largest ratio of [0.3429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	21	0.048
2	A	2	1	1.	23	0.043
3	A	2	1	1.	25	0.04
4	A	2	1	1.	27	0.037
5	A	2	1	1.	29	0.034
6	A	2	1	1.	17	0.059
7	A	3	2	1.	22	0.091
8	A	6	5	1.	25	0.2
9	A	5	5	1.	25	0.2
10	A	4	4	1.	23	0.174
11	A	4	4	1.	18	0.222
12	A	6	4	1.	25	0.16
13	A	6	4	1.	25	0.16
14	A	7	5	1.	25	0.2
15	A	6	5	1.	25	0.2
16	A	5	5	1.	25	0.2
17	A	4	4	1.	23	0.174
18	A	4	4	1.	18	0.222
19	A	6	5	1.	25	0.2
20	A	6	5	1.	25	0.2
21	A	7	6	1.	25	0.24
22	A	8	6	1.	26	0.231
23	A	7	6	1.	26	0.231
24	A	6	6	1.	24	0.25
25	A	4	4	1.	23	0.174
26	A	5	5	1.	26	0.192
27	A	3	3	1.	26	0.115
28	A	4	4	1.	26	0.154
29	A	5	4	1.	26	0.154
30	A	6	4	1.	26	0.154
31	A	4	4	1.	24	0.167
32	A	5	5	1.	36	0.139
33	A	3	3	1.	45	0.067
34	A	5	5	1.	39	0.128

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
35	A	10	8	1.	35	0.229
36	A	9	8	1.	35	0.229
37	A	8	6	1.	33	0.182
38	A	7	7	1.	28	0.25
39	A	10	10	1.	35	0.286
40	A	10	10	1.	35	0.286
41	A	11	11	1.	35	0.314
42	A	12	11	1.	35	0.314
43	A	12	10	1.	35	0.286
44	A	9	8	1.	35	0.229
45	A	8	8	1.	35	0.229
46	A	7	6	1.	33	0.182
47	A	6	6	1.	28	0.214
48	A	9	9	1.	35	0.257
49	A	10	10	1.	35	0.286
50	A	11	11	1.	35	0.314
51	A	8	8	1.	35	0.229
52	A	7	7	1.	35	0.2
53	A	6	6	1.	33	0.182
54	A	2	2	1.	28	0.071
55	A	6	6	1.	35	0.171
56	A	10	10	1.	35	0.286
57	A	11	11	1.	35	0.314
58	A	8	6	1.	35	0.171
59	A	11	8	1.	35	0.229
60	A	8	8	1.	35	0.229
61	A	7	7	1.	35	0.2
62	A	7	7	1.	35	0.2
63	A	5	5	1.	33	0.152
64	A	2	2	1.	28	0.071
65	A	3	3	1.	35	0.086
66	A	10	10	1.	35	0.286
67	A	11	11	1.	35	0.314
68	A	3	3	1.	36	0.083
69	A	6	5	1.	33	0.152
70	A	4	3	1.	35	0.086
71	A	10	8	1.	35	0.229
72	A	18	12	1.	35	0.343
73	A	3	3	1.	36	0.083
74	A	4	4	1.	31	0.129
75	A	3	3	1.	40	0.075
76	A	4	4	1.	31	0.129
77	A	12	10	1.	37	0.27
78	A	11	10	1.	37	0.27

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
79	A	10	10	1.	37	0.27
80	A	9	9	1.	37	0.243
81	A	9	9	1.	37	0.243
82	A	10	10	1.	37	0.27
83	A	9	9	1.	37	0.243
84	A	10	9	1.	37	0.243
85	A	11	10	1.	37	0.27
86	A	10	10	1.	37	0.27
87	A	9	9	1.	37	0.243
88	A	9	8	1.	37	0.216
89	A	7	7	1.	37	0.189
90	A	8	8	1.	37	0.216
91	A	9	9	1.	37	0.243
92	A	10	9	1.	37	0.243
93	A	10	10	1.	37	0.27
94	A	9	9	1.	37	0.243
95	A	9	8	1.	37	0.216
96	A	2	2	1.	37	0.054
97	B	5	5	3.25	37	0.135
98	A	8	8	1.	37	0.216
99	A	7	7	1.	37	0.189
100	A	2	2	1.29	37	0.054
101	A	9	9	1.	37	0.243
102	A	12	10	1.	37	0.27
103	A	2	2	1.	37	0.054
104	A	2	2	1.	37	0.054
105	A	8	7	1.38	37	0.189
106	A	8	8	1.	37	0.216
107	A	10	8	1.	37	0.216
108	A	2	2	1.	37	0.054
109	A	2	2	1.23	37	0.054
110	A	5	5	1.	37	0.135
111	F	0	0	N/A	0	N/A
112	A	8	3	1.	25	0.12
113	A	6	3	1.	25	0.12
114	A	4	2	1.	25	0.08
115	A	3	2	1.	23	0.087
116	A	3	2	1.	22	0.091
117	A	5	3	1.	25	0.12
118	A	6	3	1.	25	0.12
119	A	2	1	1.	23	0.043
120	A	2	2	1.	25	0.08
121	A	3	2	1.	27	0.074
122	A	5	2	1.	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
123	A	6	3	1.	25	0.12
124	A	3	3	1.	25	0.12
125	A	3	3	1.	29	0.103
126	A	3	3	1.	27	0.111
127	A	3	3	1.	29	0.103
128	A	3	3	1.01	29	0.103
129	A	3	3	1.	29	0.103
130	A	3	3	1.	29	0.103
131	A	4	3	1.	29	0.103
132	A	10	9	0.99	31	0.29
133	A	9	8	0.99	31	0.258
134	A	3	3	0.99	29	0.103
135	A	3	3	0.99	24	0.125
136	A	5	5	1.07	27	0.185
137	A	7	5	0.94	29	0.172
138	A	7	4	1.	29	0.138
139	A	31	5	1.	29	0.172
140	A	15	5	1.	29	0.172
141	A	7	4	1.	27	0.148
142	A	3	3	1.	22	0.136
143	A	0	0	0.	0	0.
144	A	7	4	1.	33	0.121
145	A	7	4	1.	34	0.118
146	A	5	5	1.	34	0.147
147	A	3	3	0.99	34	0.088
148	A	4	3	1.	34	0.088
149	A	4	4	1.	31	0.129
150	A	4	4	1.	30	0.133
151	A	7	7	1.	33	0.212
152	A	7	7	1.	33	0.212
153	A	6	6	1.	33	0.182
154	A	5	5	1.74	30	0.167
155	B	5	5	2.6	29	0.172
156	B	8	8	2.45	32	0.25
157	B	8	8	2.45	32	0.25
158	A	6	6	1.55	32	0.188
159	A	7	7	1.47	32	0.219

Chapter 3

Listing of integrals

3.1 $\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=112

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)$$

```
[Out] a*c*e*g*x + ((b*c*e*g + a*(d*e*g + c*f*g + c*e*h))*x^2)/2 + ((b*(d*e*g + c*
f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))*x^3)/3 + ((a*d*f*h + b*(d*f*g + d
*e*h + c*f*h))*x^4)/4 + (b*d*f*h*x^5)/5
```

Rubi [A] time = 0.157949, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {142}

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*(d*e*g + c*f*g + c*e*h))*x^2)/2 + ((b*(d*e*g + c*
f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))*x^3)/3 + ((a*d*f*h + b*(d*f*g + d
*e*h + c*f*h))*x^4)/4 + (b*d*f*h*x^5)/5
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \int \left(aceg+(bceg+a(deg+cfg+ceh))x+(b(deg+cfg+ceh)+a(dfg+deh\right.$$
$$= acegx+\frac{1}{2}(bceg+a(deg+cfg+ceh))x^2+\frac{1}{3}(b(deg+cfg+ceh)+a(dfg+d$$

Mathematica [A] time = 0.0493336, size = 112, normalized size = 1.

$$\frac{1}{4}x^4(adfh+bcfh+bdeh+bdfg)+\frac{1}{3}x^3(acfh+adeh+adfg+bceh+bcfg+bdeg)+\frac{1}{2}x^2(aceh+acfg+adeg+bceg)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*d*e*g + a*c*f*g + a*c*e*h)*x^2)/2 + ((b*d*e*g + b
*c*f*g + a*d*f*g + b*c*e*h + a*d*e*h + a*c*f*h)*x^3)/3 + ((b*d*f*g + b*d*e*
h + b*c*f*h + a*d*f*h)*x^4)/4 + (b*d*f*h*x^5)/5
```

Maple [A] time = 0.002, size = 109, normalized size = 1.

$$\frac{bdfhx^5}{5}+\frac{\left(\left(\left(ad+bc\right)f+bde\right)h+bdfg\right)x^4}{4}+\frac{\left(\left(acf+\left(ad+bc\right)e\right)h+\left(\left(ad+bc\right)f+bde\right)g\right)x^3}{3}+\frac{\left(aceh+\left(acf+\left(ad+\right.\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] 1/5*b*d*f*h*x^5+1/4*(((a*d+b*c)*f+b*d*e)*h+b*d*f*g)*x^4+1/3*((a*c*f+(a*d+b*
c)*e)*h+((a*d+b*c)*f+b*d*e)*g)*x^3+1/2*(a*c*e*h+(a*c*f+(a*d+b*c)*e)*g)*x^2+
a*c*e*g*x
```

Maxima [A] time = 1.14432, size = 146, normalized size = 1.3

$$\frac{1}{5}bdfhx^5+acegx+\frac{1}{4}\left(bdfg+\left(bde+(bc+ad)f\right)h\right)x^4+\frac{1}{3}\left(\left(bde+(bc+ad)f\right)g+\left(acf+(bc+ad)e\right)h\right)x^3+\frac{1}{2}\left(aceh+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*f*h*x^5 + a*c*e*g*x + 1/4*(b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^4
 + 1/3*((b*d*e + (b*c + a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^3 + 1/2*(a
*c*e*h + (a*c*f + (b*c + a*d)*e)*g)*x^2
```

Fricas [A] time = 1.08041, size = 379, normalized size = 3.38

$$\frac{1}{5}x^5hfdb+\frac{1}{4}x^4gfdb+\frac{1}{4}x^4hedb+\frac{1}{4}x^4hfcb+\frac{1}{4}x^4hfda+\frac{1}{3}x^3gedb+\frac{1}{3}x^3gfcb+\frac{1}{3}x^3hecb+\frac{1}{3}x^3gfda+\frac{1}{3}x^3heda+\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*h*f*d*b + 1/4*x^4*g*f*d*b + 1/4*x^4*h*e*d*b + 1/4*x^4*h*f*c*b + 1/4
*x^4*h*f*d*a + 1/3*x^3*g*e*d*b + 1/3*x^3*g*f*c*b + 1/3*x^3*h*e*c*b + 1/3*x^
3*g*f*d*a + 1/3*x^3*h*e*d*a + 1/3*x^3*h*f*c*a + 1/2*x^2*g*e*c*b + 1/2*x^2*g
*e*d*a + 1/2*x^2*g*f*c*a + 1/2*x^2*h*e*c*a + x*g*e*c*a
```

Sympy [A] time = 0.074263, size = 148, normalized size = 1.32

$$acegx + \frac{bdfhx^5}{5} + x^4\left(\frac{adfh}{4} + \frac{bcfh}{4} + \frac{bdeh}{4} + \frac{bdfg}{4}\right) + x^3\left(\frac{acfh}{3} + \frac{adeh}{3} + \frac{adfg}{3} + \frac{bceh}{3} + \frac{bcfg}{3} + \frac{bdeg}{3}\right) + x^2\left(\frac{acel}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] a*c*e*g*x + b*d*f*h*x**5/5 + x**4*(a*d*f*h/4 + b*c*f*h/4 + b*d*e*h/4 + b*d*
f*g/4) + x**3*(a*c*f*h/3 + a*d*e*h/3 + a*d*f*g/3 + b*c*e*h/3 + b*c*f*g/3 +
b*d*e*g/3) + x**2*(a*c*e*h/2 + a*c*f*g/2 + a*d*e*g/2 + b*c*e*g/2)
```

Giac [A] time = 1.41033, size = 203, normalized size = 1.81

$$\frac{1}{5}bdfhx^5 + \frac{1}{4}bdfgx^4 + \frac{1}{4}bcfhx^4 + \frac{1}{4}adfhx^4 + \frac{1}{4}bdhx^4e + \frac{1}{3}bcfgx^3 + \frac{1}{3}adfgx^3 + \frac{1}{3}acfhx^3 + \frac{1}{3}bdgx^3e + \frac{1}{3}bchx^3e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] 1/5*b*d*f*h*x^5 + 1/4*b*d*f*g*x^4 + 1/4*b*c*f*h*x^4 + 1/4*a*d*f*h*x^4 + 1/4
*b*d*h*x^4*e + 1/3*b*c*f*g*x^3 + 1/3*a*d*f*g*x^3 + 1/3*a*c*f*h*x^3 + 1/3*b*
d*g*x^3*e + 1/3*b*c*h*x^3*e + 1/3*a*d*h*x^3*e + 1/2*a*c*f*g*x^2 + 1/2*b*c*g
*x^2*e + 1/2*a*d*g*x^2*e + 1/2*a*c*h*x^2*e + a*c*g*x*e
```

3.2 $\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$

Optimal. Leaf size=126

$$\frac{x^2(adfh-b(-cfh-deh+dfg))}{2h^2}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}-\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+}{h^4}$$

```
[Out] ((b*(d*g - c*h)*(f*g - e*h) - a*h*(d*f*g - d*e*h - c*f*h))*x)/h^3 + ((a*d*f
*h - b*(d*f*g - d*e*h - c*f*h))*x^2)/(2*h^2) + (b*d*f*x^3)/(3*h) - ((b*g -
a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/h^4
```

Rubi [A] time = 0.211189, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {142}

$$\frac{x^2(adfh-b(-cfh-deh+dfg))}{2h^2}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}-\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+}{h^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((b*(d*g - c*h)*(f*g - e*h) - a*h*(d*f*g - d*e*h - c*f*h))*x)/h^3 + ((a*d*f
*h - b*(d*f*g - d*e*h - c*f*h))*x^2)/(2*h^2) + (b*d*f*x^3)/(3*h) - ((b*g -
a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/h^4
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx &= \int \left(\frac{b(dg-ch)(fg-eh)-ah(dfg-deh-cfh)}{h^3}+\frac{(adfh-b(dfg-deh-cfh))x}{h^2}+\frac{bdf}{h}\right.\\ &= \frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3}+\frac{(adfh-b(dfg-deh-cfh))x^2}{2h^2}+\frac{bdf}{3h}\end{aligned}$$

Mathematica [A] time = 0.0894686, size = 123, normalized size = 0.98

$$\frac{hx\left(3ah(2cfh+d(2eh-2fg+fhx))+b\left(3ch(2eh-2fg+fhx)+3deh(hx-2g)+df\left(6g^2-3ghx+2h^2x^2\right)\right)\right)-6(bg-a}{6h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] (h*x*(3*a*h*(2*c*f*h + d*(-2*f*g + 2*e*h + f*h*x)) + b*(3*d*e*h*(-2*g + h*x
) + 3*c*h*(-2*f*g + 2*e*h + f*h*x) + d*f*(6*g^2 - 3*g*h*x + 2*h^2*x^2))) -
6*(b*g - a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/(6*h^4)
```

Maple [B] time = 0.004, size = 246, normalized size = 2.

$$\frac{bdfx^3}{3h}+\frac{x^2adf}{2h}+\frac{x^2bcf}{2h}+\frac{dx^2be}{2h}-\frac{dx^2bfg}{2h^2}+\frac{acfx}{h}+\frac{adex}{h}-\frac{adfgx}{h^2}+\frac{bcex}{h}-\frac{bcfgx}{h^2}-\frac{bdegx}{h^2}+\frac{bdfg^2x}{h^3}+\frac{\ln\left(hx+\right.}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] 1/3*b*d*f*x^3/h+1/2/h*x^2*a*d*f+1/2/h*x^2*b*c*f+1/2/h*x^2*b*d*e-1/2/h^2*x^2
*b*d*f*g+1/h*a*c*f*x+1/h*a*d*e*x-1/h^2*a*d*f*g*x+1/h*b*c*e*x-1/h^2*b*c*f*g*
x-1/h^2*b*d*e*g*x+1/h^3*b*d*f*g^2*x+1/h*ln(h*x+g)*a*c*e-1/h^2*ln(h*x+g)*a*c
*f*g-1/h^2*ln(h*x+g)*a*d*e*g+1/h^3*ln(h*x+g)*a*d*f*g^2-1/h^2*ln(h*x+g)*b*c*
e*g+1/h^3*ln(h*x+g)*b*c*f*g^2+1/h^3*ln(h*x+g)*b*d*e*g^2-1/h^4*ln(h*x+g)*b*d
*f*g^3
```

Maxima [A] time = 1.15018, size = 219, normalized size = 1.74

$$\frac{2\,bdfh^2x^3-3\left(bdfgh-\left(bde+(bc+ad)f\right)h^2\right)x^2+6\left(bdfg^2-\left(bde+(bc+ad)f\right)gh+\left(acf+(bc+ad)e\right)h^2\right)x}{6\,h^3}-\frac{\left(bdf\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*(b*d*f*g*h - (b*d*e + (b*c + a*d)*f)*h^2)*x^2 + 6*
(b*d*f*g^2 - (b*d*e + (b*c + a*d)*f)*g*h + (a*c*f + (b*c + a*d)*e)*h^2)*x)/
h^3 - (b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*
c + a*d)*e)*g*h^2)*log(h*x + g)/h^4
```

Fricas [A] time = 1.21161, size = 359, normalized size = 2.85

$$\frac{2\,bdfh^3x^3-3\left(bdfgh^2-\left(bde+(bc+ad)f\right)h^3\right)x^2+6\left(bdfg^2h-\left(bde+(bc+ad)f\right)gh^2+\left(acf+(bc+ad)e\right)h^3\right)x-6\left(\right.}{6\,h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d*f*h^3*x^3 - 3*(b*d*f*g*h^2 - (b*d*e + (b*c + a*d)*f)*h^3)*x^2 +
6*(b*d*f*g^2*h - (b*d*e + (b*c + a*d)*f)*g*h^2 + (a*c*f + (b*c + a*d)*e)*h^
3)*x - 6*(b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f +
(b*c + a*d)*e)*g*h^2)*log(h*x + g))/h^4
```

Sympy [A] time = 0.972131, size = 141, normalized size = 1.12

$$\frac{bdfx^3}{3h}+\frac{x^2\left(adfh+bcfh+bdeh-bdfg\right)}{2h^2}+\frac{x\left(acfh^2+adeh^2-adfgh+bceh^2-bcfgh-bdegh+bdfg^2\right)}{h^3}+\frac{\left(ah-bg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*f*x**3/(3*h) + x**2*(a*d*f*h + b*c*f*h + b*d*e*h - b*d*f*g)/(2*h**2) +
x*(a*c*f*h**2 + a*d*e*h**2 - a*d*f*g*h + b*c*e*h**2 - b*c*f*g*h - b*d*e*g*h
 + b*d*f*g**2)/h**3 + (a*h - b*g)*(c*h - d*g)*(e*h - f*g)*log(g + h*x)/h**4
```

Giac [A] time = 1.30425, size = 281, normalized size = 2.23

$$\frac{2\,bdfh^2x^3 - 3\,bdfghx^2 + 3\,bcfh^2x^2 + 3\,adfh^2x^2 + 3\,bdh^2x^2e + 6\,bdfg^2x - 6\,bcfghx - 6\,adfghx + 6\,acfh^2x - 6\,bdghxe +}{6\,h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*b*d*f*g*h*x^2 + 3*b*c*f*h^2*x^2 + 3*a*d*f*h^2*x^2
+ 3*b*d*h^2*x^2*e + 6*b*d*f*g^2*x - 6*b*c*f*g*h*x - 6*a*d*f*g*h*x + 6*a*c*f
*h^2*x - 6*b*d*g*h*x*e + 6*b*c*h^2*x*e + 6*a*d*h^2*x*e)/h^3 - (b*d*f*g^3 -
b*c*f*g^2*h - a*d*f*g^2*h + a*c*f*g*h^2 - b*d*g^2*h*e + b*c*g*h^2*e + a*d*g
*h^2*e - a*c*h^3*e)*log(abs(h*x + g))/h^4
```

3.3 $\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=84

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}+\frac{bdx}{fh}$$

```
[Out] (b*d*x)/(f*h) + ((b*e - a*f)*(d*e - c*f)*Log[e + f*x])/(f^2*(f*g - e*h)) -
((b*g - a*h)*(d*g - c*h)*Log[g + h*x])/(h^2*(f*g - e*h))
```

Rubi [A] time = 0.0860184, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {142}

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}+\frac{bdx}{fh}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] (b*d*x)/(f*h) + ((b*e - a*f)*(d*e - c*f)*Log[e + f*x])/(f^2*(f*g - e*h)) -
((b*g - a*h)*(d*g - c*h)*Log[g + h*x])/(h^2*(f*g - e*h))
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx &= \int \left(\frac{bd}{fh}+\frac{(-be+af)(-de+cf)}{f(fg-eh)(e+fx)}+\frac{(-bg+ah)(-dg+ch)}{h(-fg+eh)(g+hx)}\right)dx\\ &= \frac{bdx}{fh}+\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}\end{aligned}$$

Mathematica [A] time = 0.0706683, size = 85, normalized size = 1.01

$$\frac{f(bdhx(fg-eh)-f(bg-ah)(dg-ch)\log(g+hx))+h^2(be-af)(de-cf)\log(e+fx)}{f^2h^2(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*h^2*Log[e + f*x] + f*(b*d*h*(f*g - e*h)*x - f*(b*g
 - a*h)*(d*g - c*h)*Log[g + h*x]))/(f^2*h^2*(f*g - e*h))
```

Maple [B] time = 0.008, size = 196, normalized size = 2.3

$$\frac{bdx}{fh}-\frac{\ln\left(fx+e\right)ac}{eh-fg}+\frac{\ln\left(fx+e\right)ade}{f\left(eh-fg\right)}+\frac{\ln\left(fx+e\right)bce}{f\left(eh-fg\right)}-\frac{\ln\left(fx+e\right)bde^{2}}{f^{2}\left(eh-fg\right)}+\frac{\ln\left(hx+g\right)ac}{eh-fg}-\frac{\ln\left(hx+g\right)adg}{h\left(eh-fg\right)}-\frac{\ln\left(hx\right.}{h\left(eh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*x/f/h-1/(e*h-f*g)*ln(f*x+e)*a*c+1/f/(e*h-f*g)*ln(f*x+e)*a*d*e+1/f/(e*h-
f*g)*ln(f*x+e)*b*c*e-1/f^2/(e*h-f*g)*ln(f*x+e)*b*d*e^2+1/(e*h-f*g)*ln(h*x+g
)*a*c-1/h/(e*h-f*g)*ln(h*x+g)*a*d*g-1/h/(e*h-f*g)*ln(h*x+g)*b*c*g+1/h^2/(e*
h-f*g)*ln(h*x+g)*b*d*g^2
```

Maxima [A] time = 1.37506, size = 140, normalized size = 1.67

$$\frac{bdx}{fh}+\frac{\left(bde^{2}+acf^{2}-(bc+ad)ef\right)\log\left(fx+e\right)}{f^{3}g-ef^{2}h}-\frac{\left(bdg^{2}+ach^{2}-(bc+ad)gh\right)\log\left(hx+g\right)}{fgh^{2}-eh^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b*d*x/(f*h) + (b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*log(f*x + e)/(f^3*g - e
*f^2*h) - (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(h*x + g)/(f*g*h^2 - e*h
^3)
```

Fricas [A] time = 1.48352, size = 242, normalized size = 2.88

$$\frac{\left(bde^{2}+acf^{2}-(bc+ad)ef\right)h^{2}\log\left(fx+e\right)+\left(bdf^{2}gh-bdefh^{2}\right)x-\left(bdf^{2}g^{2}+acf^{2}h^{2}-(bc+ad)f^{2}gh\right)\log\left(hx+g\right)}{f^{3}gh^{2}-ef^{2}h^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] ((b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*h^2*log(f*x + e) + (b*d*f^2*g*h - b*
d*e*f*h^2)*x - (b*d*f^2*g^2 + a*c*f^2*h^2 - (b*c + a*d)*f^2*g*h)*log(h*x +
g))/(f^3*g*h^2 - e*f^2*h^3)
```

Sympy [B] time = 10.4987, size = 507, normalized size = 6.04

$$\frac{bdx}{fh}+\frac{\left(ah-bg\right)\left(ch-dg\right)\log\left(x+\frac{acefh^{2}+acf^{2}gh-2adefgh-2bcefgh+bde^{2}gh+bdefg^{2}-\frac{e^{2}fh\left(ah-bg\right)\left(ch-dg\right)}{eh-fg}+\frac{2ef^{2}g\left(ah-bg\right)\left(ch-dg\right)}{eh-fg}-\frac{f^{3}g^{2}\left(ah-bg\right)\left(ch-dg\right)}{h\left(eh-fg\right)}}{2acf^{2}h^{2}-adefh^{2}-adf^{2}gh-bcefh^{2}-bcf^{2}gh+bde^{2}h^{2}+bdf^{2}g^{2}}\right)}{h^{2}\left(eh-fg\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*x/(f*h) + (a*h - b*g)*(c*h - d*g)*log(x + (a*c*e*f*h**2 + a*c*f**2*g*h
- 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h + b*d*e*f*g**2 - e**2*f*h*(a
*h - b*g)*(c*h - d*g)/(e*h - f*g) + 2*e*f**2*g*(a*h - b*g)*(c*h - d*g)/(e*h
 - f*g) - f**3*g**2*(a*h - b*g)*(c*h - d*g)/(h*(e*h - f*g)))/(2*a*c*f**2*h*
*2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**2 - b*c*f**2*g*h + b*d*e**2*h
**2 + b*d*f**2*g**2))/(h**2*(e*h - f*g)) - (a*f - b*e)*(c*f - d*e)*log(x +
(a*c*e*f*h**2 + a*c*f**2*g*h - 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h
 + b*d*e*f*g**2 + e**2*h**3*(a*f - b*e)*(c*f - d*e)/(f*(e*h - f*g)) - 2*e*g
*h**2*(a*f - b*e)*(c*f - d*e)/(e*h - f*g) + f*g**2*h*(a*f - b*e)*(c*f - d*e
)/(e*h - f*g))/(2*a*c*f**2*h**2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**
2 - b*c*f**2*g*h + b*d*e**2*h**2 + b*d*f**2*g**2))/(f**2*(e*h - f*g))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.4 $\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)} dx$

Optimal. Leaf size=108

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

```
[Out] -(((b*c - a*d)*Log[c + d*x])/((d*e - c*f)*(d*g - c*h))) + ((b*e - a*f)*Log[
e + f*x])/((d*e - c*f)*(f*g - e*h)) - ((b*g - a*h)*Log[g + h*x])/((d*g - c*
h)*(f*g - e*h))
```

Rubi [A] time = 0.110342, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {148}

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*x])/((d*e - c*f)*(d*g - c*h))) + ((b*e - a*f)*Log[
e + f*x])/((d*e - c*f)*(f*g - e*h)) - ((b*g - a*h)*Log[g + h*x])/((d*g - c*
h)*(f*g - e*h))
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)} dx = \int \left(\frac{d(-bc+ad)}{(de-cf)(dg-ch)(c+dx)}+\frac{f(-be+af)}{(de-cf)(-fg+eh)(e+fx)}+\frac{h(-bg+ah)}{(dg-ch)(fg-eh)(g+\right.$$
$$=-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

Mathematica [A] time = 0.0862102, size = 102, normalized size = 0.94

$$\frac{(bc-ad)\log(c+dx)(fg-eh)-(be-af)(dg-ch)\log(e+fx)+(bg-ah)(de-cf)\log(g+hx)}{(de-cf)(dg-ch)(eh-fg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*c - a*d)*(f*g - e*h)*Log[c + d*x] - (b*e - a*f)*(d*g - c*h)*Log[e + f*x
] + (d*e - c*f)*(b*g - a*h)*Log[g + h*x])/((d*e - c*f)*(d*g - c*h)*(-(f*g)
+ e*h))
```

Maple [A] time = 0.007, size = 179, normalized size = 1.7

$$\frac{\ln\left(dx+c\right)ad}{\left(cf-de\right)\left(ch-dg\right)}-\frac{\ln\left(dx+c\right)bc}{\left(cf-de\right)\left(ch-dg\right)}-\frac{\ln\left(fx+e\right)af}{\left(cf-de\right)\left(eh-fg\right)}+\frac{\ln\left(fx+e\right)be}{\left(cf-de\right)\left(eh-fg\right)}+\frac{\ln\left(hx+g\right)ah}{\left(ch-dg\right)\left(eh-fg\right)}-\frac{\ln(}{(ch-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] 1/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)*a*d-1/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)*b*c-1/(
c*f-d*e)/(e*h-f*g)*ln(f*x+e)*a*f+1/(c*f-d*e)/(e*h-f*g)*ln(f*x+e)*b*e+1/(c*h
-d*g)/(e*h-f*g)*ln(h*x+g)*a*h-1/(c*h-d*g)/(e*h-f*g)*ln(h*x+g)*b*g
```

Maxima [A] time = 1.64723, size = 181, normalized size = 1.68

$$-\frac{(bc-ad)\log\left(dx+c\right)}{\left(d^2e-cdf\right)g-\left(cde-c^2f\right)h}+\frac{\left(be-af\right)\log\left(fx+e\right)}{\left(def-cf^2\right)g-\left(de^2-cef\right)h}-\frac{\left(bg-ah\right)\log\left(hx+g\right)}{dfg^2+ceh^2-\left(de+cf\right)gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*log(d*x + c)/((d^2*e - c*d*f)*g - (c*d*e - c^2*f)*h) + (b*e -
a*f)*log(f*x + e)/((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h) - (b*g - a*h)*log
(h*x + g)/(d*f*g^2 + c*e*h^2 - (d*e + c*f)*g*h)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(dx + c)\left(fx + e\right)\left(hx + g\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

3.5 $\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=163

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)) + (f^2*Log[e + f*x])/((b*e - a*f)*
(d*e - c*f)*(f*g - e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)*(f*g
 - e*h))
```

Rubi [A] time = 0.211911, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {180}

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)) + (f^2*Log[e + f*x])/((b*e - a*f)*
(d*e - c*f)*(f*g - e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)*(f*g
 - e*h))
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx=\int \left(\frac{b^3}{(bc-ad)(be-af)(bg-ah)(a+bx)}-\frac{d^3}{(bc-ad)(-de+cf)(-dg+ch)(c+\cdots}\right.$$

$$=\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e\cdots}{(be-af)(de-c\cdots}$$

Mathematica [A] time = 0.246551, size = 164, normalized size = 1.01

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(cf-de)(ch-dg)}-\frac{f^2\log(e+fx)}{(be-af)(de-cf)(eh-fg)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) - (f^2*Log[e + f*x])/((b*e -
```

```
 a*f)*(d*e - c*f)*(-(f*g) + e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g -
c*h)*(f*g - e*h))
```

Maple [A] time = 0.007, size = 164, normalized size = 1.

$$\frac{d^2\ln(dx+c)}{(ad-bc)(cf-de)(ch-dg)}-\frac{f^2\ln(fx+e)}{(af-be)(cf-de)(eh-fg)}+\frac{h^2\ln(hx+g)}{(ch-dg)(ah-bg)(eh-fg)}-\frac{b^2\ln(bx+a)}{(ad-bc)(af-be)(ah-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] d^2/(a*d-b*c)/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)-f^2/(a*f-b*e)/(c*f-d*e)/(e*h-f*
g)*ln(f*x+e)+h^2/(c*h-d*g)/(a*h-b*g)/(e*h-f*g)*ln(h*x+g)-b^2/(a*d-b*c)/(a*f
-b*e)/(a*h-b*g)*ln(b*x+a)
```

Maxima [A] time = 1.28156, size = 419, normalized size = 2.57

$$\frac{b^2\log(bx+a)}{\left(\left(b^3c-ab^2d\right)e-\left(ab^2c-a^2bd\right)f\right)g-\left(\left(ab^2c-a^2bd\right)e-\left(a^2bc-a^3d\right)f\right)h}-\frac{d^2\log(dx+c)}{\left(\left(bcd^2-ad^3\right)e-\left(bc^2d-acd^2\right)f\right)g-\left(\left(bc^2d-a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(((b^3*c - a*b^2*d)*e - (a*b^2*c - a^2*b*d)*f)*g - ((a*b^2
*c - a^2*b*d)*e - (a^2*b*c - a^3*d)*f)*h) - d^2*log(d*x + c)/(((b*c*d^2 - a
*d^3)*e - (b*c^2*d - a*c*d^2)*f)*g - ((b*c^2*d - a*c*d^2)*e - (b*c^3 - a*c^
2*d)*f)*h) + f^2*log(f*x + e)/((b*d*e^2*f + a*c*f^3 - (b*c + a*d)*e*f^2)*g
- (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h) - h^2*log(h*x + g)/(b*d*f*g^
3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*c + a*d)*e)*g*h
^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] Timed out
```

3.6 $\int \frac{x}{(1+x)(2+x)(3+x)} dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rubi [A] time = 0.0107876, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {148}

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)(2+x)(3+x)}\,dx &= \int \left(-\frac{1}{2(1+x)}+\frac{2}{2+x}-\frac{3}{2(3+x)}\right)dx\\ &= -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0062417, size = 23, normalized size = 1.

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Maple [A] time = 0.006, size = 20, normalized size = 0.9

$$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Maxima [A] time = 2.15871, size = 26, normalized size = 1.13

$$-\frac{3}{2}\log\left(x+3\right)+2\log\left(x+2\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Fricas [A] time = 1.68537, size = 66, normalized size = 2.87

$$-\frac{3}{2}\log\left(x+3\right)+2\log\left(x+2\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A] time = 0.115222, size = 20, normalized size = 0.87

$$-\frac{\log\left(x+1\right)}{2}+2\log\left(x+2\right)-\frac{3\log\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Giac [A] time = 2.2248, size = 30, normalized size = 1.3

$$-\frac{3}{2}\log\left(|x+3|\right)+2\log\left(|x+2|\right)-\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

3.7 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx$

Optimal. Leaf size=43

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rubi [A] time = 0.0366784, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 148}

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx &= \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3}\,dx\\ &= \int \left(\frac{20}{3993(-6+x)}+\frac{24}{275(3+5x)^3}-\frac{201}{3025(3+5x)^2}+\frac{1493}{99825(3+5x)}\right)dx\\ &= -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}\end{aligned}$$

Mathematica [A] time = 0.020165, size = 33, normalized size = 0.77

$$\frac{\frac{99(335x+157)}{(5x+3)^2}+2500\log(x-6)+1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

Maple [A] time = 0.007, size = 34, normalized size = 0.8

$$\frac{20\ln(x-6)}{3993}-\frac{12}{1375\,(3+5x)^2}+\frac{201}{45375+75625x}+\frac{1493\ln(3+5x)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)/(x-6)/(3+5*x)^3,x)
```

```
[Out] 20/3993*ln(x-6)-12/1375/(3+5*x)^2+201/15125/(3+5*x)+1493/499125*ln(3+5*x)
```

Maxima [A] time = 1.23242, size = 46, normalized size = 1.07

$$\frac{3\,(335x+157)}{15125\,(25x^2+30x+9)}+\frac{1493}{499125}\log(5x+3)+\frac{20}{3993}\log(x-6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

Fricas [A] time = 1.6749, size = 170, normalized size = 3.95

$$\frac{1493\,(25x^2+30x+9)\log(5x+3)+2500\,(25x^2+30x+9)\log(x-6)+33165x+15543}{499125\,(25x^2+30x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

Sympy [A] time = 0.142484, size = 32, normalized size = 0.74

$$\frac{1005x+471}{378125x^2+453750x+136125}+\frac{20\log(x-6)}{3993}+\frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

Giac [A] time = 2.16757, size = 42, normalized size = 0.98

$$\frac{3\,(335\,x+157)}{15125\,(5\,x+3)^2}+\frac{1493}{499125}\,\log\left(|5\,x+3|\right)+\frac{20}{3993}\,\log\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

3.8 $\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal. Leaf size=227

$$\frac{2(c+dx)^{3/2}\left(2\left(3a^2bd^2(45de-16cf)+20a^3d^3f-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf)\right)+3bdx\left(21abd^2e-4(bc-ac\right.\right.}{315d^4}$$

```
[Out] 2*a^3*e*Sqrt[c + d*x] + (2*(3*b*d*e - 2*b*c*f + 2*a*d*f)*(a + b*x)^2*(c + d
*x)^(3/2))/(21*d^2) + (2*f*(a + b*x)^3*(c + d*x)^(3/2))/(9*d) + (2*(c + d*x
)^(3/2)*(2*(20*a^3*d^3*f + 3*a^2*b*d^2*(45*d*e - 16*c*f) - 9*a*b^2*c*d*(7*d
*e - 4*c*f) + 4*b^3*c^2*(3*d*e - 2*c*f)) + 3*b*d*(21*a*b*d^2*e - 4*(b*c - a
*d)*(3*b*d*e - 2*b*c*f + 2*a*d*f))*x))/(315*d^4) - 2*a^3*Sqrt[c]*e*ArcTanh[
Sqrt[c + d*x]/Sqrt[c]]
```

Rubi [A] time = 0.257237, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(c+dx)^{3/2}\left(2\left(3a^2bd^2(45de-16cf)+20a^3d^3f-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf)\right)+3bdx\left(21abd^2e-4(bc-ac\right.\right.}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```


```
[Out] 2*a^3*e*Sqrt[c + d*x] + (2*(3*b*d*e - 2*b*c*f + 2*a*d*f)*(a + b*x)^2*(c + d
*x)^(3/2))/(21*d^2) + (2*f*(a + b*x)^3*(c + d*x)^(3/2))/(9*d) + (2*(c + d*x
)^(3/2)*(2*(20*a^3*d^3*f + 3*a^2*b*d^2*(45*d*e - 16*c*f) - 9*a*b^2*c*d*(7*d
*e - 4*c*f) + 4*b^3*c^2*(3*d*e - 2*c*f)) + 3*b*d*(21*a*b*d^2*e - 4*(b*c - a
*d)*(3*b*d*e - 2*b*c*f + 2*a*d*f))*x))/(315*d^4) - 2*a^3*Sqrt[c]*e*ArcTanh[
Sqrt[c + d*x]/Sqrt[c]]
```


Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2\int \frac{(a+bx)^2\sqrt{c+dx}\left(\frac{9ade}{2}+\frac{3}{2}(3bde-2bcf+2adf)x\right)}{x}\,dx}{9d}$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{4\int \frac{(a+bx)\sqrt{c+dx}(\cdots}{\cdots}}{\cdots}$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2(c+dx)^{3/2}\left(2\cdots\right.}{\cdots}$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \cdots$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \cdots$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \cdots$$

Mathematica [A] time = 0.263712, size = 205, normalized size = 0.9

$$\frac{2\left(3de\left(35b(c+dx)^{3/2}\left(3a^2d^2-3abcd+b^2c^2\right)+105a^3d^3\sqrt{c+dx}-105a^3\sqrt{c}d^3\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)-21b^2(c+dx)^{5/2}(2bc-3a\cdots\right.\right.}{315\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(-(f*(c + d*x)^(3/2)*(105*(b*c - a*d)^3 - 189*b*(b*c - a*d)^2*(c + d*x)
+ 135*b^2*(b*c - a*d)*(c + d*x)^2 - 35*b^3*(c + d*x)^3)) + 3*d*e*(105*a^3*d
^3*Sqrt[c + d*x] + 35*b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*(c + d*x)^(3/2) -
 21*b^2*(2*b*c - 3*a*d)*(c + d*x)^(5/2) + 15*b^3*(c + d*x)^(7/2) - 105*a^3*
Sqrt[c]*d^3*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])))/(315*d^4)
```

Maple [A] time = 0.008, size = 301, normalized size = 1.3

$$2\,\frac{1}{d^4}\left(1/9\,fb^3\,(dx+c)^{9/2}+3/7\,(dx+c)^{7/2}\,ab^2df-3/7\,(dx+c)^{7/2}\,b^3cf+1/7\,(dx+c)^{7/2}\,b^3de+3/5\,(dx+c)^{5/2}\,a^2bd^2f-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^4*(1/9*f*b^3*(d*x+c)^(9/2)+3/7*(d*x+c)^(7/2)*a*b^2*d*f-3/7*(d*x+c)^(7/2
)*b^3*c*f+1/7*(d*x+c)^(7/2)*b^3*d*e+3/5*(d*x+c)^(5/2)*a^2*b*d^2*f-6/5*(d*x+
c)^(5/2)*a*b^2*c*d*f+3/5*(d*x+c)^(5/2)*a*b^2*d^2*e+3/5*(d*x+c)^(5/2)*b^3*c^
2*f-2/5*(d*x+c)^(5/2)*b^3*c*d*e+1/3*(d*x+c)^(3/2)*a^3*d^3*f-(d*x+c)^(3/2)*a
^2*b*c*d^2*f+(d*x+c)^(3/2)*a^2*b*d^3*e+(d*x+c)^(3/2)*a*b^2*c^2*d*f-(d*x+c)^
(3/2)*a*b^2*c*d^2*e-1/3*(d*x+c)^(3/2)*b^3*c^3*f+1/3*(d*x+c)^(3/2)*b^3*c^2*d
*e+a^3*d^4*e*(d*x+c)^(1/2)-a^3*c^(1/2)*d^4*e*arctanh((d*x+c)^(1/2)/c^(1/2))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82981, size = 1419, normalized size = 6.25

$$\left[\frac{315\,a^3\sqrt{c}d^4e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\left(35\,b^3d^4fx^4+5\left(9\,b^3d^4e+\left(b^3cd^3+27\,ab^2d^4\right)f\right)x^3+3\left(3\left(b^3cd^3+21\,ab^2d^4\right)e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*a^3*sqrt(c)*d^4*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(35*b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3
+ 3*(3*(b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a
^2*b*d^4)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 10
5*a^3*d^4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c
*d^3)*f - (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^
3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))
/d^4, 2/315*(315*a^3*sqrt(-c)*d^4*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (35*
b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3 + 3*(3*(
b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a^2*b*d^4
)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 105*a^3*d^
4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c*d^3)*f
- (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^3*d - 36
```

```
*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))/d^4]
```

Sympy [A] time = 26.773, size = 274, normalized size = 1.21

$$\frac{2a^3ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2a^3e\sqrt{c+dx} + \frac{2b^3f(c+dx)^{\frac{9}{2}}}{9d^4} + \frac{2(c+dx)^{\frac{7}{2}}\left(3ab^2df - 3b^3cf + b^3de\right)}{7d^4} + \frac{2(c+dx)^{\frac{5}{2}}\left(3a^2bd^2f - 6a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**3*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**3*e*sqrt(c + d*x) +
 2*b**3*f*(c + d*x)**(9/2)/(9*d**4) + 2*(c + d*x)**(7/2)*(3*a*b**2*d*f - 3*
b**3*c*f + b**3*d*e)/(7*d**4) + 2*(c + d*x)**(5/2)*(3*a**2*b*d**2*f - 6*a*b
**2*c*d*f + 3*a*b**2*d**2*e + 3*b**3*c**2*f - 2*b**3*c*d*e)/(5*d**4) + 2*(c
 + d*x)**(3/2)*(a**3*d**3*f - 3*a**2*b*c*d**2*f + 3*a**2*b*d**3*e + 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e - b**3*c**3*f + b**3*c**2*d*e)/(3*d**4)
```

Giac [A] time = 2.37989, size = 456, normalized size = 2.01

$$\frac{2\,a^3c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}} + \frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^3d^{32}f - 135\,(dx+c)^{\frac{7}{2}}b^3cd^{32}f + 189\,(dx+c)^{\frac{5}{2}}b^3c^2d^{32}f - 105\,(dx+c)^{\frac{3}{2}}b^3c^3d^{32}f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/315*(35*(d*x + c)^(9/
2)*b^3*d^32*f - 135*(d*x + c)^(7/2)*b^3*c*d^32*f + 189*(d*x + c)^(5/2)*b^3*
c^2*d^32*f - 105*(d*x + c)^(3/2)*b^3*c^3*d^32*f + 135*(d*x + c)^(7/2)*a*b^2
*d^33*f - 378*(d*x + c)^(5/2)*a*b^2*c*d^33*f + 315*(d*x + c)^(3/2)*a*b^2*c^
2*d^33*f + 189*(d*x + c)^(5/2)*a^2*b*d^34*f - 315*(d*x + c)^(3/2)*a^2*b*c*d
^34*f + 105*(d*x + c)^(3/2)*a^3*d^35*f + 45*(d*x + c)^(7/2)*b^3*d^33*e - 12
6*(d*x + c)^(5/2)*b^3*c*d^33*e + 105*(d*x + c)^(3/2)*b^3*c^2*d^33*e + 189*(
d*x + c)^(5/2)*a*b^2*d^34*e - 315*(d*x + c)^(3/2)*a*b^2*c*d^34*e + 315*(d*x
 + c)^(3/2)*a^2*b*d^35*e + 315*sqrt(d*x + c)*a^3*d^36*e)/d^36
```

3.9 $\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal. Leaf size=146

$$\frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf)\right)+3bdx(4adf-4bcf+7bde)\right)}{105d^3}+2a^2e\sqrt{c+dx}-2a^2\sqrt{}$$

```
[Out] 2*a^2*e*Sqrt[c + d*x] + (2*f*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + (2*(c + d
*x)^(3/2)*(2*(10*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) + 7*a*b*d*(5*d*e - 2*c*f
)) + 3*b*d*(7*b*d*e - 4*b*c*f + 4*a*d*f)*x))/(105*d^3) - 2*a^2*Sqrt[c]*e*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rubi [A] time = 0.0981034, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf)\right)+3bdx(4adf-4bcf+7bde)\right)}{105d^3}+2a^2e\sqrt{c+dx}-2a^2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```


```
[Out] 2*a^2*e*Sqrt[c + d*x] + (2*f*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + (2*(c + d
*x)^(3/2)*(2*(10*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) + 7*a*b*d*(5*d*e - 2*c*f
)) + 3*b*d*(7*b*d*e - 4*b*c*f + 4*a*d*f)*x))/(105*d^3) - 2*a^2*Sqrt[c]*e*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]]
```


Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2\int \frac{(a+bx)\sqrt{c+dx}\left(\frac{7ade}{2}+\frac{1}{2}(7bde-4bcf+4adf)x\right)}{x}\,dx}{7d}$$

$$= \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-4cf)+7abd(5de-2cf)\right.\right.}{105d^3}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-4cf)+7\right.\right.}{10}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-4cf)+7\right.\right.}{10}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-4cf)+7\right.\right.}{10}$$

Mathematica [A] time = 0.17611, size = 145, normalized size = 0.99

$$\frac{2\left(7de\left(\sqrt{c+dx}\left(15a^2d^2+10abd(c+dx)+b^2\left(-2c^2+cdx+3d^2x^2\right)\right)-15a^2\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)+f(c+dx)^{3/2}\left(-42b(c\right.\right.}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(f*(c + d*x)^(3/2)*(35*(b*c - a*d)^2 - 42*b*(b*c - a*d)*(c + d*x) + 15*b
^2*(c + d*x)^2) + 7*d*e*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + d*x) + b
^2*(-2*c^2 + c*d*x + 3*d^2*x^2)) - 15*a^2*Sqrt[c]*d^2*ArcTanh[Sqrt[c + d*x]
/Sqrt[c]])))/(105*d^3)
```

Maple [A] time = 0.008, size = 176, normalized size = 1.2

$$2\,\frac{1}{d^3}\left(1/7\,b^2f\,(dx+c)^{7/2}+2/5\,(dx+c)^{5/2}\,abdf-2/5\,(dx+c)^{5/2}\,b^2cf+1/5\,(dx+c)^{5/2}\,b^2de+1/3\,(dx+c)^{3/2}\,a^2d^2f-2/3\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^3*(1/7*b^2*f*(d*x+c)^(7/2)+2/5*(d*x+c)^(5/2)*a*b*d*f-2/5*(d*x+c)^(5/2)*
b^2*c*f+1/5*(d*x+c)^(5/2)*b^2*d*e+1/3*(d*x+c)^(3/2)*a^2*d^2*f-2/3*(d*x+c)^(
3/2)*a*b*c*d*f+2/3*(d*x+c)^(3/2)*a*b*d^2*e+1/3*(d*x+c)^(3/2)*b^2*c^2*f-1/3*
(d*x+c)^(3/2)*b^2*c*d*e+a^2*d^3*e*(d*x+c)^(1/2)-a^2*c^(1/2)*d^3*e*arctanh((
d*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83759, size = 914, normalized size = 6.26

$$\left[\frac{105\,a^2\sqrt{c}d^3e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\left(15\,b^2d^3fx^3+3\left(7\,b^2d^3e+\left(b^2cd^2+14\,abd^3\right)f\right)x^2-7\left(2\,b^2c^2d-10\,abcd^2-15\right.\right.}{105\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*a^2*sqrt(c)*d^3*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2*c*d^2 + 14*a*b*d^3)*f)*x^2 -
7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e + (8*b^2*c^3 - 28*a*b*c^2*d +
 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3)*e - (4*b^2*c^2*d - 14*a*b*c*
d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3, 2/105*(105*a^2*sqrt(-c)*d^3*e*a
rctan(sqrt(d*x + c)*sqrt(-c)/c) + (15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2
*c*d^2 + 14*a*b*d^3)*f)*x^2 - 7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e
 + (8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3
)*e - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3]
```

Sympy [A] time = 18.8006, size = 167, normalized size = 1.14

$$\frac{2a^2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^2e\sqrt{c+dx}+\frac{2b^2f\left(c+dx\right)^{\frac{7}{2}}}{7d^3}+\frac{2\left(c+dx\right)^{\frac{5}{2}}\left(2abdf-2b^2cf+b^2de\right)}{5d^3}+\frac{2\left(c+dx\right)^{\frac{3}{2}}\left(a^2d^2f-2a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**2*e*sqrt(c + d*x) +
 2*b**2*f*(c + d*x)**(7/2)/(7*d**3) + 2*(c + d*x)**(5/2)*(2*a*b*d*f - 2*b**
```

```
2*c*f + b**2*d*e)/(5*d**3) + 2*(c + d*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
+ 2*a*b*d**2*e + b**2*c**2*f - b**2*c*d*e)/(3*d**3)
```

Giac [A] time = 1.73379, size = 271, normalized size = 1.86

$$\frac{2\,a^2c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^2d^{18}f-42\,(dx+c)^{\frac{5}{2}}b^2cd^{18}f+35\,(dx+c)^{\frac{3}{2}}b^2c^2d^{18}f+42\,(dx+c)^{\frac{5}{2}}abd^{19}f-70\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/105*(15*(d*x + c)^(7/
2)*b^2*d^18*f - 42*(d*x + c)^(5/2)*b^2*c*d^18*f + 35*(d*x + c)^(3/2)*b^2*c^
2*d^18*f + 42*(d*x + c)^(5/2)*a*b*d^19*f - 70*(d*x + c)^(3/2)*a*b*c*d^19*f
+ 35*(d*x + c)^(3/2)*a^2*d^20*f + 21*(d*x + c)^(5/2)*b^2*d^19*e - 35*(d*x +
 c)^(3/2)*b^2*c*d^19*e + 70*(d*x + c)^(3/2)*a*b*d^20*e + 105*sqrt(d*x + c)*
a^2*d^21*e)/d^21
```

3.10 $\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}+2ae\sqrt{c+dx}-2a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] 2*a*e*Sqrt[c + d*x] - (2*(c + d*x)^(3/2)*(2*b*c*f - 5*d*(b*e + a*f) - 3*b*d
*f*x))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rubi [A] time = 0.0244644, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {147, 50, 63, 208}

$$-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}+2ae\sqrt{c+dx}-2a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a*e*Sqrt[c + d*x] - (2*(c + d*x)^(3/2)*(2*b*c*f - 5*d*(b*e + a*f) - 3*b*d
*f*x))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = -\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+(ae)\int \frac{\sqrt{c+dx}}{x}\,dx$$
$$= 2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+(ace)\int \frac{1}{x\sqrt{c+dx}}\,dx$$
$$= 2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+\frac{(2ace)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x\right.}{d}$$
$$= 2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}-2a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.150035, size = 81, normalized size = 1.05

$$\frac{2\sqrt{c+dx}(5ad(cf+3de+dfx)-b(c+dx)(2cf-5de-3dfx))}{15d^2}-2a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*(c + d*x)*(-5*d*e + 2*c*f - 3*d*f*x)) + 5*a*d*(3*d*e
+ c*f + d*f*x)))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Maple [A] time = 0.006, size = 89, normalized size = 1.2

$$2\frac{1}{d^2}\left(1/5\,fb\,(dx+c)^{5/2}+1/3\,(dx+c)^{3/2}\,adf-1/3\,(dx+c)^{3/2}\,bcf+1/3\,(dx+c)^{3/2}\,bde+ad^2e\sqrt{dx+c}-a\sqrt{c}d^2e\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^2*(1/5*f*b*(d*x+c)^(5/2)+1/3*(d*x+c)^(3/2)*a*d*f-1/3*(d*x+c)^(3/2)*b*c*
f+1/3*(d*x+c)^(3/2)*b*d*e+a*d^2*e*(d*x+c)^(1/2)-a*c^(1/2)*d^2*e*arctanh((d*
x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.349, size = 517, normalized size = 6.71

$$\left[\frac{15\,a\sqrt{c}d^2e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\left(3\,bd^2fx^2+5\left(bcd+3\,ad^2\right)e-\left(2\,bc^2-5\,acd\right)f+\left(5\,bd^2e+\left(bcd+5\,ad^2\right)f\right)x\right)\sqrt{d}}{15\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a*sqrt(c)*d^2*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*
(3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e
 + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/d^2, 2/15*(15*a*sqrt(-c)*d^2*e*ar
ctan(sqrt(d*x + c)*sqrt(-c)/c) + (3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (
2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/
d^2]
```

Sympy [A] time = 18.4181, size = 92, normalized size = 1.19

$$\frac{2ace\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2ae\sqrt{c+dx}+\frac{2bf\,(c+dx)^{\frac{5}{2}}}{5d^2}+\frac{2\,(c+dx)^{\frac{3}{2}}\left(adf-bcf+bde\right)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a*e*sqrt(c + d*x) + 2*b*f
*(c + d*x)**(5/2)/(5*d**2) + 2*(c + d*x)**(3/2)*(a*d*f - b*c*f + b*d*e)/(3*
d**2)
```

Giac [A] time = 1.47718, size = 142, normalized size = 1.84

$$\frac{2\,ac\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}bd^8f-5\,(dx+c)^{\frac{3}{2}}bcd^8f+5\,(dx+c)^{\frac{3}{2}}ad^9f+5\,(dx+c)^{\frac{3}{2}}bd^9e+15\,\sqrt{dx+c}ad^{10}\right.}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/15*(3*(d*x + c)^(5/2)*b
*d^8*f - 5*(d*x + c)^(3/2)*b*c*d^8*f + 5*(d*x + c)^(3/2)*a*d^9*f + 5*(d*x +
 c)^(3/2)*b*d^9*e + 15*sqrt(d*x + c)*a*d^10*e)/d^10
```

3.11 $\int \frac{\sqrt{c+dx}(e+fx)}{x} dx$

Optimal. Leaf size=54

$$2e\sqrt{c+dx} - 2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \frac{2f(c+dx)^{3/2}}{3d}$$

```
[Out] 2*e*Sqrt[c + d*x] + (2*f*(c + d*x)^(3/2))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]]
```

Rubi [A] time = 0.0165239, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {80, 50, 63, 208}

$$2e\sqrt{c+dx} - 2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \frac{2f(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[c + d*x] + (2*f*(c + d*x)^(3/2))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2f(c+dx)^{3/2}}{3d} + e\int \frac{\sqrt{c+dx}}{x}\,dx$$
$$= 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} + (ce)\int \frac{1}{x\sqrt{c+dx}}\,dx$$
$$= 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} + \frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} - 2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.0461209, size = 55, normalized size = 1.02

$$e\left(2\sqrt{c+dx} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right) + \frac{2f(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*f*(c + d*x)^(3/2))/(3*d) + e*(2*Sqrt[c + d*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])
```

Maple [A] time = 0.008, size = 46, normalized size = 0.9

$$2\frac{1}{d}\left(1/3\,f\,(dx+c)^{3/2} + de\sqrt{dx+c} - \sqrt{c}de\text{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d*(1/3*f*(d*x+c)^(3/2)+d*e*(d*x+c)^(1/2)-c^(1/2)*d*e*arctanh((d*x+c)^(1/2
)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32366, size = 279, normalized size = 5.17

$$\left[\frac{3\sqrt{c}de\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right) + 2\left(dfx+3de+cf\right)\sqrt{dx+c}}{3d}, \frac{2\left(3\sqrt{-c}de\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) + \left(dfx+3de+cf\right)\sqrt{dx+\right.}{3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*d*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*(d*f*x
 + 3*d*e + c*f)*sqrt(d*x + c))/d, 2/3*(3*sqrt(-c)*d*e*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + (d*f*x + 3*d*e + c*f)*sqrt(d*x + c))/d]
```

Sympy [A] time = 3.87157, size = 54, normalized size = 1.

$$\frac{2ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2e\sqrt{c+dx} + \frac{2f\,(c+dx)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*e*sqrt(c + d*x) + 2*f*(c +
d*x)**(3/2)/(3*d)
```

Giac [A] time = 1.53136, size = 77, normalized size = 1.43

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}} + \frac{2\left((dx+c)^{\frac{3}{2}}d^2f + 3\,\sqrt{dx+c}d^3e\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/3*((d*x + c)^(3/2)*d^2*f
+ 3*sqrt(d*x + c)*d^3*e)/d^3
```

3.12 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{bc-ad}(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

Rubi [A] time = 0.113531, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {154, 156, 63, 208}

$$\frac{2\sqrt{bc-ad}(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx &= \frac{2f\sqrt{c+dx}}{b}+\frac{2\int \frac{\frac{bce}{2}+\frac{1}{2}(bde+bcf-adf)x}{x(a+bx)\sqrt{c+dx}}\,dx}{b}\\
&= \frac{2f\sqrt{c+dx}}{b}+\frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a}-\frac{((bc-ad)(be-af))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{ab}\\
&= \frac{2f\sqrt{c+dx}}{b}+\frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{ad}-\frac{(2(bc-ad)(be-af))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,d\right.}{abd}\\
&= \frac{2f\sqrt{c+dx}}{b}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2\sqrt{bc-ad}(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.114611, size = 101, normalized size = 1.

$$\frac{2\sqrt{bc-ad}(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

Maple [B] time = 0.011, size = 196, normalized size = 1.9

$$2\frac{f\sqrt{dx+c}}{b}-2\frac{e\sqrt{c}}{a}\text{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)-2\frac{adf}{b\sqrt{(ad-bc)b}}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)+2\frac{cf}{\sqrt{(ad-bc)b}}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x)
```

```
[Out] 2*f*(d*x+c)^(1/2)/b-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a-2*a/b/((a*
d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d*f+2/((a*d-b*c
)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c*f+2/((a*d-b*c)*b)^
(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d*e-2/a*b/((a*d-b*c)*b)^(
1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69275, size = 1013, normalized size = 10.03

$$\left[\frac{b\sqrt{c}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{dx+c}af-(be-af)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab}, \frac{b\sqrt{c}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2}{x}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c
)*a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d
*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), (b*sqrt(c)*e*log((d*x - 2
*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c)*a*f + 2*(b*e - a*f)*sqrt
(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))
/(a*b), (2*b*sqrt(-c)*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 2*sqrt(d*x + c)*
a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x
 + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), 2*(b*sqrt(-c)*e*arctan(sqrt
(d*x + c)*sqrt(-c)/c) + sqrt(d*x + c)*a*f + (b*e - a*f)*sqrt(-(b*c - a*d)/b
)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [A] time = 17.0919, size = 97, normalized size = 0.96

$$\frac{2f\sqrt{c+dx}}{b}+\frac{2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2(ad-bc)(af-be)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a),x)
```

```
[Out] 2*f*sqrt(c + d*x)/b + 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/(a*sqrt(-c)) - 2*(
a*d - b*c)*(a*f - b*e)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(a*b**2*sqrt
((a*d - b*c)/b))
```

Giac [A] time = 1.43571, size = 151, normalized size = 1.5

$$\frac{2c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a\sqrt{-c}}+\frac{2\sqrt{dx+c}f}{b}+\frac{2\left(abcf-a^2df-b^2ce+abde\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a*sqrt(-c)) + 2*sqrt(d*x + c)*f/b + 2
*(a*b*c*f - a^2*d*f - b^2*c*e + a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

3.13 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2} dx$

Optimal. Leaf size=127

$$\frac{\left(2b^2ce-ad(af+be)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}$$

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(a*b*(a + b*x)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]])/a^2 + ((2*b^2*c*e - a*d*(b*e + a*f))*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[b*c - a*d]])/(a^2*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.109307, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {149, 156, 63, 208}

$$\frac{\left(2b^2ce-ad(af+be)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(a*b*(a + b*x)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]])/a^2 + ((2*b^2*c*e - a*d*(b*e + a*f))*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[b*c - a*d]])/(a^2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{\int \frac{-bce-\frac{1}{2}d(be+af)x}{x(a+bx)\sqrt{c+dx}}\,dx}{ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^2} + \frac{\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{a^2b}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^2d} + \frac{\left(2\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\right)\text{Subst}}{a^2bd}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{\left(2b^2ce-ad(be+af)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.468364, size = 124, normalized size = 0.98

$$\frac{-\frac{\left(a^2df+abde-2b^2ce\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{a\sqrt{c+dx}(be-af)}{b(a+bx)} - 2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((a*(b*e - a*f)*Sqrt[c + d*x])/(b*(a + b*x)) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]] - ((-2*b^2*c*e + a*b*d*e + a^2*d*f)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/a^2
```

Maple [A] time = 0.013, size = 192, normalized size = 1.5

$$-2\,\frac{e\sqrt{c}}{a^2}\text{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) - \frac{df}{b\,(bdx+ad)}\sqrt{dx+c} + \frac{de}{a\,(bdx+ad)}\sqrt{dx+c} + \frac{df}{b}\arctan\left(b\sqrt{dx+c}\frac{1}{\sqrt{(ad-bc)\,b}}\right)\frac{1}{\sqrt{(ad-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x)
```

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a^2-d/b*(d*x+c)^(1/2)/(b*d*x+a*
d)*f+d/a*(d*x+c)^(1/2)/(b*d*x+a*d)*e+d/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+
c)^(1/2)/((a*d-b*c)*b)^(1/2))*f+d/a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1
/2)/((a*d-b*c)*b)^(1/2))*e-2/a^2*b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/
2)/((a*d-b*c)*b)^(1/2))*c*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91043, size = 2114, normalized size = 16.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)
*e)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)
*sqrt(d*x + c))/(b*x + a)) + 2*((b^4*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*
d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*((a*b^3*c -
a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b^2
*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2
*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
+ a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + ((b^4*c - a*b^3*d)*e*x + (a*b^3*c -
 a^2*b^2*d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + ((a*b
^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c -
a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), 1/2*(4*((b^4*c - a*b^3*d)*e*x + (a*
b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (a^3*d*f
- (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) + 2*((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*
x + c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2
*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + 2*((b^4
*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + ((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x
+ c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x)]
```

Sympy [B] time = 52.5411, size = 1204, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*f*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x -
 2*b**3*c*d*x) + a*d**2*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*f*sqrt(-1/(b*(a*
d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1
/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x
))/(2*b) - 2*b*c*d*e*sqrt(c + d*x)/(2*a**3*d**2 - 2*a**2*b*c*d + 2*a**2*b*d
**2*x - 2*a*b**2*c*d*x) - c*d*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c*d*f*sqrt(-1/(b*(a*d
```

```
 - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/
(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x)
)/2 + 2*c*d*f*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**
2*c*d*x) - d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*
d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(
b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*
log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c
)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*d**2*
e*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2
*d*f*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c)) + b*c*d*e*s
qrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a
*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3))
+ sqrt(c + d*x))/(2*a) - b*c*d*e*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*a) - 2*c*e*atan(sqrt(c
 + d*x)/sqrt(a*d/b - c))/(a**2*sqrt(a*d/b - c)) + 2*c*e*atan(sqrt(c + d*x)/
sqrt(-c))/(a**2*sqrt(-c))
```

Giac [A] time = 1.67258, size = 192, normalized size = 1.51

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a^2\sqrt{-c}}+\frac{\left(a^2df-2\,b^2ce+abde\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2b}-\frac{\sqrt{dx+c}adf-\sqrt{dx+c}bde}{((dx+c)b-bc+ad)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^2*sqrt(-c)) + (a^2*d*f - 2*b^2*c*e
+ a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2*b) - (sqrt(d*x + c)*a*d*f - sqrt(d*x + c)*b*d*e)/(((d*x + c)*b - b*c
 + a*d)*a*b)
```

3.14 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx$

Optimal. Leaf size=208

$$\frac{\left(3a^2bd^2e+a^3d^2f-12ab^2cde+8b^3c^2e\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx}\left(a^2(-d)f-3abde+4b^2ce\right)}{4a^2b(a+bx)(bc-ad)}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}$$

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(2*a*b*(a + b*x)^2) + ((4*b^2*c*e - 3*a*b*d*e -
 a^2*d*f)*Sqrt[c + d*x])/(4*a^2*b*(b*c - a*d)*(a + b*x)) - (2*Sqrt[c]*e*Arc
Tanh[Sqrt[c + d*x]/Sqrt[c]])/a^3 + ((8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b
*d^2*e + a^3*d^2*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*a^
3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.273771, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {149, 151, 156, 63, 208}

$$\frac{\left(3a^2bd^2e+a^3d^2f-12ab^2cde+8b^3c^2e\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx}\left(a^2(-d)f-3abde+4b^2ce\right)}{4a^2b(a+bx)(bc-ad)}-\frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(2*a*b*(a + b*x)^2) + ((4*b^2*c*e - 3*a*b*d*e -
 a^2*d*f)*Sqrt[c + d*x])/(4*a^2*b*(b*c - a*d)*(a + b*x)) - (2*Sqrt[c]*e*Arc
Tanh[Sqrt[c + d*x]/Sqrt[c]])/a^3 + ((8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b
*d^2*e + a^3*d^2*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*a^
3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} - \frac{\int \frac{-2bce-\frac{1}{2}d(3be+af)x}{x(a+bx)^2\sqrt{c+dx}}\,dx}{2ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{\int \frac{-2bc(bc-ad)e-\frac{1}{4}d\left(4b^2ce-ad(3be+af)\right)x}{x(a+bx)\sqrt{c+dx}}\,dx}{2a^2b(bc-ad)}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^3} - \frac{\left(8b^3c^2e-12ab^2cde+\right.}{}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(2ce)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^3d} -$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3} + \frac{\left(8b^3c^2e-12ab^2\right.}{}$$

Mathematica [A] time = 0.635833, size = 260, normalized size = 1.25

$$\frac{\frac{\left(3a^2bd^2e+a^3d^2f-12ab^2cde+8b^3c^2e\right)\left(\sqrt{b}\sqrt{c+dx}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\right)}{2a^2b^{3/2}(ad-bc)} - \frac{(c+dx)^{3/2}\left(a^2df-5abde+4b^2ce\right)}{2a(a+bx)(ad-bc)} + \frac{4e(bc-ad)\left(\sqrt{c+dx}-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)}{a^2} + \frac{(\cdots}{}}{2a(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] (((b*e - a*f)*(c + d*x)^(3/2))/(a + b*x)^2 - ((4*b^2*c*e - 5*a*b*d*e + a^2*
d*f)*(c + d*x)^(3/2))/(2*a*(-(b*c) + a*d)*(a + b*x)) + (4*(b*c - a*d)*e*(Sq
rt[c + d*x] - Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]))/a^2 + ((8*b^3*c^2*e
- 12*a*b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*(Sqrt[b]*Sqrt[c + d*x] - Sqrt
[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]]))/(2*a^2*b^(3/
2)*(-(b*c) + a*d)))/(2*a*(b*c - a*d))
```

Maple [B] time = 0.016, size = 424, normalized size = 2.

$$-2\,\frac{e\sqrt{c}}{a^3}\mathrm{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) + \frac{d^2f}{4\,(bdx+ad)^2\,(ad-bc)}(dx+c)^{\frac{3}{2}} + \frac{3\,d^2be}{4\,a\,(bdx+ad)^2\,(ad-bc)}(dx+c)^{\frac{3}{2}} - \frac{b^2cde}{a^2\,(bdx+ad)^2\,(ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x)
```

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a^3+1/4*d^2/(b*d*x+a*d)^2/(a*d-
b*c)*(d*x+c)^(3/2)*f+3/4*d^2/a/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)*b*e-d/
a^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)*b^2*c*e-1/4*d^2/(b*d*x+a*d)^2/b*(
d*x+c)^(1/2)*f+5/4*d^2/a/(b*d*x+a*d)^2*(d*x+c)^(1/2)*e-d/a^2/(b*d*x+a*d)^2*
b*(d*x+c)^(1/2)*c*e+1/4*d^2/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c
)^(1/2)/((a*d-b*c)*b)^(1/2))*f+3/4*d^2/a/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arct
an(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*e-3*d/a^2/(a*d-b*c)*b/((a*d-b*c)*b)
^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c*e+2/a^3/(a*d-b*c)*b^2/
((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^2*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.49477, size = 4552, normalized size = 21.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*
d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d
^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^2*c - a*
b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x +
 a)) - 8*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^
2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*
e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*((6*a^2*b^4*c^2
 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5
*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d
 - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2
*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2
*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*
c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*
d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3
*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x +
 c)/(b*d*x + b*c)) - 4*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*
b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - ((6
*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b
^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e -
(a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c
*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4
```

```
*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), 1/8*(16*((b^6*c^2 - 2*a*b^5*c*d
 + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (
a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x + c)
*sqrt(-c)/c) - (a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*
a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*
(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a
^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d +
 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b
^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*
d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2
+ (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*
b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - 8*((b^6*c^2 - 2*a*b^5*
c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x
+ (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x +
 c)*sqrt(-c)/c) - ((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*
a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d
+ 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*
b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*
b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74538, size = 405, normalized size = 1.95

$$-\frac{\left(a^3d^2f+8\,b^3c^2e-12\,ab^2cde+3\,a^2bd^2e\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^3b^2c-a^4bd\right)\sqrt{-b^2c+abd}}+\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a^3\sqrt{-c}}-\frac{(dx+c)^{\frac{3}{2}}a^2bd^2f+\sqrt{dx+c}a^2bcd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*d^2*f + 8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b*d^2*e)*arctan(sqrt
(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c - a^4*b*d)*sqrt(-b^2*c + a*b*
d)) + 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^3*sqrt(-c)) - 1/4*((d*x + c)^
(3/2)*a^2*b*d^2*f + sqrt(d*x + c)*a^2*b*c*d^2*f - sqrt(d*x + c)*a^3*d^3*f -
 4*(d*x + c)^(3/2)*b^3*c*d*e + 4*sqrt(d*x + c)*b^3*c^2*d*e + 3*(d*x + c)^(3
/2)*a*b^2*d^2*e - 9*sqrt(d*x + c)*a*b^2*c*d^2*e + 5*sqrt(d*x + c)*a^2*b*d^3
*e)/((a^2*b^2*c - a^3*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

3.15 $\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$

Optimal. Leaf size=226

$$-\frac{2(a+bx)^{3/2}\left(2\left(-12a^2bd^2(3cf+de)+8a^3d^3f+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de)\right)-3bdx\left(4(bc-ad)(-2ad\right.\right.}{315b^4}$$

```
[Out] 2*c^3*e*Sqrt[a + b*x] + (2*(3*b*d*e + 2*b*c*f - 2*a*d*f)*(a + b*x)^(3/2)*(c
 + d*x)^2)/(21*b^2) + (2*f*(a + b*x)^(3/2)*(c + d*x)^3)/(9*b) - (2*(a + b*x
)^(3/2)*(2*(8*a^3*d^3*f - 12*a^2*b*d^2*(d*e + 3*c*f) - 5*b^3*c^2*(27*d*e +
4*c*f) + 3*a*b^2*c*d*(21*d*e + 16*c*f)) - 3*b*d*(21*b^2*c*d*e + 4*(b*c - a*
d)*(3*b*d*e + 2*b*c*f - 2*a*d*f))*x))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.251778, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {153, 147, 50, 63, 208}

$$-\frac{2(a+bx)^{3/2}\left(2\left(-12a^2bd^2(3cf+de)+8a^3d^3f+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de)\right)-3bdx\left(4(bc-ad)(-2ad\right.\right.}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```


```
[Out] 2*c^3*e*Sqrt[a + b*x] + (2*(3*b*d*e + 2*b*c*f - 2*a*d*f)*(a + b*x)^(3/2)*(c
 + d*x)^2)/(21*b^2) + (2*f*(a + b*x)^(3/2)*(c + d*x)^3)/(9*b) - (2*(a + b*x
)^(3/2)*(2*(8*a^3*d^3*f - 12*a^2*b*d^2*(d*e + 3*c*f) - 5*b^3*c^2*(27*d*e +
4*c*f) + 3*a*b^2*c*d*(21*d*e + 16*c*f)) - 3*b*d*(21*b^2*c*d*e + 4*(b*c - a*
d)*(3*b*d*e + 2*b*c*f - 2*a*d*f))*x))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```


Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{2\int \frac{\sqrt{a+bx}(c+dx)^2\left(\frac{9bce}{2}+\frac{3}{2}(3bde+2bcf-2adf)x\right)}{x}\,dx}{9b}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{4\int \frac{\sqrt{a+bx}(c+dx)(\ldots}{\ldots}}{\ldots}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \frac{2(a+bx)^{3/2}\left(2\ldots\right.}{\ldots}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \ldots$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \ldots$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \ldots$$

Mathematica [A] time = 0.273006, size = 204, normalized size = 0.9

$$\frac{2\left(3be\left(35d(a+bx)^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)+105b^3c^3\sqrt{a+bx}-105\sqrt{a}b^3c^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+21d^2(a+bx)^{5/2}(3bc-2a\ldots\right.\right.}{31\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] (2*(f*(a + b*x)^(3/2)*(105*(b*c - a*d)^3 + 189*d*(b*c - a*d)^2*(a + b*x) +
135*d^2*(b*c - a*d)*(a + b*x)^2 + 35*d^3*(a + b*x)^3) + 3*b*e*(105*b^3*c^3*
Sqrt[a + b*x] + 35*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*(a + b*x)^(3/2) + 21
*d^2*(3*b*c - 2*a*d)*(a + b*x)^(5/2) + 15*d^3*(a + b*x)^(7/2) - 105*Sqrt[a]
*b^3*c^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/(315*b^4)
```

Maple [A] time = 0.008, size = 301, normalized size = 1.3

$$2\frac{1}{b^4}\left(1/9\,fd^3\,(bx+a)^{9/2}-3/7\,(bx+a)^{7/2}\,ad^3f+3/7\,(bx+a)^{7/2}\,bcd^2f+1/7\,(bx+a)^{7/2}\,bd^3e+3/5\,(bx+a)^{5/2}\,a^2d^3f-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^4*(1/9*f*d^3*(b*x+a)^(9/2)-3/7*(b*x+a)^(7/2)*a*d^3*f+3/7*(b*x+a)^(7/2)*
b*c*d^2*f+1/7*(b*x+a)^(7/2)*b*d^3*e+3/5*(b*x+a)^(5/2)*a^2*d^3*f-6/5*(b*x+a)
^(5/2)*a*b*c*d^2*f-2/5*(b*x+a)^(5/2)*a*b*d^3*e+3/5*(b*x+a)^(5/2)*b^2*c^2*d*
f+3/5*(b*x+a)^(5/2)*b^2*c*d^2*e-1/3*(b*x+a)^(3/2)*a^3*d^3*f+(b*x+a)^(3/2)*a
^2*b*c*d^2*f+1/3*(b*x+a)^(3/2)*a^2*b*d^3*e-(b*x+a)^(3/2)*a*b^2*c^2*d*f-(b*x
+a)^(3/2)*a*b^2*c*d^2*e+1/3*(b*x+a)^(3/2)*b^3*c^3*f+(b*x+a)^(3/2)*b^3*c^2*d
*e+b^4*c^3*e*(b*x+a)^(1/2)-a^(1/2)*b^4*c^3*e*arctanh((b*x+a)^(1/2)/a^(1/2))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42747, size = 1419, normalized size = 6.28

$$\left[\frac{315\,\sqrt{a}b^4c^3e\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(35\,b^4d^3fx^4+5\left(9\,b^4d^3e+\left(27\,b^4cd^2+ab^3d^3\right)f\right)x^3+3\left(3\left(21\,b^4cd^2+ab^3d^3\right)e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*sqrt(a)*b^4*c^3*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(35*b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3
+ 3*(3*(21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2
*b^2*d^3)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*
a^3*b*d^3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4
*d^3)*f + (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*
c^3 + 63*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))
/b^4, 2/315*(315*sqrt(-a)*b^4*c^3*e*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (35*
b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3 + 3*(3*(
21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2*b^2*d^3
)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*a^3*b*d^
3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4*d^3)*f
+ (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*c^3 + 63
```

```
*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))/b^4]
```

Sympy [A] time = 26.1571, size = 274, normalized size = 1.21

$$\frac{2ac^3e\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2c^3e\sqrt{a+bx} + \frac{2d^3f\,(a+bx)^{\frac{9}{2}}}{9b^4} + \frac{2\,(a+bx)^{\frac{7}{2}}\left(-3ad^3f+3bcd^2f+bd^3e\right)}{7b^4} + \frac{2\,(a+bx)^{\frac{5}{2}}\left(3a^2d^3f-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**3*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**3*e*sqrt(a + b*x) +
 2*d**3*f*(a + b*x)**(9/2)/(9*b**4) + 2*(a + b*x)**(7/2)*(-3*a*d**3*f + 3*b
*c*d**2*f + b*d**3*e)/(7*b**4) + 2*(a + b*x)**(5/2)*(3*a**2*d**3*f - 6*a*b*
c*d**2*f - 2*a*b*d**3*e + 3*b**2*c**2*d*f + 3*b**2*c*d**2*e)/(5*b**4) + 2*(
a + b*x)**(3/2)*(-a**3*d**3*f + 3*a**2*b*c*d**2*f + a**2*b*d**3*e - 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e + b**3*c**3*f + 3*b**3*c**2*d*e)/(3*b**4)
```

Giac [A] time = 2.56003, size = 456, normalized size = 2.02

$$\frac{2\,ac^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}} + \frac{2\left(105\,(bx+a)^{\frac{3}{2}}b^{35}c^3f + 189\,(bx+a)^{\frac{5}{2}}b^{34}c^2df - 315\,(bx+a)^{\frac{3}{2}}ab^{34}c^2df + 135\,(bx+a)^{\frac{7}{2}}b^{33}cd^2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^3*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/315*(105*(b*x + a)^(3
/2)*b^35*c^3*f + 189*(b*x + a)^(5/2)*b^34*c^2*d*f - 315*(b*x + a)^(3/2)*a*b
^34*c^2*d*f + 135*(b*x + a)^(7/2)*b^33*c*d^2*f - 378*(b*x + a)^(5/2)*a*b^33
*c*d^2*f + 315*(b*x + a)^(3/2)*a^2*b^33*c*d^2*f + 35*(b*x + a)^(9/2)*b^32*d
^3*f - 135*(b*x + a)^(7/2)*a*b^32*d^3*f + 189*(b*x + a)^(5/2)*a^2*b^32*d^3*
f - 105*(b*x + a)^(3/2)*a^3*b^32*d^3*f + 315*sqrt(b*x + a)*b^36*c^3*e + 315
*(b*x + a)^(3/2)*b^35*c^2*d*e + 189*(b*x + a)^(5/2)*b^34*c*d^2*e - 315*(b*x
 + a)^(3/2)*a*b^34*c*d^2*e + 45*(b*x + a)^(7/2)*b^33*d^3*e - 126*(b*x + a)^
(5/2)*a*b^33*d^3*e + 105*(b*x + a)^(3/2)*a^2*b^33*d^3*e)/b^36
```

3.16 $\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx$

Optimal. Leaf size=145

$$\frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(2cf+de)+5b^2c(2cf+7de)\right)+3bdx(-4adf+4bcf+7bde)\right)}{105b^3}+2c^2e\sqrt{a+bx}-2\sqrt{a}c^2e$$

```
[Out] 2*c^2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + (2*(a + b
*x)^(3/2)*(2*(4*a^2*d^2*f - 7*a*b*d*(d*e + 2*c*f) + 5*b^2*c*(7*d*e + 2*c*f)
) + 3*b*d*(7*b*d*e + 4*b*c*f - 4*a*d*f)*x))/(105*b^3) - 2*Sqrt[a]*c^2*e*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0938083, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(2cf+de)+5b^2c(2cf+7de)\right)+3bdx(-4adf+4bcf+7bde)\right)}{105b^3}+2c^2e\sqrt{a+bx}-2\sqrt{a}c^2e$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] 2*c^2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + (2*(a + b
*x)^(3/2)*(2*(4*a^2*d^2*f - 7*a*b*d*(d*e + 2*c*f) + 5*b^2*c*(7*d*e + 2*c*f)
) + 3*b*d*(7*b*d*e + 4*b*c*f - 4*a*d*f)*x))/(105*b^3) - 2*Sqrt[a]*c^2*e*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2\int \frac{\sqrt{a+bx}(c+dx)\left(\frac{7bce}{2}+\frac{1}{2}(7bde+4bcf-4adf)x\right)}{x}\,dx}{7b}$$

$$= \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de+2cf)+5b^2c(7de+2cf)\right)\right.}{105b^3}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de+2cf)+5b\right.\right.}{105}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de+2cf)+5b\right.\right.}{105}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de+2cf)+5b\right.\right.}{105}$$

Mathematica [A] time = 0.162919, size = 146, normalized size = 1.01

$$\frac{2\left(7be\left(15b^2c^2\sqrt{a+bx}-15\sqrt{a}b^2c^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+5d(a+bx)^{3/2}(2bc-ad)+3d^2(a+bx)^{5/2}\right)+f(a+bx)^{3/2}\left(42d(a+b\right.}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] (2*(f*(a + b*x)^(3/2)*(35*(b*c - a*d)^2 + 42*d*(b*c - a*d)*(a + b*x) + 15*d
^2*(a + b*x)^2) + 7*b*e*(15*b^2*c^2*Sqrt[a + b*x] + 5*d*(2*b*c - a*d)*(a +
b*x)^(3/2) + 3*d^2*(a + b*x)^(5/2) - 15*Sqrt[a]*b^2*c^2*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])))/(105*b^3)
```

Maple [A] time = 0.007, size = 176, normalized size = 1.2

$$2\frac{1}{b^3}\left(1/7\,d^2f\,(bx+a)^{7/2}-2/5\,(bx+a)^{5/2}\,ad^2f+2/5\,(bx+a)^{5/2}\,bcdf+1/5\,(bx+a)^{5/2}\,bd^2e+1/3\,(bx+a)^{3/2}\,a^2d^2f-2/3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^3*(1/7*d^2*f*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a*d^2*f+2/5*(b*x+a)^(5/2)*
b*c*d*f+1/5*(b*x+a)^(5/2)*b*d^2*e+1/3*(b*x+a)^(3/2)*a^2*d^2*f-2/3*(b*x+a)^(
3/2)*a*b*c*d*f-1/3*(b*x+a)^(3/2)*a*b*d^2*e+1/3*(b*x+a)^(3/2)*b^2*c^2*f+2/3*
(b*x+a)^(3/2)*b^2*c*d*e+b^3*c^2*e*(b*x+a)^(1/2)-a^(1/2)*b^3*c^2*e*arctanh((
b*x+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36354, size = 914, normalized size = 6.3

$$\left[\frac{105\sqrt{a}b^3c^2e\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15b^3d^2fx^3+3\left(7b^3d^2e+\left(14b^3cd+ab^2d^2\right)f\right)x^2+7\left(15b^3c^2+10ab^2cd-2a\right.\right.}{105b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(a)*b^3*c^2*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*b^3*c*d + a*b^2*d^2)*f)*x^2 +
7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e + (35*a*b^2*c^2 - 28*a^2*b*c*
d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2)*e + (35*b^3*c^2 + 14*a*b^2*c
*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3, 2/105*(105*sqrt(-a)*b^3*c^2*e*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*
b^3*c*d + a*b^2*d^2)*f)*x^2 + 7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e
 + (35*a*b^2*c^2 - 28*a^2*b*c*d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2
)*e + (35*b^3*c^2 + 14*a*b^2*c*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3]
```

Sympy [A] time = 18.6783, size = 167, normalized size = 1.15

$$\frac{2ac^2e\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2c^2e\sqrt{a+bx}+\frac{2d^2f\left(a+bx\right)^{\frac{7}{2}}}{7b^3}+\frac{2\left(a+bx\right)^{\frac{5}{2}}\left(-2ad^2f+2bcdf+bd^2e\right)}{5b^3}+\frac{2\left(a+bx\right)^{\frac{3}{2}}\left(a^2d^2f-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**2*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**2*e*sqrt(a + b*x) +
 2*d**2*f*(a + b*x)**(7/2)/(7*b**3) + 2*(a + b*x)**(5/2)*(-2*a*d**2*f + 2*b
```

```
*c*d*f + b*d**2*e)/(5*b**3) + 2*(a + b*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
 - a*b*d**2*e + b**2*c**2*f + 2*b**2*c*d*e)/(3*b**3)
```

Giac [A] time = 1.75434, size = 271, normalized size = 1.87

$$\frac{2\,ac^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(35\,(bx+a)^{\frac{3}{2}}b^{20}c^2f+42\,(bx+a)^{\frac{5}{2}}b^{19}cdf-70\,(bx+a)^{\frac{3}{2}}ab^{19}cdf+15\,(bx+a)^{\frac{7}{2}}b^{18}d^2f-42\,(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^2*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/105*(35*(b*x + a)^(3/
2)*b^20*c^2*f + 42*(b*x + a)^(5/2)*b^19*c*d*f - 70*(b*x + a)^(3/2)*a*b^19*c
*d*f + 15*(b*x + a)^(7/2)*b^18*d^2*f - 42*(b*x + a)^(5/2)*a*b^18*d^2*f + 35
*(b*x + a)^(3/2)*a^2*b^18*d^2*f + 105*sqrt(b*x + a)*b^21*c^2*e + 70*(b*x +
a)^(3/2)*b^20*c*d*e + 21*(b*x + a)^(5/2)*b^19*d^2*e - 35*(b*x + a)^(3/2)*a*
b^19*d^2*e)/b^21
```

3.17 $\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}+2ce\sqrt{a+bx}-2\sqrt{a}ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2*c*e*Sqrt[a + b*x] - (2*(a + b*x)^(3/2)*(2*a*d*f - 5*b*(d*e + c*f) - 3*b*d
*f*x))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0235912, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {147, 50, 63, 208}

$$-\frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}+2ce\sqrt{a+bx}-2\sqrt{a}ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] - (2*(a + b*x)^(3/2)*(2*a*d*f - 5*b*(d*e + c*f) - 3*b*d
*f*x))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx = -\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+(ce)\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+(ace)\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= 2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+\frac{(2ace)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{b}$$
$$= 2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}-2\sqrt{a}ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.123062, size = 87, normalized size = 1.13

$$\frac{2(a+bx)^{3/2}(-adf+bcf+bde)}{3b^2}+\frac{2df(a+bx)^{5/2}}{5b^2}+2ce\sqrt{a+bx}-2\sqrt{a}ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] + (2*(b*d*e + b*c*f - a*d*f)*(a + b*x)^(3/2))/(3*b^2) +
 (2*d*f*(a + b*x)^(5/2))/(5*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]]
```

Maple [A] time = 0.007, size = 89, normalized size = 1.2

$$2\frac{1}{b^2}\left(1/5\,df\,(bx+a)^{5/2}-1/3\,(bx+a)^{3/2}\,adf+1/3\,(bx+a)^{3/2}\,bcf+1/3\,(bx+a)^{3/2}\,bde+b^2ce\sqrt{bx+a}-\sqrt{a}b^2ce\text{Artanh}\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^2*(1/5*d*f*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a*d*f+1/3*(b*x+a)^(3/2)*b*c*
f+1/3*(b*x+a)^(3/2)*b*d*e+b^2*c*e*(b*x+a)^(1/2)-a^(1/2)*b^2*c*e*arctanh((b*
x+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.22554, size = 517, normalized size = 6.71

$$\left[\frac{15\sqrt{a}b^2ce\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3b^2dfx^2+5\left(3b^2c+abd\right)e+\left(5abc-2a^2d\right)f+\left(5b^2de+\left(5b^2c+abd\right)f\right)x\right)\sqrt{b}}{15b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(a)*b^2*c*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e
 + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/b^2, 2/15*(15*sqrt(-a)*b^2*c*e*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (
5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/
b^2]
```

Sympy [A] time = 18.1863, size = 92, normalized size = 1.19

$$\frac{2ace\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2ce\sqrt{a+bx}+\frac{2df\left(a+bx\right)^{\frac{5}{2}}}{5b^2}+\frac{2\left(a+bx\right)^{\frac{3}{2}}\left(-adf+bcf+bde\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c*e*sqrt(a + b*x) + 2*d*f
*(a + b*x)**(5/2)/(5*b**2) + 2*(a + b*x)**(3/2)*(-a*d*f + b*c*f + b*d*e)/(3
*b**2)
```

Giac [A] time = 2.75584, size = 142, normalized size = 1.84

$$\frac{2ac\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(5\left(bx+a\right)^{\frac{3}{2}}b^9cf+3\left(bx+a\right)^{\frac{5}{2}}b^8df-5\left(bx+a\right)^{\frac{3}{2}}ab^8df+15\sqrt{bx+a}b^{10}ce+5\left(bx+a\right)^{\frac{3}{2}}b^9d\right.}{15b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/15*(5*(b*x + a)^(3/2)*b
^9*c*f + 3*(b*x + a)^(5/2)*b^8*d*f - 5*(b*x + a)^(3/2)*a*b^8*d*f + 15*sqrt(
b*x + a)*b^10*c*e + 5*(b*x + a)^(3/2)*b^9*d*e)/b^10
```

3.18 $\int \frac{\sqrt{a+bx}(e+fx)}{x} \, dx$

Optimal. Leaf size=54

$$2e\sqrt{a+bx} - 2\sqrt{a}e \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

```
[Out] 2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0158209, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {80, 50, 63, 208}

$$2e\sqrt{a+bx} - 2\sqrt{a}e \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}}{3b} + e\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} + (ae)\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} + \frac{(2ae)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} - 2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.0462837, size = 55, normalized size = 1.02

$$e\left(2\sqrt{a+bx} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] (2*f*(a + b*x)^(3/2))/(3*b) + e*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])
```

Maple [A] time = 0.006, size = 46, normalized size = 0.9

$$2\,\frac{1}{b}\left(1/3\,f\,(bx+a)^{3/2} + \sqrt{bx+a}be - \sqrt{a}eb\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b*(1/3*f*(b*x+a)^(3/2)+(b*x+a)^(1/2)*b*e-a^(1/2)*e*b*arctanh((b*x+a)^(1/2
)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29334, size = 279, normalized size = 5.17

$$\left[\frac{3\sqrt{a}be\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\left(bfx+3be+af\right)\sqrt{bx+a}}{3b}, \frac{2\left(3\sqrt{-a}be\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \left(bfx+3be+af\right)\sqrt{bx}\right.}{3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(a)*b*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(b*f*x
 + 3*b*e + a*f)*sqrt(b*x + a))/b, 2/3*(3*sqrt(-a)*b*e*arctan(sqrt(b*x + a)*
sqrt(-a)/a) + (b*f*x + 3*b*e + a*f)*sqrt(b*x + a))/b]
```

Sympy [A] time = 3.90109, size = 54, normalized size = 1.

$$\frac{2ae\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2e\sqrt{a+bx} + \frac{2f\,(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*e*sqrt(a + b*x) + 2*f*(a +
b*x)**(3/2)/(3*b)
```

Giac [A] time = 2.64331, size = 77, normalized size = 1.43

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}} + \frac{2\left((bx+a)^{\frac{3}{2}}b^2 f + 3\,\sqrt{bx+a}b^3 e\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/3*((b*x + a)^(3/2)*b^2*f
+ 3*sqrt(b*x + a)*b^3*e)/b^3
```

3.19 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{bc-ad}(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}+\frac{2f\sqrt{a+bx}}{d}$$

```
[Out] (2*f*Sqrt[a + b*x])/d + (2*Sqrt[b*c - a*d]*(d*e - c*f)*ArcTan[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])/c
```

Rubi [A] time = 0.11897, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {154, 156, 63, 208, 205}

$$\frac{2\sqrt{bc-ad}(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}+\frac{2f\sqrt{a+bx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d + (2*Sqrt[b*c - a*d]*(d*e - c*f)*ArcTan[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])/c
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx &= \frac{2f\sqrt{a+bx}}{d}+\frac{2\int \frac{\frac{ade}{2}+\frac{1}{2}(bde-bcf+adf)x}{x\sqrt{a+bx}(c+dx)}\,dx}{d}\\ &= \frac{2f\sqrt{a+bx}}{d}+\frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c}+\frac{((bc-ad)(de-cf))\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx}{cd}\\ &= \frac{2f\sqrt{a+bx}}{d}+\frac{(2ae)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{bc}+\frac{(2(bc-ad)(de-cf))\,\mathrm{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\right.}{bcd}\\ &= \frac{2f\sqrt{a+bx}}{d}+\frac{2\sqrt{bc-ad}(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}\end{aligned}$$

Mathematica [A] time = 0.218331, size = 100, normalized size = 0.99

$$\frac{-\frac{2\sqrt{bc-ad}(cf-de)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}}+\frac{2cf\sqrt{a+bx}}{d}-2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] ((2*c*f*Sqrt[a + b*x])/d - (2*Sqrt[b*c - a*d]*(-(d*e) + c*f)*ArcTan[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/d^(3/2) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/c
```

Maple [A] time = 0.013, size = 103, normalized size = 1.

$$2\,\frac{f\sqrt{bx+a}}{d}-2\,\frac{acdf-ad^2e-bc^2f+bcde}{dc\sqrt{(ad-bc)\,d}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}\,d}{\sqrt{(ad-bc)\,d}}\right)-2\,\frac{e\sqrt{a}}{c}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x)
```

```
[Out] 2*f*(b*x+a)^(1/2)/d-2/d*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/((a*d-b*c)*d)^(
1/2)*arctanh((b*x+a)^(1/2)*d/((a*d-b*c)*d)^(1/2))-2*e*arctanh((b*x+a)^(1/2)
/a^(1/2))*a^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64713, size = 1013, normalized size = 10.03

$$\left[\frac{\sqrt{a}de\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}cf-\left(de-cf\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{bdx-bc+2ad-2\sqrt{bx+a}d\sqrt{-\frac{bc-ad}{d}}}{dx+c}\right)}{cd}, \frac{2\sqrt{-a}de\arctan\left(\frac{\sqrt{b}}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a
)*c*f - (d*e - c*f)*sqrt(-(b*c - a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(
b*x + a)*d*sqrt(-(b*c - a*d)/d))/(d*x + c)))/(c*d), (2*sqrt(-a)*d*e*arctan(
sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt(-(b*c -
a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(b*x + a)*d*sqrt(-(b*c - a*d)/d))/
(d*x + c)))/(c*d), (sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x
) + 2*sqrt(b*x + a)*c*f - 2*(d*e - c*f)*sqrt((b*c - a*d)/d)*arctan(-sqrt(b*
x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d), 2*(sqrt(-a)*d*e*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt((b*c - a*d)/
d)*arctan(-sqrt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d)]
```

Sympy [A] time = 15.252, size = 100, normalized size = 0.99

$$\frac{2ae\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{c\sqrt{-a}}+\frac{2f\sqrt{a+bx}}{d}+\frac{2\left(ad-bc\right)\left(cf-de\right)\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-\frac{ad-bc}{d}}}\right)}{cd^2\sqrt{-\frac{ad-bc}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c),x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/(c*sqrt(-a)) + 2*f*sqrt(a + b*x)/d + 2*(
a*d - b*c)*(c*f - d*e)*atan(sqrt(a + b*x)/sqrt(-(a*d - b*c)/d))/(c*d**2*sqr
t(-(a*d - b*c)/d))
```

Giac [A] time = 2.62353, size = 151, normalized size = 1.5

$$\frac{2a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}c}+\frac{2\sqrt{bx+a}f}{d}-\frac{2\left(bc^2f-acdf-bcde+ad^2e\right)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c) + 2*sqrt(b*x + a)*f/d - 2
*(b*c^2*f - a*c*d*f - b*c*d*e + a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d
- a*d^2))/(sqrt(b*c*d - a*d^2)*c*d)
```

3.20 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} \, dx$

Optimal. Leaf size=128

$$-\frac{\left(2ad^2e-bc(cf+de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(c*d*(c + d*x)) - ((2*a*d^2*e - b*c*(d*e + c*f)
)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2)*Sqrt[b*c -
a*d]) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Rubi [A] time = 0.115998, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {149, 156, 63, 208, 205}

$$-\frac{\left(2ad^2e-bc(cf+de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(c*d*(c + d*x)) - ((2*a*d^2*e - b*c*(d*e + c*f)
)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2)*Sqrt[b*c -
a*d]) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx &= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\int \frac{-ade-\frac{1}{2}b(de+cf)x}{x\sqrt{a+bx}(c+dx)}\,dx}{cd} \\
&= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^2} - \frac{\left(2ad^2e-bc(de+cf)\right)\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx}{2c^2d} \\
&= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(2ae)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^2} - \frac{\left(2ad^2e-bc(de+cf)\right)\mathrm{Subst}\left(\int \frac{}{c-}\right.}{bc^2d} \\
&= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\left(2ad^2e-bc(de+cf)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.191439, size = 122, normalized size = 0.95

$$
\frac{\frac{\left(bc(cf+de)-2ad^2e\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}} + \frac{c\sqrt{a+bx}(de-cf)}{d(c+dx)} - 2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((c*(d*e - c*f)*Sqrt[a + b*x])/(d*(c + d*x)) + ((-2*a*d^2*e + b*c*(d*e + c*
f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[b*c - a*
d]) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Maple [A] time = 0.016, size = 137, normalized size = 1.1

$$
2\,b\left(-\frac{1}{bc^2}\left(1/2\,\frac{bc\left(cf-de\right)\sqrt{bx+a}}{d\left((bx+a)\,d-ad+bc\right)}-1/2\,\frac{2\,ad^2e-bc^2f-bcde}{d\sqrt{(ad-bc)\,d}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}\,d}{\sqrt{(ad-bc)\,d}}\right)\right)-\frac{\sqrt{a}e}{bc^2}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x)
```

```
[Out] 2*b*(-1/b/c^2*(1/2*b*c*(c*f-d*e)/d*(b*x+a)^(1/2)/((b*x+a)*d-a*d+b*c)-1/2*(2
*a*d^2*e-b*c^2*f-b*c*d*e)/d/((a*d-b*c)*d)^(1/2)*arctanh((b*x+a)^(1/2)*d/((a
*d-b*c)*d)^(1/2)))-a^(1/2)*e/b/c^2*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95887, size = 2118, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c^3*f + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3
)*e)*x)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d
^2)*sqrt(b*x + a))/(d*x + c)) - 2*((b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c
*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*((b*c^2*d
^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x + a))/(b*c^4*d^2 - a*c^
3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), 1/2*(4*((b*c*d^3 - a*d^4)*e*x + (b*c^2*
d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (b*c^3*f + (b
*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(-b*c*d
+ a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(
d*x + c)) + 2*((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x
+ a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f + (b*
c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*c*d -
a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - ((b*c*d^3
- a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*
sqrt(a) + 2*a)/x) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqr
t(b*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f
 + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*
c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 2*((
b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b
*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x)]
```

Sympy [B] time = 54.0157, size = 1149, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**2,x)
```

```
[Out] 2*a*b*d*e*sqrt(a + b*x)/(2*a*b*c**2*d + 2*a*b*c*d**2*x - 2*b**2*c**3 - 2*b*
*2*c**2*d*x) - a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a
*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d
*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log
(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)
) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2*a*b*f*sqrt(
```

```
a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*x) + a*b*d*e*
sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*
b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + s
qrt(a + b*x))/(2*c) - a*b*d*e*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt
(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*s
qrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*c) - 2*a*e*atan(sqrt(a + b*x)
/sqrt(-a + b*c/d))/(c**2*sqrt(-a + b*c/d)) + 2*a*e*atan(sqrt(a + b*x)/sqrt(
-a))/(c**2*sqrt(-a)) + 2*b**2*c*f*sqrt(a + b*x)/(2*a*b*c*d**2 + 2*a*b*d**3*
x - 2*b**2*c**2*d - 2*b**2*c*d**2*x) + b**2*c*f*sqrt(1/(d*(a*d - b*c)**3))*
log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)
**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*d) - b**2*
c*f*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2
*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3))
+ sqrt(a + b*x))/(2*d) - b**2*e*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*s
qrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**
2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + b**2*e*sqrt(1/(d*(a*d - b
*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*
d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2
*b**2*e*sqrt(a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*
x) + 2*b*f*atan(sqrt(a + b*x)/sqrt(-a + b*c/d))/(d**2*sqrt(-a + b*c/d))
```

Giac [A] time = 2.68528, size = 192, normalized size = 1.5

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}c^2}+\frac{\left(bc^2f+bcde-2\,ad^2e\right)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}c^2d}-\frac{\sqrt{bx+a}bcf-\sqrt{bx+a}bde}{(bc+(bx+a)d-ad)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^2) + (b*c^2*f + b*c*d*e -
2*a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)
*c^2*d) - (sqrt(b*x + a)*b*c*f - sqrt(b*x + a)*b*d*e)/((b*c + (b*x + a)*d -
 a*d)*c*d)
```

3.21 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3} \, dx$

Optimal. Leaf size=205

$$-\frac{\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{a+bx}\left(4ad^2e-bc(cf+3de)\right)}{4c^2d(c+dx)(bc-ad)}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}$$

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(2*c*d*(c + d*x)^2) - ((4*a*d^2*e - b*c*(3*d*e
+ c*f))*Sqrt[a + b*x])/(4*c^2*d*(b*c - a*d)*(c + d*x)) - ((12*a*b*c*d^2*e -
 8*a^2*d^3*e - b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b
*c - a*d]])/(4*c^3*d^(3/2)*(b*c - a*d)^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c^3
```

Rubi [A] time = 0.279463, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {149, 151, 156, 63, 208, 205}

$$-\frac{\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{a+bx}\left(4ad^2e-bc(cf+3de)\right)}{4c^2d(c+dx)(bc-ad)}-\frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(2*c*d*(c + d*x)^2) - ((4*a*d^2*e - b*c*(3*d*e
+ c*f))*Sqrt[a + b*x])/(4*c^2*d*(b*c - a*d)*(c + d*x)) - ((12*a*b*c*d^2*e -
 8*a^2*d^3*e - b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b
*c - a*d]])/(4*c^3*d^(3/2)*(b*c - a*d)^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c^3
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx &= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\int \frac{-2ade-\frac{1}{2}b(3de+cf)x}{x\sqrt{a+bx}(c+dx)^2}\,dx}{2cd} \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{\int \frac{2ad(bc-ad)e-\frac{1}{4}b\left(4ad^2e-bc(3de+cf)\right)x}{x\sqrt{a+bx}(c+dx)}\,dx}{2c^2d(bc-ad)} \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^3} - \frac{\left(12abcd^2e-8a^2d^3e-\right.}{} \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^3} - \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} - \frac{\left(12abcd^2e-8a^2d^3e-b^2c^2(3de+cf)\right)\text{ta}}{4c^3d^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.55004, size = 259, normalized size = 1.26

$$\frac{\frac{2\left(\frac{\left(8a^2d^3e-12abcd^2e+b^2c^2(cf+3de)\right)\left(\sqrt{d}\sqrt{a+bx}-\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{4d^{3/2}}+2e(bc-ad)^2\left(\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{a+bx}\right)\right)}{c^2(bc-ad)} - \frac{(a+bx)^{3/2}\left(4ad^2e+bc(cf-5de)\right)}{2c(c+dx)(bc-ad)} + \frac{(a+bx)^{3/2}(de}{(c+dx)^2}}{2c(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] (((d*e - c*f)*(a + b*x)^(3/2))/(c + d*x)^2 - ((4*a*d^2*e + b*c*(-5*d*e + c*
f))*(a + b*x)^(3/2))/(2*c*(b*c - a*d)*(c + d*x)) + (2*(((-12*a*b*c*d^2*e +
8*a^2*d^3*e + b^2*c^2*(3*d*e + c*f))*(Sqrt[d]*Sqrt[a + b*x] - Sqrt[b*c - a*
d]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(4*d^(3/2)) + 2*(b*c -
 a*d)^2*e*(-Sqrt[a + b*x] + Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/(c^2*
```

```
(b*c - a*d)))/(2*c*(-(b*c) + a*d))
```

Maple [A] time = 0.014, size = 221, normalized size = 1.1

$$2\,b^{2}\left(-\frac{1}{c^{3}b^{2}}\left(\frac{1}{\left(\left(bx+a\right)d-ad+bc\right)^{2}}\left(-1/8\,\frac{bc\left(4\,ad^{2}e-bc^{2}f-3\,bcde\right)\left(bx+a\right)^{3/2}}{ad-bc}+1/8\,\frac{\left(4\,ad^{2}e+bc^{2}f-5\,bcde\right)bc\sqrt{bx}}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x)
```

```
[Out] 2*b^2*(-1/b^2/c^3*((-1/8*b*c*(4*a*d^2*e-b*c^2*f-3*b*c*d*e)/(a*d-b*c)*(b*x+a
)^(3/2)+1/8*(4*a*d^2*e+b*c^2*f-5*b*c*d*e)*b*c/d*(b*x+a)^(1/2))/((b*x+a)*d-a
*d+b*c)^2-1/8*(8*a^2*d^3*e-12*a*b*c*d^2*e+b^2*c^3*f+3*b^2*c^2*d*e)/(a*d-b*c
)/d/((a*d-b*c)*d)^(1/2)*arctanh((b*x+a)^(1/2)*d/((a*d-b*c)*d)^(1/2)))-a^(1/
2)*e/b^2/c^3*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.97522, size = 4551, normalized size = 22.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d
^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*
d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*
d^2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x
+ c)) + 8*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2
*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)
*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*((5*b^2*c^4*d^
2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*
c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^
2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^
5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 -
2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), 1/8*(16*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*
d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2
 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5
)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*
```

```
f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*d^
2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x +
c)) + 2*((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d -
3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*
c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 -
2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x
^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2*c^5*f + (
b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2 + (3*b^2*
c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b^2*c^3*d^2
 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*
d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 4*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a
^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*
d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sq
rt(a) + 2*a)/x) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^
2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^
4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*
c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*
c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2
*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2
 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b
^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan
(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 8*((b^2*c^2*d^4 - 2*a*b
*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x +
 (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2
*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4
 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c
^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c
^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.82932, size = 406, normalized size = 1.98

$$\frac{\left(b^2c^3f+3\,b^2c^2de-12\,abcd^2e+8\,a^2d^3e\right)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{4\left(bc^4d-ac^3d^2\right)\sqrt{bcd-ad^2}}+\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}c^3}-\sqrt{bx+a}ab^3c^3f-(bx+a)^{\frac{3}{2}}b^2c^2df-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e + 8*a^2*d^3*e)*arctan(sqrt(
b*x + a)*d/sqrt(b*c*d - a*d^2))/((b*c^4*d - a*c^3*d^2)*sqrt(b*c*d - a*d^2))
 + 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^3) - 1/4*(sqrt(b*x + a)
*b^3*c^3*f - (b*x + a)^(3/2)*b^2*c^2*d*f - sqrt(b*x + a)*a*b^2*c^2*d*f - 5*
```

```
sqrt(b*x + a)*b^3*c^2*d*e - 3*(b*x + a)^(3/2)*b^2*c*d^2*e + 9*sqrt(b*x + a)
*a*b^2*c*d^2*e + 4*(b*x + a)^(3/2)*a*b*d^3*e - 4*sqrt(b*x + a)*a^2*b*d^3*e)
/((b*c^3*d - a*c^2*d^2)*(b*c + (b*x + a)*d - a*d)^2)
```

3.22 $\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{1-ax}(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\sqrt{ax}}{64a^4}-\frac{75\sin^{-1}(1-2ax)}{128a^4}$$

```
[Out] (-75*Sqrt[a*x]*Sqrt[1 - a*x])/(64*a^4) - (25*(a*x)^(3/2)*Sqrt[1 - a*x])/(32
*a^4) - (5*(a*x)^(5/2)*Sqrt[1 - a*x])/(8*a^4) - ((a*x)^(7/2)*Sqrt[1 - a*x])
/(4*a^4) - (75*ArcSin[1 - 2*a*x])/(128*a^4)
```

Rubi [A] time = 0.0448675, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\sqrt{ax}}{64a^4}-\frac{75\sin^{-1}(1-2ax)}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-75*Sqrt[a*x]*Sqrt[1 - a*x])/(64*a^4) - (25*(a*x)^(3/2)*Sqrt[1 - a*x])/(32
*a^4) - (5*(a*x)^(5/2)*Sqrt[1 - a*x])/(8*a^4) - ((a*x)^(7/2)*Sqrt[1 - a*x])
/(4*a^4) - (75*ArcSin[1 - 2*a*x])/(128*a^4)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx &= \frac{\int \frac{(ax)^{5/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^3} \\
&= -\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{15\int \frac{(ax)^{5/2}}{\sqrt{1-ax}}\,dx}{8a^3} \\
&= -\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{25\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{16a^3} \\
&= -\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{64a^3} \\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{128a^3} \\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{128a^3} \\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right.}{128a} \\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\sin^{-1}(1-2ax)}{128a^4}
\end{aligned}$$

Mathematica [A] time = 0.0846198, size = 89, normalized size = 0.8

$$\frac{\sqrt{ax}\left(16a^4x^4+24a^3x^3+10a^2x^2+25ax-75\right)+75\sqrt{x}\sqrt{1-ax}\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-75 + 25*a*x + 10*a^2*x^2 + 24*a^3*x^3 + 16*a^4*x^4) + 75*Sqrt[
x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]])/(64*a^(7/2)*Sqrt[-(a*x*(-1 + a*x)
)])
```

Maple [C] time = 0.03, size = 132, normalized size = 1.2

$$-\frac{x\text{csgn}(a)}{128\,a^3}\sqrt{-ax+1}\left(32\,\text{csgn}(a)\,x^3a^3\sqrt{-x\,(ax-1)\,a}+80\,a^2x^2\sqrt{-x\,(ax-1)\,a}\text{csgn}(a)+100\,\sqrt{-x\,(ax-1)\,a}\text{csgn}(a)\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/128*(-a*x+1)^(1/2)*x*(32*csgn(a)*x^3*a^3*(-x*(a*x-1)*a)^(1/2)+80*a^2*x^2
*(-x*(a*x-1)*a)^(1/2)*csgn(a)+100*(-x*(a*x-1)*a)^(1/2)*csgn(a)*x*a+150*(-x*
(a*x-1)*a)^(1/2)*csgn(a)-75*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/
2)))*csgn(a)/a^3/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26349, size = 165, normalized size = 1.49

$$-\frac{\left(16\,a^3x^3+40\,a^2x^2+50\,ax+75\right)\sqrt{ax}\sqrt{-ax+1}+75\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*((16*a^3*x^3 + 40*a^2*x^2 + 50*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) + 7
5*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a^4
```

Sympy [C] time = 25.7371, size = 484, normalized size = 4.36

$$a\left(\begin{cases}-\frac{35i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{64a^5}-\frac{ix^{\frac{9}{2}}}{4\sqrt{a}\sqrt{ax-1}}-\frac{ix^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{ax-1}}-\frac{7ix^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{ax-1}}-\frac{35ix^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{ax-1}}+\frac{35i\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1\\ \frac{35\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{64a^5}+\frac{x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-ax+1}}+\frac{x^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{-ax+1}}+\frac{7x^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{-ax+1}}+\frac{35x^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{-ax+1}}-\frac{35\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{5i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4}\cdots\\ \frac{5\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4}+\cdots\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-35*I*acosh(sqrt(a)*sqrt(x))/(64*a**5) - I*x**(9/2)/(4*sqrt(a)
*sqrt(a*x - 1)) - I*x**(7/2)/(24*a**(3/2)*sqrt(a*x - 1)) - 7*I*x**(5/2)/(96
*a**(5/2)*sqrt(a*x - 1)) - 35*I*x**(3/2)/(192*a**(7/2)*sqrt(a*x - 1)) + 35*
I*sqrt(x)/(64*a**(9/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (35*asin(sqrt(a)*sqrt
(x))/(64*a**5) + x**(9/2)/(4*sqrt(a)*sqrt(-a*x + 1)) + x**(7/2)/(24*a**(3/2
)*sqrt(-a*x + 1)) + 7*x**(5/2)/(96*a**(5/2)*sqrt(-a*x + 1)) + 35*x**(3/2)/(
192*a**(7/2)*sqrt(-a*x + 1)) - 35*sqrt(x)/(64*a**(9/2)*sqrt(-a*x + 1)), Tru
e)) + Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(
a)*sqrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(
24*a**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*
```

```
x) > 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x
+ 1)) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqr
t(-a*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True))
```

Giac [A] time = 1.77946, size = 85, normalized size = 0.77

$$-\frac{\left(2\left(4\,ax\left(\frac{2\,x}{a^2}+\frac{5}{a^3}\right)+\frac{25}{a^3}\right)ax+\frac{75}{a^3}\right)\sqrt{ax}\sqrt{-ax+1}-\frac{75\,\arcsin\left(\sqrt{ax}\right)}{a^3}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*((2*(4*a*x*(2*x/a^2 + 5/a^3) + 25/a^3)*a*x + 75/a^3)*sqrt(a*x)*sqrt(-
a*x + 1) - 75*arcsin(sqrt(a*x))/a^3)/a
```

3.23 $\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{1-ax}(ax)^{5/2}}{3a^3}-\frac{11\sqrt{1-ax}(ax)^{3/2}}{12a^3}-\frac{11\sqrt{1-ax}\sqrt{ax}}{8a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}$$

```
[Out] (-11*Sqrt[a*x]*Sqrt[1 - a*x])/(8*a^3) - (11*(a*x)^(3/2)*Sqrt[1 - a*x])/(12*
a^3) - ((a*x)^(5/2)*Sqrt[1 - a*x])/(3*a^3) - (11*ArcSin[1 - 2*a*x])/(16*a^3
)
```

Rubi [A] time = 0.0335764, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}(ax)^{5/2}}{3a^3}-\frac{11\sqrt{1-ax}(ax)^{3/2}}{12a^3}-\frac{11\sqrt{1-ax}\sqrt{ax}}{8a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-11*Sqrt[a*x]*Sqrt[1 - a*x])/(8*a^3) - (11*(a*x)^(3/2)*Sqrt[1 - a*x])/(12*
a^3) - ((a*x)^(5/2)*Sqrt[1 - a*x])/(3*a^3) - (11*ArcSin[1 - 2*a*x])/(16*a^3
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx &= \frac{\int \frac{(ax)^{3/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^2}\\
&= -\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{6a^2}\\
&= -\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{8a^2}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{16a^2}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{16a^2}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{16a^4}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0311233, size = 81, normalized size = 0.93

$$
\frac{\sqrt{ax}\left(8a^3x^3+14a^2x^2+11ax-33\right)+33\sqrt{x}\sqrt{1-ax}\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{24a^{5/2}\sqrt{-ax(ax-1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-33 + 11*a*x + 14*a^2*x^2 + 8*a^3*x^3) + 33*Sqrt[x]*Sqrt[1 - a*
x]*ArcSin[Sqrt[a]*Sqrt[x]])/(24*a^(5/2)*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] time = 0.013, size = 111, normalized size = 1.3

$$
\frac{x\text{csgn}\,(a)}{48\,a^2}\sqrt{-ax+1}\Big(-16\,a^2x^2\sqrt{-x\,(ax-1)}\,a\text{csgn}\,(a)-44\,\sqrt{-x\,(ax-1)}\,a\text{csgn}\,(a)\,xa-66\,\sqrt{-x\,(ax-1)}\,a\text{csgn}\,(a)+33
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] 1/48*(-a*x+1)^(1/2)*x*(-16*a^2*x^2*(-x*(a*x-1)*a)^(1/2)*csgn(a)-44*(-x*(a*x
-1)*a)^(1/2)*csgn(a)*x*a-66*(-x*(a*x-1)*a)^(1/2)*csgn(a)+33*arctan(1/2*csgn
(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/a^2/(a*x)^(1/2)/(-x*(a*x-1)*a)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37876, size = 146, normalized size = 1.68

$$-\frac{\left(8\,a^{2}x^{2}+22\,ax+33\right)\sqrt{ax}\sqrt{-ax+1}+33\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{24\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*((8*a^2*x^2 + 22*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) + 33*arctan(sqrt(
a*x)*sqrt(-a*x + 1)/(a*x)))/a^3
```

Sympy [C] time = 18.821, size = 393, normalized size = 4.52

$$a\left(\begin{cases}-\frac{5i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^{4}}-\frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}}-\frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}}-\frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}}+\frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{5\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^{4}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}}+\frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}}+\frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}}-\frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{3i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3}}-\frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}}-\frac{}{4a^{\frac{3}{2}}} \\ \frac{3\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}}+\frac{x}{4a^{\frac{3}{2}}\sqrt{}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(a)*s
qrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(24*a
**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*x) >
 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x + 1)
) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqrt(-a
*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True)) + Piecewise((-3*I*
acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*sqrt(a*x - 1)) - I*
x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(5/2)*sqrt(a*x - 1)
), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**(5/2)/(2*sqrt(a)*s
qrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3*sqrt(x)/(4*a**(5/
2)*sqrt(-a*x + 1)), True))
```

Giac [A] time = 2.58971, size = 72, normalized size = 0.83

$$-\frac{\left(2\,ax\left(\frac{4\,x}{a}+\frac{11}{a^2}\right)+\frac{33}{a^2}\right)\sqrt{ax}\sqrt{-ax+1}-\frac{33\,\arcsin\left(\sqrt{ax}\right)}{a^2}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*a*x*(4*x/a + 11/a^2) + 33/a^2)*sqrt(a*x)*sqrt(-a*x + 1) - 33*arcs
in(sqrt(a*x))/a^2)/a
```

3.24 $\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{1-ax}(ax)^{3/2}}{2a^2}-\frac{7\sqrt{1-ax}\sqrt{ax}}{4a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}$$

```
[Out] (-7*Sqrt[a*x]*Sqrt[1 - a*x])/(4*a^2) - ((a*x)^(3/2)*Sqrt[1 - a*x])/(2*a^2)
- (7*ArcSin[1 - 2*a*x])/(8*a^2)
```

Rubi [A] time = 0.0232313, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}(ax)^{3/2}}{2a^2}-\frac{7\sqrt{1-ax}\sqrt{ax}}{4a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-7*Sqrt[a*x]*Sqrt[1 - a*x])/(4*a^2) - ((a*x)^(3/2)*Sqrt[1 - a*x])/(2*a^2)
- (7*ArcSin[1 - 2*a*x])/(8*a^2)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx &= \frac{\int \frac{\sqrt{ax}(1+ax)}{\sqrt{1-ax}}\,dx}{a}\\
&= -\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}+\frac{7\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{4a}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}+\frac{7\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{8a}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}+\frac{7\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{8a}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}-\frac{7\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{8a^3}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0267346, size = 73, normalized size = 1.16

$$\frac{\sqrt{ax}\left(2a^2x^2+5ax-7\right)+7\sqrt{x}\sqrt{1-ax}\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-7 + 5*a*x + 2*a^2*x^2) + 7*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a
]*Sqrt[x]])/(4*a^(3/2)*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] time = 0.011, size = 90, normalized size = 1.4

$$-\frac{x\mathrm{csgn}\left(a\right)}{8\,a}\sqrt{-ax+1}\left(4\,\sqrt{-x\left(ax-1\right)a}\mathrm{csgn}\left(a\right)xa+14\,\sqrt{-x\left(ax-1\right)a}\mathrm{csgn}\left(a\right)-7\,\arctan\left(1/2\,\frac{\mathrm{csgn}\left(a\right)\left(2\,ax-1\right)}{\sqrt{-x\left(ax-1\right)a}}\right)\right)\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/8*(-a*x+1)^(1/2)*x/a*(4*(-x*(a*x-1)*a)^(1/2)*csgn(a)*x*a+14*(-x*(a*x-1)*
a)^(1/2)*csgn(a)-7*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn
(a)/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31869, size = 124, normalized size = 1.97

$$-\frac{(2\,ax+7)\sqrt{ax}\sqrt{-ax+1}+7\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) + 7*arctan(sqrt(a*x)*sqrt(-a*x +
 1)/(a*x)))/a^2
```

Sympy [C] time = 14.3845, size = 269, normalized size = 4.27

$$a\left(\begin{cases}-\frac{3i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3}-\frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}}-\frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}}+\frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{3\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}}+\frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}}-\frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2}-\frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax|>\\ \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-3*I*acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*s
qrt(a*x - 1)) - I*x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(
5/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**
(5/2)/(2*sqrt(a)*sqrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3
*sqrt(x)/(4*a**(5/2)*sqrt(-a*x + 1)), True)) + Piecewise((-I*acosh(sqrt(a)*
sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3/2), Abs(a*x) > 1), (asin(sqrt
(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a*x + 1)) - sqrt(x)/(a**(3/2)*s
qrt(-a*x + 1)), True))
```

Giac [A] time = 2.49052, size = 54, normalized size = 0.86

$$-\frac{\sqrt{ax}\sqrt{-ax+1}\left(2\,x+\frac{7}{a}\right)-\frac{7\,\arcsin\left(\sqrt{ax}\right)}{a}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")

[Out] -1/4*(sqrt(a*x)*sqrt(-a*x + 1)*(2*x + 7/a) - 7*arcsin(sqrt(a*x))/a)/a
```

3.25 $\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}$$

```
[Out] -((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (3*ArcSin[1 - 2*a*x])/(2*a)
```

Rubi [A] time = 0.0125165, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {80, 53, 619, 216}

$$-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]

[Out] -((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (3*ArcSin[1 - 2*a*x])/(2*a)
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx &= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{2a^2} \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0242423, size = 61, normalized size = 1.65

$$\frac{\sqrt{a}x(ax-1)+3\sqrt{x}\sqrt{1-ax}\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-1 + a*x) + 3*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]])/(S
qrt[a]*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] time = 0.015, size = 70, normalized size = 1.9

$$-\frac{x\text{csgn}\,(a)}{2}\sqrt{-ax+1}\left(2\sqrt{-x\,(ax-1)\,a}\text{csgn}\,(a)-3\,\arctan\left(1/2\,\frac{\text{csgn}\,(a)\,(2\,ax-1)}{\sqrt{-x\,(ax-1)\,a}}\right)\right)\frac{1}{\sqrt{ax}}\frac{1}{\sqrt{-x\,(ax-1)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(-a*x+1)^(1/2)*x*(2*(-x*(a*x-1)*a)^(1/2)*csgn(a)-3*arctan(1/2*csgn(a)*
(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30267, size = 100, normalized size = 2.7

$$-\frac{\sqrt{ax}\sqrt{-ax+1}+3\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) + 3*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a
```

Sympy [C] time = 7.76718, size = 133, normalized size = 3.59

$$a\left(\begin{cases} -\frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2} - \frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax| > 1 \\ \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2} + \frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}} - \frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{2i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for } |ax| > 1 \\ \frac{2\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-I*acosh(sqrt(a)*sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3
/2), Abs(a*x) > 1), (asin(sqrt(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a
*x + 1)) - sqrt(x)/(a**(3/2)*sqrt(-a*x + 1)), True)) + Piecewise((-2*I*acos
h(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*sqrt(x))/a, True))
```

Giac [A] time = 2.54551, size = 38, normalized size = 1.03

$$-\frac{\sqrt{ax}\sqrt{-ax+1} - 3\arcsin\left(\sqrt{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) - 3*arcsin(sqrt(a*x)))/a
```

3.26 $\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)$$

```
[Out] (-2*Sqrt[1 - a*x])/Sqrt[a*x] - ArcSin[1 - 2*a*x]
```

Rubi [A] time = 0.0160123, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {16, 78, 53, 619, 216}

$$-\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x])/Sqrt[a*x] - ArcSin[1 - 2*a*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx &= a\int \frac{1+ax}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{a} \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \sin^{-1}(1-2ax)
\end{aligned}
$$

Mathematica [A] time = 0.0239233, size = 53, normalized size = 1.83

$$\frac{2\left(ax+\sqrt{a}\sqrt{x}\sqrt{1-ax}\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)-1\right)}{\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (2*(-1 + a*x + Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]]))/Sqrt
[-(a*x*(-1 + a*x))]
```

Maple [C] time = 0.016, size = 69, normalized size = 2.4

$$\text{csgn}(a)\left(\arctan\left(\frac{\text{csgn}(a)(2ax-1)}{2}\frac{1}{\sqrt{-x(ax-1)a}}\right)xa-2\sqrt{-x(ax-1)a}\,\text{csgn}(a)\right)\sqrt{-ax+1}\frac{1}{\sqrt{ax}}\frac{1}{\sqrt{-x(ax-1)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] (arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2))*x*a-2*(-x*(a*x-1)*a)^(1
/2)*csgn(a))*(-a*x+1)^(1/2)*csgn(a)/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.31399, size = 111, normalized size = 3.83

$$-\frac{2\left(ax\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)+\sqrt{ax}\sqrt{-ax+1}\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(a*x*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)) + sqrt(a*x)*sqrt(-a*x + 1))/
(a*x)
```

Sympy [C] time = 10.317, size = 71, normalized size = 2.45

$$a\left(\begin{cases}-\frac{2i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for } |ax|>1\\ \frac{2\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}\right)+\begin{cases}-2\sqrt{-1+\frac{1}{ax}} & \text{for } \frac{1}{|ax|}>1\\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*I*acosh(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*
sqrt(x))/a, True)) + Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2
*I*sqrt(1 - 1/(a*x)), True))
```

Giac [A] time = 2.97911, size = 59, normalized size = 2.03

$$-\frac{\sqrt{-ax+1}-1}{\sqrt{ax}}+\frac{\sqrt{ax}}{\sqrt{-ax+1}-1}+2\arcsin\left(\sqrt{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-a*x + 1) - 1)/sqrt(a*x) + sqrt(a*x)/(sqrt(-a*x + 1) - 1) + 2*arcsin
(sqrt(a*x))
```

3.27 $\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=45

$$-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

```
[Out] (-2*a*Sqrt[1 - a*x])/(3*(a*x)^(3/2)) - (10*a*Sqrt[1 - a*x])/(3*Sqrt[a*x])
```

Rubi [A] time = 0.0098901, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {16, 78, 37}

$$-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a*Sqrt[1 - a*x])/(3*(a*x)^(3/2)) - (10*a*Sqrt[1 - a*x])/(3*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx &= a^2\int \frac{1+ax}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}+\frac{1}{3}\left(5a^2\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}\end{aligned}$$

Mathematica [A] time = 0.012325, size = 29, normalized size = 0.64

$$-\frac{2\sqrt{-ax(ax-1)}(5ax+1)}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 5*a*x))/(3*a*x^2)
```

Maple [A] time = 0.003, size = 25, normalized size = 0.6

$$-\frac{10\,ax+2}{3\,x}\sqrt{-ax+1}\frac{1}{\sqrt{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/3*(5*a*x+1)/x/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45223, size = 69, normalized size = 1.53

$$-\frac{2\,(5\,ax+1)\sqrt{ax}\sqrt{-ax+1}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^2)
```

Sympy [C] time = 8.86366, size = 107, normalized size = 2.38

$$a\left(\begin{cases} -2\sqrt{-1+\frac{1}{ax}} & \text{for}\ \frac{1}{|ax|}>1 \\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise} \end{cases}\right)+\begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|}>1 \\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**2/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2*I*sqrt(1 - 1/(a*x)
), True)) + Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*
x), 1/Abs(a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3
*x), True))
```

Giac [B] time = 1.57237, size = 119, normalized size = 2.64

$$-\frac{\frac{a^2\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{21\,a^2\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(a^2+\frac{21\,a\left(\sqrt{-ax+1}-1\right)^2}{x}\right)(ax)^{\frac{3}{2}}}{\left(\sqrt{-ax+1}-1\right)^3}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(a^2*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 21*a^2*(sqrt(-a*x + 1) - 1)
/sqrt(a*x) - (a^2 + 21*a*(sqrt(-a*x + 1) - 1)^2/x)*(a*x)^(3/2)/(sqrt(-a*x +
 1) - 1)^3)/a
```

3.28 $\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=73

$$-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

```
[Out] (-2*a^2*Sqrt[1 - a*x])/(5*(a*x)^(5/2)) - (6*a^2*Sqrt[1 - a*x])/(5*(a*x)^(3/
2)) - (12*a^2*Sqrt[1 - a*x])/(5*Sqrt[a*x])
```

Rubi [A] time = 0.0190258, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 78, 45, 37}

$$-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^2*Sqrt[1 - a*x])/(5*(a*x)^(5/2)) - (6*a^2*Sqrt[1 - a*x])/(5*(a*x)^(3/
2)) - (12*a^2*Sqrt[1 - a*x])/(5*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx &= a^3\int \frac{1+ax}{(ax)^{7/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}+\frac{1}{5}\left(9a^3\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}+\frac{1}{5}\left(6a^3\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0146271, size = 37, normalized size = 0.51

$$
-\frac{2\sqrt{-ax(ax-1)}\left(6a^2x^2+3ax+1\right)}{5ax^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 3*a*x + 6*a^2*x^2))/(5*a*x^3)
```

Maple [A] time = 0.004, size = 33, normalized size = 0.5

$$
-\frac{12\,a^2x^2+6\,ax+2}{5\,x^2}\sqrt{-ax+1}\frac{1}{\sqrt{ax}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/5*(6*a^2*x^2+3*a*x+1)/x^2/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49853, size = 85, normalized size = 1.16

$$
-\frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{ax}\sqrt{-ax+1}}{5\,ax^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^3)
```

Sympy [C] time = 12.7437, size = 189, normalized size = 2.59

$$a\left(\begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3} - \frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|} > 1 \\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3} - \frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15} - \frac{8a\sqrt{-1+\frac{1}{ax}}}{15x} - \frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|} > 1 \\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15} - \frac{8ia\sqrt{1-\frac{1}{ax}}}{15x} - \frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**3/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*x), 1/Abs(
a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3*x), True)
) + Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*
x) - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1
/(a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2
), True))
```

Giac [B] time = 1.8686, size = 176, normalized size = 2.41

$$-\frac{\frac{a^3\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}} + \frac{15\,a^3\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}} + \frac{110\,a^3\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} - \frac{\left(a^3 + \frac{15\,a^2\left(\sqrt{-ax+1}-1\right)^2}{x} + \frac{110\,a\left(\sqrt{-ax+1}-1\right)^4}{x^2}\right)(ax)^{\frac{5}{2}}}{\left(\sqrt{-ax+1}-1\right)^5}}{80\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80*(a^3*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 15*a^3*(sqrt(-a*x + 1) - 1)
^3/(a*x)^(3/2) + 110*a^3*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (a^3 + 15*a^2*(sq
rt(-a*x + 1) - 1)^2/x + 110*a*(sqrt(-a*x + 1) - 1)^4/x^2)*(a*x)^(5/2)/(sqrt
(-a*x + 1) - 1)^5)/a
```

3.29 $\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=97

$$-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

```
[Out] (-2*a^3*Sqrt[1 - a*x])/(7*(a*x)^(7/2)) - (26*a^3*Sqrt[1 - a*x])/(35*(a*x)^(
5/2)) - (104*a^3*Sqrt[1 - a*x])/(105*(a*x)^(3/2)) - (208*a^3*Sqrt[1 - a*x])
/(105*Sqrt[a*x])
```

Rubi [A] time = 0.0266511, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 78, 45, 37}

$$-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^3*Sqrt[1 - a*x])/(7*(a*x)^(7/2)) - (26*a^3*Sqrt[1 - a*x])/(35*(a*x)^(
5/2)) - (104*a^3*Sqrt[1 - a*x])/(105*(a*x)^(3/2)) - (208*a^3*Sqrt[1 - a*x])
/(105*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rubi steps

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = a^4\int \frac{1+ax}{(ax)^{9/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} + \frac{1}{7}\left(13a^4\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} + \frac{1}{35}\left(52a^4\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} - \frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}} + \frac{1}{105}\left(104a^4\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} - \frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}} - \frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}$$

Mathematica [A] time = 0.0166348, size = 45, normalized size = 0.46

$$-\frac{2\sqrt{-ax(ax-1)}\left(104a^3x^3+52a^2x^2+39ax+15\right)}{105ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*
a*x^4)
```

Maple [A] time = 0.003, size = 41, normalized size = 0.4

$$-\frac{208\,a^3x^3+104\,a^2x^2+78\,ax+30}{105\,x^3}\sqrt{-ax+1}\frac{1}{\sqrt{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/105*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44396, size = 111, normalized size = 1.14

$$-\frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{ax}\sqrt{-ax+1}}{105\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt(a*x)*sqrt(-a*x + 1)/(a
*x^4)
```

Sympy [C] time = 18.8721, size = 274, normalized size = 2.82

$$a\left(\begin{cases} -\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|}>1 \\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise} \end{cases}\right)+\begin{cases} -\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for}\ \frac{1}{|ax|}>1 \\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**4/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*x)
 - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1/(
a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2),
 True)) + Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(
a*x))/(35*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*
x**3), 1/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1
 - 1/(a*x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a
*x))/(7*x**3), True))
```

Giac [B] time = 2.89268, size = 236, normalized size = 2.43

$$-\frac{\frac{15\,a^4\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{231\,a^4\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{1435\,a^4\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{7875\,a^4\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(15\,a^4+\frac{231\,a^3\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{1435\,a^2\left(\sqrt{-ax+1}-1\right)^4}{x^2}+\frac{7875\,a\left(\sqrt{-ax\ldots}\right.}{x^3}\ldots}{\left(\sqrt{-ax+1}-1\right)^7}}{6720\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6720*(15*a^4*(sqrt(-a*x + 1) - 1)^7/(a*x)^(7/2) + 231*a^4*(sqrt(-a*x + 1
) - 1)^5/(a*x)^(5/2) + 1435*a^4*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 7875*a
^4*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (15*a^4 + 231*a^3*(sqrt(-a*x + 1) - 1)^
2/x + 1435*a^2*(sqrt(-a*x + 1) - 1)^4/x^2 + 7875*a*(sqrt(-a*x + 1) - 1)^6/x
^3)*(a*x)^(7/2)/(sqrt(-a*x + 1) - 1)^7)/a
```

3.30 $\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=121

$$-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

```
[Out] (-2*a^4*Sqrt[1 - a*x])/(9*(a*x)^(9/2)) - (34*a^4*Sqrt[1 - a*x])/(63*(a*x)^(
7/2)) - (68*a^4*Sqrt[1 - a*x])/(105*(a*x)^(5/2)) - (272*a^4*Sqrt[1 - a*x])/
(315*(a*x)^(3/2)) - (544*a^4*Sqrt[1 - a*x])/(315*Sqrt[a*x])
```

Rubi [A] time = 0.0384211, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 78, 45, 37}

$$-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^4*Sqrt[1 - a*x])/(9*(a*x)^(9/2)) - (34*a^4*Sqrt[1 - a*x])/(63*(a*x)^(
7/2)) - (68*a^4*Sqrt[1 - a*x])/(105*(a*x)^(5/2)) - (272*a^4*Sqrt[1 - a*x])/
(315*(a*x)^(3/2)) - (544*a^4*Sqrt[1 - a*x])/(315*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

1]

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx &= a^5\int \frac{1+ax}{(ax)^{11/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}+\frac{1}{9}\left(17a^5\right)\int \frac{1}{(ax)^{9/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}+\frac{1}{21}\left(34a^5\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}+\frac{1}{105}\left(136a^5\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}+\frac{1}{315}\left(272a^5\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0185621, size = 53, normalized size = 0.44

$$
-\frac{2\sqrt{-ax(ax-1)}\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)}{315ax^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*
a^4*x^4))/(315*a*x^5)
```

Maple [A] time = 0.005, size = 49, normalized size = 0.4

$$
-\frac{544\,a^4x^4+272\,a^3x^3+204\,a^2x^2+170\,ax+70}{315\,x^4}\sqrt{-ax+1}\frac{1}{\sqrt{ax}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)/x^4/(a*x)^(1/2)*(-a*
x+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40138, size = 131, normalized size = 1.08

$$-\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{ax}\sqrt{-ax+1}}{315\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt(a*x)*sq
rt(-a*x + 1)/(a*x^5)
```

Sympy [C] time = 29.9426, size = 359, normalized size = 2.97

$$a\left(\begin{cases}-\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{256a^4\sqrt{-1+\frac{1}{ax}}}{315}-\frac{128a^3\sqrt{-1+\frac{1}{ax}}}{315x}-\frac{32a^2\sqrt{-1+\frac{1}{ax}}}{105x^2}-\ldots\\ -\frac{256ia^4\sqrt{1-\frac{1}{ax}}}{315}-\frac{128ia^3\sqrt{1-\frac{1}{ax}}}{315x}-\frac{32ia^2\sqrt{1-\frac{1}{ax}}}{105x^2}-\ldots\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**5/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(a*x))/(3
5*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*x**3), 1
/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1 - 1/(a*
x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a*x))/(7*
x**3), True)) + Piecewise((-256*a**4*sqrt(-1 + 1/(a*x))/315 - 128*a**3*sqrt
(-1 + 1/(a*x))/(315*x) - 32*a**2*sqrt(-1 + 1/(a*x))/(105*x**2) - 16*a*sqrt(
-1 + 1/(a*x))/(63*x**3) - 2*sqrt(-1 + 1/(a*x))/(9*x**4), 1/Abs(a*x) > 1), (
-256*I*a**4*sqrt(1 - 1/(a*x))/315 - 128*I*a**3*sqrt(1 - 1/(a*x))/(315*x) -
32*I*a**2*sqrt(1 - 1/(a*x))/(105*x**2) - 16*I*a*sqrt(1 - 1/(a*x))/(63*x**3)
 - 2*I*sqrt(1 - 1/(a*x))/(9*x**4), True))
```

Giac [B] time = 2.92981, size = 293, normalized size = 2.42

$$-\frac{\frac{35\,a^5\left(\sqrt{-ax+1}-1\right)^9}{(ax)^{\frac{9}{2}}}+\frac{585\,a^5\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{4032\,a^5\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{17640\,a^5\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{83790\,a^5\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(35\,a^5+\frac{585\,a^4\left(\sqrt{-ax+1}-1\right)^2}{x}\ldots\right.}{\ldots}}{80640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80640*(35*a^5*(sqrt(-a*x + 1) - 1)^9/(a*x)^(9/2) + 585*a^5*(sqrt(-a*x +
1) - 1)^7/(a*x)^(7/2) + 4032*a^5*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 17640
*a^5*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 83790*a^5*(sqrt(-a*x + 1) - 1)/sq
rt(a*x) - (35*a^5 + 585*a^4*(sqrt(-a*x + 1) - 1)^2/x + 4032*a^3*(sqrt(-a*x
+ 1) - 1)^4/x^2 + 17640*a^2*(sqrt(-a*x + 1) - 1)^6/x^3 + 83790*a*(sqrt(-a*x
 + 1) - 1)^8/x^4)*(a*x)^(9/2)/(sqrt(-a*x + 1) - 1)^9)/a
```

3.31 $\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rubi [A] time = 0.0074949, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {151, 12, 92, 203}

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\int \frac{2a}{\sqrt{-1+x}x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+(2a)\int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\sqrt{1+x}\right)\\ &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0147942, size = 48, normalized size = 1.23

$$\frac{2a\sqrt{x^2-1}x\tan^{-1}\left(\sqrt{x^2-1}\right)-x^2+1}{\sqrt{x-1}x\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] (1 - x^2 + 2*a*x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])/(Sqrt[-1 + x]*x*Sqr
t[1 + x])
```

Maple [A] time = 0.015, size = 44, normalized size = 1.1

$$\frac{1}{x}\left(-2\,ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{-1+x}\sqrt{1+x}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/(
x^2-1)^(1/2)
```

Maxima [A] time = 3.18987, size = 28, normalized size = 0.72

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Fricas [A] time = 1.3243, size = 104, normalized size = 2.67

$$\frac{4\,ax\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.86463, size = 58, normalized size = 1.49

$$-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^{2}\right)-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^{4}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
1))^4 + 4)
```

3.32 $\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rubi [A] time = 0.0114988, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {188, 151, 12, 92, 203}

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 188

```
Int[(u_)^(m_.)*(v_)^(n_.)*(w_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[ExpandToS
um[u, x]^m*ExpandToSum[v, x]^n*ExpandToSum[w, x]^p*ExpandToSum[z, x]^q, x]
/; FreeQ[{m, n, p, q}, x] && LinearQ[{u, v, w, z}, x] &&  !LinearMatchQ[{u,
 v, w, z}, x]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx &= \int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\int \frac{2a}{\sqrt{-1+x}x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+(2a)\int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+(2a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\sqrt{1+x}\right)\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0058114, size = 48, normalized size = 1.23

$$\frac{2a\sqrt{x^2-1}x\tan^{-1}\left(\sqrt{x^2-1}\right)-x^2+1}{\sqrt{x-1}x\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] (1 - x^2 + 2*a*x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])/(Sqrt[-1 + x]*x*Sqr
t[1 + x])
```

Maple [A] time = 0.004, size = 44, normalized size = 1.1

$$\frac{1}{x}\left(-2\,ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{-1+x}\sqrt{1+x}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/(
x^2-1)^(1/2)
```

Maxima [A] time = 1.76616, size = 28, normalized size = 0.72

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Fricas [A] time = 1.32775, size = 104, normalized size = 2.67

$$\frac{4\,ax\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="f
ricas")
```

```
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2ax-1}{x^2\sqrt{x-1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-(-a*x+1)**2)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral((2*a*x - 1)/(x**2*sqrt(x - 1)*sqrt(x + 1)), x)
```

Giac [A] time = 2.33417, size = 58, normalized size = 1.49

$$-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

3.33 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{a}(aBe+A(b-be))\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right),\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}-\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}$$

```
[Out] (-2*a^(3/2)*B*EllipticE[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e
)/(1 - c)])/(b^2*Sqrt[1 - c]*(1 - e)) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*El
lipticF[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e)/(1 - c)])/(b^2
*Sqrt[1 - c]*(1 - e))
```

Rubi [A] time = 0.108071, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {158, 113, 119}

$$\frac{2\sqrt{a}(aBe+A(b-be))F\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}-\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(-1 + e
)*x)/a]),x]
```

```
[Out] (-2*a^(3/2)*B*EllipticE[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e
)/(1 - c)])/(b^2*Sqrt[1 - c]*(1 - e)) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*El
lipticF[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e)/(1 - c)])/(b^2
*Sqrt[1 - c]*(1 - e))
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
```

```
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}}\,dx}{b(1-e)} + \left(A+\frac{aBe}{b-be}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e-$$

$$= -\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)} + \frac{2\sqrt{a}\left(A+\frac{aBe}{b-be}\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right.\right.}{b\sqrt{1-c}}$$

Mathematica [C] time = 1.44704, size = 309, normalized size = 2.13

$$-\frac{2\sqrt{\frac{a}{c-1}}(a+bx)^{3/2}\left(\frac{i(e-1)\sqrt{\frac{\frac{a}{a+bx}+c-1}{c-1}}\sqrt{\frac{\frac{a}{a+bx}+e-1}{e-1}}(aBc+A(b-bc))\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{c-1}}}{\sqrt{a+bx}}\right),\frac{c-1}{e-1}\right)}{\sqrt{a+bx}} - B\sqrt{\frac{a}{c-1}}\left(\frac{a}{a+bx}+c-1\right)\left(\frac{a}{a+bx}+e-1\right)\right.}{ab^2(e-1)\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(
-1 + e)*x)/a]),x]
```

```
[Out] (-2*Sqrt[a/(-1 + c)]*(a + b*x)^(3/2)*(-(B*Sqrt[a/(-1 + c)]*(-1 + c + a/(a +
 b*x))*(-1 + e + a/(a + b*x))) - (I*a*B*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x)
)/(-1 + c)]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[
a/(-1 + c)]/Sqrt[a + b*x]], (-1 + c)/(-1 + e)])/Sqrt[a + b*x] + (I*(a*B*c +
 A*(b - b*c))*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x))/(-1 + c)]*Sqrt[(-1 + e +
 a/(a + b*x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + c)]/Sqrt[a + b*x]]
, (-1 + c)/(-1 + e)])/Sqrt[a + b*x]))/(a*b^2*(-1 + e)*Sqrt[c + (b*(-1 + c)*
x)/a]*Sqrt[e + (b*(-1 + e)*x)/a])
```

Maple [B] time = 0.047, size = 624, normalized size = 4.3

$$-2\frac{a}{\sqrt{bx+a}(-1+e)(c-1)^2b^2}\left(A\text{EllipticF}\left(\sqrt{-\frac{(-1+e)(bcx+ac-bx)}{a(c-e)}},\sqrt{-\frac{c-e}{-1+e}}\right)bc^2 - A\text{EllipticF}\left(\sqrt{-\frac{(-1+e}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(c+b*(c-1)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x)
```

```
[Out] -2*(A*EllipticF((-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^(1/2),(-(c-e)/(-1+e))^(1/
2))*b*c^2-A*EllipticF((-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^(1/2),(-(c-e)/(-1+e
))^(1/2))*b*c*e-B*EllipticF((-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^(1/2),(-(c-e)
/(-1+e))^(1/2))*a*c^2+B*EllipticF((-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^(1/2),(
-(c-e)/(-1+e))^(1/2))*a*c*e-A*EllipticF((-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^(
1/2),(-(c-e)/(-1+e))^(1/2))*b*c+A*EllipticF((-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e
```

```
))^(1/2),(-(c-e)/(-1+e))^(1/2))*b*e+B*EllipticF((-(-1+e)*(b*c*x+a*c-b*x)/a/
(c-e))^(1/2),(-(c-e)/(-1+e))^(1/2))*a*c-B*EllipticF((-(-1+e)*(b*c*x+a*c-b*x
)/a/(c-e))^(1/2),(-(c-e)/(-1+e))^(1/2))*a*e-B*EllipticE((-(-1+e)*(b*c*x+a*c
-b*x)/a/(c-e))^(1/2),(-(c-e)/(-1+e))^(1/2))*a*c+B*EllipticE((-(-1+e)*(b*c*x
+a*c-b*x)/a/(c-e))^(1/2),(-(c-e)/(-1+e))^(1/2))*a*e)*((c-1)*(b*e*x+a*e-b*x)
/a/(c-e))^(1/2)*(-(b*x+a)*(c-1)/a)^(1/2)*(-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^
(1/2)*a/(b*x+a)^(1/2)/((b*c*x+a*c-b*x)/a)^(1/2)/((b*e*x+a*e-b*x)/a)^(1/2)/(
-1+e)/(c-1)^2/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*(e - 1)*x
/a + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Ba^2x+Aa^2\right)\sqrt{bx+a}\sqrt{\frac{ac+(bc-b)x}{a}}\sqrt{\frac{ae+(be-b)x}{a}}}{a^3ce-\left(b^3c-b^3-\left(b^3c-b^3\right)e\right)x^3-\left(2\,ab^2c-ab^2-\left(3\,ab^2c-2\,ab^2\right)e\right)x^2-\left(a^2bc-\left(3\,a^2bc-a^2b\right)e\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((B*a^2*x + A*a^2)*sqrt(b*x + a)*sqrt((a*c + (b*c - b)*x)/a)*sqrt((
a*e + (b*e - b)*x)/a)/(a^3*c*e - (b^3*c - b^3 - (b^3*c - b^3)*e)*x^3 - (2*a
*b^2*c - a*b^2 - (3*a*b^2*c - 2*a*b^2)*e)*x^2 - (a^2*b*c - (3*a^2*b*c - a^2
*b)*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(c+b*(-1+c)*x/a)**(1/2)/(e+b*(-1+e)*x/a)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx + A}{\sqrt{bx + a}\sqrt{\frac{b(c-1)x}{a} + c}\sqrt{\frac{b(e-1)x}{a} + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*(e - 1)*x
/a + e)), x)
```

3.34 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal. Leaf size=221

$$\frac{2\sqrt{a}(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right),-\frac{ad}{(1-e)(bc-ad)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}-\frac{2aB\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|-\right.}{b^2\sqrt{d}(1-e)\sqrt{c+dx}}$$

```
[Out] (-2*a*B*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticE[ArcSin
[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], -(((b*c - a*d)*(1 - e))/(a*d)
)])/(b^2*Sqrt[d]*(1 - e)*Sqrt[c + d*x]) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticF[ArcSin[(Sqrt[1 - e]*Sqrt[a + b*x]
)/Sqrt[a]], -((a*d)/((b*c - a*d)*(1 - e)))])/(b^2*(1 - e)^(3/2)*Sqrt[c + d*
x])
```

Rubi [A] time = 0.199431, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {158, 114, 113, 121, 119}

$$\frac{2\sqrt{a}(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right)|-\frac{ad}{(bc-ad)(1-e)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}-\frac{2aB\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|-\frac{(bc-ad)(1}{ad}\right.}{b^2\sqrt{d}(1-e)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]),x]
```



```
[Out] (-2*a*B*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticE[ArcSin
[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], -(((b*c - a*d)*(1 - e))/(a*d)
)])/(b^2*Sqrt[d]*(1 - e)*Sqrt[c + d*x]) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticF[ArcSin[(Sqrt[1 - e]*Sqrt[a + b*x]
)/Sqrt[a]], -((a*d)/((b*c - a*d)*(1 - e)))])/(b^2*(1 - e)^(3/2)*Sqrt[c + d*
x])
```


Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
```

```
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b(1-e)} + \left(A+\frac{aBe}{b-be}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$$

$$= \frac{\left(\left(A+\frac{aBe}{b-be}\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx}{\sqrt{c+dx}} - \frac{\left(aB\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+\frac{b(-\ldots}{}}\right.}{b(1-e)\ldots}$$

$$= -\frac{2aB\sqrt{-bc+ad}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}(1-e)\sqrt{c+dx}} + \frac{2\sqrt{a}(aBe+A(b\ldots}{\ldots}$$

Mathematica [C] time = 2.17453, size = 312, normalized size = 1.41

$$-\frac{2\sqrt{\frac{a}{e-1}}(a+bx)^{3/2}\left(\frac{id\sqrt{\frac{\frac{a}{a+bx}+e-1}{e-1}}(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{d(a+bx)}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{e-1}}}{\sqrt{a+bx}}\right),\frac{(e-1)(bc-ad)}{ad}\right)}{\sqrt{a+bx}} - \frac{bB\sqrt{\frac{a}{e-1}}(c+dx)(ae+b(e-1)x)}{(a+bx)^2} - \frac{iaBd\sqrt{\frac{\frac{a}{a+bx}+e\ldots}{e-1}}\ldots}{\ldots}\right.}{ab^2d\sqrt{c+dx}\sqrt{\frac{b(e-1)x}{a}+e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]
),x]
```

```
[Out] (-2*Sqrt[a/(-1 + e)]*(a + b*x)^(3/2)*(-((b*B*Sqrt[a/(-1 + e)]*(c + d*x)*(a*
e + b*(-1 + e)*x))/(a + b*x)^2) - (I*a*B*d*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[a/(-1 + e)
```

```
]/Sqrt[a + b*x]], ((b*c - a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x] + (I*d*(a*B*
e + A*(b - b*e))*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(-1 + e + a/(a + b*
x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + e)]/Sqrt[a + b*x]], ((b*c -
a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x]))/(a*b^2*d*Sqrt[c + d*x]*Sqrt[e + (b*(
-1 + e)*x)/a])
```

Maple [B] time = 0.039, size = 940, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2)*(
-(b*x+a)*(-1+e)/a)^(1/2)*(-(d*x+c)*b*(-1+e)/(a*d*e-b*c*e+b*c))^(1/2)*(A*Ell
ipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^
(1/2))*a*b*d*e^2-A*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((
a*d*e-b*c*e+b*c)/d/a)^(1/2))*b^2*c*e^2-B*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*
e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a^2*d*e^2+B*EllipticF((d
*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a*
b*c*e^2-A*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c
*e+b*c)/d/a)^(1/2))*a*b*d*e+2*A*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b
*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*b^2*c*e+B*EllipticF((d*(b*e*x+a*e
-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a^2*d*e-2*B*E
llipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a
)^(1/2))*a*b*c*e-B*EllipticE((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((
a*d*e-b*c*e+b*c)/d/a)^(1/2))*a^2*d*e+B*EllipticE((d*(b*e*x+a*e-b*x)/(a*d*e-
b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a*b*c*e-A*EllipticF((d*(b*
e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*b^2*c+
B*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/
d/a)^(1/2))*a*b*c-B*EllipticE((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),(
(a*d*e-b*c*e+b*c)/d/a)^(1/2))*a*b*c)/((b*e*x+a*e-b*x)/a)^(1/2)/(b*d*x^2+a*d
*x+b*c*x+a*c)/(-1+e)^2/b^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx + A}{\sqrt{bx + a}\sqrt{dx + c}\sqrt{\frac{b(e-1)x}{a} + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bax + Aa)\sqrt{bx + a}\sqrt{dx + c}\sqrt{\frac{ae+(be-b)x}{a}}}{a^2ce + \left(b^2de - b^2d\right)x^3 - \left(b^2c + abd - \left(b^2c + 2\,abd\right)e\right)x^2 - \left(abc - \left(2\,abc + a^2d\right)e\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*a*x + A*a)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt((a*e + (b*e - b)*x)
/a)/(a^2*c*e + (b^2*d*e - b^2*d)*x^3 - (b^2*c + a*b*d - (b^2*c + 2*a*b*d)*e
)*x^2 - (a*b*c - (2*a*b*c + a^2*d)*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(e+b*(-1+e)*x/a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx + A}{\sqrt{bx + a}\sqrt{dx + c}\sqrt{\frac{b(e-1)x}{a} + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
x)
```

3.35 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$

Optimal. Leaf size=281

$$\frac{522167393\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{23328\sqrt{2x-5}}+\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4-\frac{427\sqrt{2-3x}\sqrt{2x-}}{29}$$

```
[Out] (-1182926269*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1603800 - (1224313
9*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/356400 - (17561*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/8910 - (427*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/2970 + (2*Sqrt[2 - 3*x]*Sqrt[
-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^4)/55 - (6489123157*Sqrt[11]*Sqrt[-5 + 2*
x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(699840*Sqrt[5 - 2*
x]) + (522167393*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(23328*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.389425, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {161, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4-\frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3}{2970}-\frac{17561\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}{8910}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```


```
[Out] (-1182926269*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1603800 - (1224313
9*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/356400 - (17561*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/8910 - (427*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/2970 + (2*Sqrt[2 - 3*x]*Sqrt[
-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^4)/55 - (6489123157*Sqrt[11]*Sqrt[-5 + 2*
x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(699840*Sqrt[5 - 2*
x]) + (522167393*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(23328*Sqrt[-5 + 2*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
```

```
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
```

```
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 + \frac{1}{55}\int \frac{(7+5x)^3\left(-3-1190x+854\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7$$

$$= -\frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2970}$$

$$= -\frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{8910}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{356400}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{356400}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{356400}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{356400}$$

Mathematica [A] time = 0.416079, size = 135, normalized size = 0.48

$$\frac{57438413230\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+24\sqrt{2-3x}\sqrt{4x+1}\left(29160000x^5+67338000x^4-16773\right.}{15396480\sqrt{2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(3325071575 - 797747975*x - 670058262*x^2 -
 167736600*x^3 + 67338000*x^4 + 29160000*x^5) - 71380354727*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 57438413230*Sqrt
[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(15396
480*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.047, size = 160, normalized size = 0.6

$$\frac{1}{184757760x^3-538876800x^2+161663040x+76982400}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\left(4199040000x^7+7947072000x^6+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/7698240*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(4199040000*x^7+7947072
000*x^6+86157619845*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*Elli
pticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-71380354727*11^(1/2)*(2-3*x)^(1/2
)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))
-28894190400*x^5-88040305728*x^4-70646534280*x^3+542756583588*x^2-180358343
100*x-79801717800)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^3\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(125\,x^3+525\,x^2+735\,x+343\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt
(-3*x + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^3\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

3.36 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx$

Optimal. Leaf size=243

$$\frac{5592499\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{3888\sqrt{2x-5}}+\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3-\frac{61}{270}\sqrt{2-3x}\sqrt{2x$$

```
[Out] (-5256763*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/97200 - (8141*Sqrt[2
- 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/2700 - (61*Sqrt[2 - 3*x]*Sqr
t[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/270 + (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/45 - (17746949*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipti
cE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(29160*Sqrt[5 - 2*x]) + (5592
499*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3])/(3888*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.299877, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {161, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3-\frac{61}{270}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{8141\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7}{2700}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```


```
[Out] (-5256763*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/97200 - (8141*Sqrt[2
- 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/2700 - (61*Sqrt[2 - 3*x]*Sqr
t[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/270 + (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/45 - (17746949*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipti
cE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(29160*Sqrt[5 - 2*x]) + (5592
499*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3])/(3888*Sqrt[-5 + 2*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
```

```
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
```

```
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 + \frac{1}{45}\int \frac{(7+5x)^2\left(-3-1190x+\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1-}}$$

$$= -\frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}$$

$$= -\frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700} - \frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7}{2700}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7}{2700}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7}{2700}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7}{2700}$$

Mathematica [A] time = 0.267728, size = 130, normalized size = 0.53

$$\frac{27962495\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+6\sqrt{2-3x}\sqrt{4x+1}\left(216000x^4+147600x^3-1649952x^2-\right.}{116640\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(6902575 - 2933650*x - 1649952*x^2 + 147600*
x^3 + 216000*x^4) - 35493898*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3] + 27962495*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(116640*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.011, size = 155, normalized size = 0.6

$$\frac{1}{2799360x^3-8164800x^2+2449440x+1166400}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\left(15552000x^6+83887485\sqrt{11}\sqrt{2-3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/116640*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(15552000*x^6+83887485*1
1^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^
(1/2),1/2*I*2^(1/2))-70987796*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^
(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))+4147200*x^5-125816544*x
```

```
^4-163495440*x^3+604794324*x^2-171873450*x-82830900)/(24*x^3-70*x^2+21*x+10
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25\,x^2+70\,x+49\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

3.37 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx$

Optimal. Leaf size=193

$$\frac{72479\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{756\sqrt{2x-5}}+\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}+\frac{136}{105}\sqrt{2-3x}\sqrt{2x-5}(4x$$

```
[Out] (-20911*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3780 + (136*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/105 + (5*Sqrt[2 - 3*x]*(-5 + 2*x)^(3/2)*
(1 + 4*x)^(3/2))/28 - (954811*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*S
qrt[2 - 3*x])/Sqrt[11]], -1/2])/(22680*Sqrt[5 - 2*x]) + (72479*Sqrt[11/6]*S
qrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(756*Sqrt[-5
 + 2*x])
```

Rubi [A] time = 0.0774144, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {154, 158, 114, 113, 121, 119}

$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}+\frac{136}{105}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{20911\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{3780}+\frac{72479\sqrt{\frac{11}{6}}\sqrt{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```


```
[Out] (-20911*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3780 + (136*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/105 + (5*Sqrt[2 - 3*x]*(-5 + 2*x)^(3/2)*
(1 + 4*x)^(3/2))/28 - (954811*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*S
qrt[2 - 3*x])/Sqrt[11]], -1/2])/(22680*Sqrt[5 - 2*x]) + (72479*Sqrt[11/6]*S
qrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(756*Sqrt[-5
 + 2*x])
```


Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
```

```
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) && !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) && !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2}+\frac{1}{28}\int \frac{\left(\frac{1249}{2}-1088x\right)\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{2-3x}}$$

$$= \frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}+\frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2}-\frac{1}{840}$$

$$= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780}+\frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}+\frac{5}{28}\sqrt{}$$

$$= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780}+\frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}+\frac{5}{28}\sqrt{}$$

$$= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780}+\frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}+\frac{5}{28}\sqrt{}$$

$$= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780}+\frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}+\frac{5}{28}\sqrt{}$$

Mathematica [A] time = 0.232161, size = 125, normalized size = 0.65

$$\frac{724790\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+24\sqrt{2-3x}\sqrt{4x+1}\left(5400x^3-6066x^2-37975x+48475\right)}{45360\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(48475 - 37975*x - 6066*x^2 + 5400*x^3) - 9
54811*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3] + 724790*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(45360*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.012, size = 150, normalized size = 0.8

$$\frac{1}{544320\,x^3-1587600\,x^2+476280\,x+226800}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(1087185\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\text{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/22680*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1087185*11^(1/2)*(2-3*x)
^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(
1/2))-954811*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2
/11*(22-33*x)^(1/2),1/2*I*2^(1/2))+777600*x^5-1197504*x^4-5234040*x^3+94044
84*x^2-1997100*x-1163400)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

3.38 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx$

Optimal. Leaf size=162

$$\frac{121\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{18\sqrt{2x-5}}+\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{22}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-$$

```
[Out] (-22*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/45 + (Sqrt[2 - 3*x]*Sqrt[-
5 + 2*x]*(1 + 4*x)^(3/2))/10 - (847*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(270*Sqrt[5 - 2*x]) + (121*Sqrt[11/6]
*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(18*Sqrt[-
5 + 2*x])
```

Rubi [A] time = 0.0627527, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {101, 154, 158, 114, 113, 121, 119}

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{22}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{121\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{18\sqrt{2x-5}}-\frac{847\sqrt{11}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

```
[Out] (-22*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/45 + (Sqrt[2 - 3*x]*Sqrt[-
5 + 2*x]*(1 + 4*x)^(3/2))/10 - (847*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(270*Sqrt[5 - 2*x]) + (121*Sqrt[11/6]
*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(18*Sqrt[-
5 + 2*x])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
```

```
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{1}{10}\int \frac{\left(\frac{99}{2}-44x\right)\sqrt{1+4x}}{\sqrt{2-3x}\sqrt{-5+2x}}\,dx$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{1}{90}\int \frac{-}{\sqrt{2-3x}}$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{847}{90}\int \frac{\sqrt{-}}{\sqrt{2-3}}$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{\left(121\sqrt{\frac{11}{2}}\sqrt{5-}\right.}{}$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{847\sqrt{11}\sqrt{-5+}}{}$$

Mathematica [A] time = 0.170029, size = 120, normalized size = 0.74

$$\frac{605\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right) + 6\sqrt{2-3x}\sqrt{4x+1}\left(72x^2-250x+175\right) - 847\sqrt{66}\sqrt{5-2x}E\left(\text{si}\right.}{540\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(175 - 250*x + 72*x^2) - 847*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 605*Sqrt[66]*Sqr
t[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(540*Sqrt[-5 +
 2*x])
```

Maple [A] time = 0.008, size = 145, normalized size = 0.9

$$\frac{1}{12960\,x^3-37800\,x^2+11340\,x+5400}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(1815\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\text{EllipticF}\left(2/11\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/540*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1815*11^(1/2)*(2-3*x)^(1/2
)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))
-1694*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22
-33*x)^(1/2),1/2*I*2^(1/2))+5184*x^4-20160*x^3+19236*x^2-2250*x-2100)/(24*x
^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

3.39 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx$

Optimal. Leaf size=182

$$-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{375\sqrt{2x-5}}+\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{427\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2}}{\sqrt{}}\right.\right.}{225\sqrt{5-2x}}$$

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/15 - (427*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(225*Sqrt[5 - 2*
x]) - (1253*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(375*Sqrt[-5 + 2*x]) - (2691*Sqrt[5 - 2*x]*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(125*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.212519, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {161, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{375\sqrt{2x-5}}-\frac{427\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```


```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/15 - (427*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(225*Sqrt[5 - 2*
x]) - (1253*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(375*Sqrt[-5 + 2*x]) - (2691*Sqrt[5 - 2*x]*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(125*Sqrt[11]*Sqrt[-5 + 2*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
```

```
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{15}\int \frac{-3-1190x+854x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$$

$$= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{15}\int \frac{-\frac{11928}{25}+\frac{854x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx+\frac{27807}{125}\int$$

$$= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{1253}{375}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx+\frac{427}{75}\int$$

$$= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{\left(1253\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{375\sqrt{-5+2x}}-\frac{(55}{}$$

$$= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{427\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}}-\frac{1253}{}$$

Mathematica [A] time = 0.765967, size = 141, normalized size = 0.77

$$\frac{\sqrt{2x-5}\left(-3759\sqrt{11}\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+1650\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}-23485\sqrt{11}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\right.\right.}{12375\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

```
[Out] (Sqrt[-5 + 2*x]*(1650*Sqrt[2 - 3*x]*Sqrt[5 - 2*x]*Sqrt[1 + 4*x] - 23485*Sqr
t[11]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 3759*Sqrt[11]*E
llipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 24219*Sqrt[11]*Ellipti
cPi[55/124, -ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(12375*Sqrt[5 - 2*
x])
```

Maple [A] time = 0.02, size = 183, normalized size = 1.

$$-\frac{1}{297000\,x^3-866250\,x^2+259875\,x+123750}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(3759\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x),x)
```

```
[Out] -1/12375*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(3759*11^(1/2)*(2-3*x)^(
1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/
2))+23485*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11
*(22-33*x)^(1/2),1/2*I*2^(1/2))-24219*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*
```

```
(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))-39600*x
^3+115500*x^2-34650*x-16500)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{5x+7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

3.40 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$\frac{152\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{125\sqrt{2x-5}}-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}+\frac{6\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\right.}{25\sqrt{5-2x}}$$

```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*(7 + 5*x)) + (6*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt
[5 - 2*x]) + (152*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt
[1 + 4*x]], 1/3])/(125*Sqrt[-5 + 2*x]) + (26859*Sqrt[5 - 2*x]*EllipticPi[55
/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(7750*Sqrt[11]*Sqrt[-5 + 2
*x])
```

Rubi [A] time = 0.21081, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {160, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}+\frac{152\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{125\sqrt{2x-5}}+\frac{6\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*(7 + 5*x)) + (6*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt
[5 - 2*x]) + (152*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt
[1 + 4*x]], 1/3])/(125*Sqrt[-5 + 2*x]) + (26859*Sqrt[5 - 2*x]*EllipticPi[55
/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(7750*Sqrt[11]*Sqrt[-5 + 2
*x])
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
```

```
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{1}{10}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{1}{10}\int \frac{\frac{1204}{25}-\frac{72x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx-\frac{8953}{250}\int \frac{}{\sqrt{2}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}-\frac{18}{25}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx+\frac{152}{125}\int \frac{1}{\sqrt{2-3x}\sqrt{-5}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{\left(152\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{125\sqrt{-5+2x}}+\frac{(895}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{6\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}}+\frac{152\sqrt{\frac{2}{33}}}{}$$

Mathematica [A] time = 0.751783, size = 132, normalized size = 0.7

$$\frac{\sqrt{2x-5}\left(\frac{3\sqrt{11}\left(9424\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+20460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+26859\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}}-\frac{51150\sqrt{2-3x}\sqrt{4x+1}}{5x+7}\right)}{255750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((-51150*Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(7 + 5*x) + (3*Sqrt[1
1]*(20460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 9424*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 26859*EllipticPi[55/124, -A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/255750
```

Maple [B] time = 0.025, size = 320, normalized size = 1.7

$$\frac{1}{\left(2046000\,x^3-5967500\,x^2+1790250\,x+852500\right)(7+5\,x)}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\Bigl(47120\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^2,x)
```

```
[Out] 1/85250*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(47120*11^(1/2)*(2-3*x)^(
1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/
2))*x+102300*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2
```

```
/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-134295*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^
(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x
+65968*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(2
2-33*x)^(1/2),1/2*I*2^(1/2))+143220*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4
*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-188013*11^(1/2)*(
2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55
/124,1/2*I*2^(1/2))-409200*x^3+1193500*x^2-358050*x-170500)/(24*x^3-70*x^2+
21*x+10)/(7+5*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{25x^2+70x+49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 + 70*x + 49), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

3.41 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx$

Optimal. Leaf size=227

$$\frac{397\sqrt{\frac{3}{22}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{89125\sqrt{2x-5}}+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{556140(5x+7)}-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}-\frac{895\cdots}{\cdots}$$

```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(10*(7 + 5*x)^2) + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(556140*(7 + 5*x)) - (8953*Sqrt[11]
*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(13903
50*Sqrt[5 - 2*x]) + (397*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/1
1]*Sqrt[1 + 4*x]], 1/3])/(89125*Sqrt[-5 + 2*x]) - (14832503*Sqrt[5 - 2*x]*E
llipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(287339000*Sqr
t[11]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.308533, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {160, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{556140(5x+7)}-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}+\frac{397\sqrt{\frac{3}{22}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{89125\sqrt{2x-5}}-\frac{8953\sqrt{11}\sqrt{2x\cdots}}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```


```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(10*(7 + 5*x)^2) + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(556140*(7 + 5*x)) - (8953*Sqrt[11]
*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(13903
50*Sqrt[5 - 2*x]) + (397*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/1
1]*Sqrt[1 + 4*x]], 1/3])/(89125*Sqrt[-5 + 2*x]) - (14832503*Sqrt[5 - 2*x]*E
llipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(287339000*Sqr
t[11]*Sqrt[-5 + 2*x])
```


Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
```

```
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
```

```
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{1}{20}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\int \frac{-106729-199200x+2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4}}}{1112280}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\int \frac{-\frac{2500104}{25}+\frac{214872x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4}}}{1112280}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{1191\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}}}{178250}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\left(1191\sqrt{5-2x}\right)\int -}{89125\sqrt{22}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} - \frac{8953\sqrt{11}\sqrt{-5+2x}}{13903}$$

Mathematica [A] time = 0.696315, size = 136, normalized size = 0.6

$$\frac{\sqrt{2x-5}\left(\frac{\sqrt{11}\left(5759676\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-61059460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-44497509\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}} + \frac{17050\sqrt{2-3x}\sqrt{4x+1}(44}{(5x+7)^2}\right.}{9482187000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7057 + 44765*x))/(7 +
5*x)^2 + (Sqrt[11]*(-61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] + 5759676*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 4449
7509*EllipticPi[55/124, -ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5
 - 2*x]))/9482187000
```

Maple [B] time = 0.025, size = 461, normalized size = 2.

$$\frac{1}{\left(227572488000\,x^{3}-663753090000\,x^{2}+199125927000\,x+94821870000\right)(7+5\,x)^{2}}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\,\Big(143$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^3,x)
```

```
[Out] 1/9482187000*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(143991900*11^(1/2)*
(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/
2*I*2^(1/2))*x^2-1526486500*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1
/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x^2+1112437725*11^(1/2)*(
2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55
/124,1/2*I*2^(1/2))*x^2+403177320*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x
+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-4274162200*11^(1/
2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2)
,1/2*I*2^(1/2))*x+3114825630*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(
1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x+282224124*11^(
1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/
2),1/2*I*2^(1/2))-2991913540*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(
1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))+2180377941*11^(1/2)*(2-3
*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/12
4,1/2*I*2^(1/2))+18317838000*x^4-50539303100*x^3+7605578750*x^2+10159191350
*x+1203218500)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{125\,x^{3}+525\,x^{2}+735\,x+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3 + 525*x^2 + 73
5*x + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

3.42 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx$

Optimal. Leaf size=263

$$\frac{24957247\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{4956597750\sqrt{66}\sqrt{2x-5}}+\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{30929169960(5x+7)}+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{\ldots}}{1668420(5x+7)\ldots}$$

```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^3) + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1668420*(7 + 5*x)^2) + (16830401*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(30929169960*(7 + 5*x)) - (16830
401*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(77322924900*Sqrt[5 - 2*x]) + (24957247*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(4956597750*Sqrt[66]*Sqrt[-5 + 2*x]) + (
15664616449*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[
11]], -1/2])/(15980071146000*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.397013, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {160, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{30929169960(5x+7)}+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1668420(5x+7)^2}-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}+\frac{24957247\sqrt{5-2x}\ldots}{4956597\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```


```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^3) + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1668420*(7 + 5*x)^2) + (16830401*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(30929169960*(7 + 5*x)) - (16830
401*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(77322924900*Sqrt[5 - 2*x]) + (24957247*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(4956597750*Sqrt[66]*Sqrt[-5 + 2*x]) + (
15664616449*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[
11]], -1/2])/(15980071146000*Sqrt[11]*Sqrt[-5 + 2*x])
```


Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
```

```
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
```

```
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{1}{30}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{\int \frac{-401471+855020\cdots}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{\cdots}}}{33368\cdots}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-\cdots}}{3092916\cdots}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-\cdots}}{3092916\cdots}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-\cdots}}{3092916\cdots}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-\cdots}}{3092916\cdots}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-\cdots}}{3092916\cdots}$$

Mathematica [A] time = 0.753849, size = 141, normalized size = 0.54

$$\frac{\sqrt{2x-5}\left(\frac{\sqrt{11}\left(120693246492\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-114783334820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)+46993849347\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}}+\frac{1705\cdots}{\cdots}\right.}{527342347818000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-75460017 + 2007981640
*x + 420760025*x^2))/(7 + 5*x)^3 + (Sqrt[11]*(-114783334820*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 120693246492*EllipticF[ArcSin[(2*Sqr
t[2 - 3*x])/Sqrt[11]], -1/2] + 46993849347*EllipticPi[55/124, -ArcSin[(2*Sq
rt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/527342347818000
```

Maple [B] time = 0.026, size = 602, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^4,x)
```

```
[Out] 1/527342347818000*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(15086655811500
*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x
)^(1/2),1/2*I*2^(1/2))*x^3-14347916852500*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1
/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x^3-5874231
168375*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(
22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x^3+63363954408300*11^(1/2)*(2-3*x)^(1
/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2
))*x^2-60261250780500*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*El
lipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x^2-24671770907175*11^(1/2)*(2-
3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/1
24,1/2*I*2^(1/2))*x^2+88709536171620*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(
4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-84365751092700
*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x
)^(1/2),1/2*I*2^(1/2))*x-34540479270045*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2
)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x+413
97783546756*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/
11*(22-33*x)^(1/2),1/2*I*2^(1/2))-39370683843260*11^(1/2)*(2-3*x)^(1/2)*(5-
2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-1611
8890326021*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/
11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))+172175002230000*x^5+31948899725050
0*x^4-2276751199345150*x^3+880758940754000*x^2+315342410533150*x-1286593289
8500)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{625\,x^4+3500\,x^3+7350\,x^2+6860\,x+2401},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4 + 3500*x^3 + 7
350*x^2 + 6860*x + 2401), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**4,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

3.43 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$

Optimal. Leaf size=570

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2-3abh^2(cf+de)+b^2(-(dg(fg-eh)-ch(2eh+fg)))\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right.\right.}{3b^3d\sqrt{fh}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*b) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], (
(d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^2*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(
g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a^2*d*f*h^2 - 3*a*b*(d*e
 + c*f)*h^2 - b^2*(d*g*(f*g - e*h) - c*h*(f*g + 2*e*h)))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqr
t[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^3*d
*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]
*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)
]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d
*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^3*Sqrt[f]*Sq
rt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] time = 1.28584, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {161, 1607, 169, 538, 537, 158, 114, 113, 121, 120}

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2-3abh^2(cf+de)+b^2(-(dg(fg-eh)-ch(2eh+fg)))\right)F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch}\right.}{3b^3d\sqrt{fh}\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```


```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*b) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], (
(d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^2*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(
g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a^2*d*f*h^2 - 3*a*b*(d*e
 + c*f)*h^2 - b^2*(d*g*(f*g - e*h) - c*h*(f*g + 2*e*h)))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqr
t[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^3*d
*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]
*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)
]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d
*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^3*Sqrt[f]*Sq
rt[e + f*x]*Sqrt[g + h*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
```

```
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
```

```
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} + \frac{\int \frac{3bceg-a(deg+cfg+ceh)+2(b(deg+cfg+ceh)-a(dfg+deh+cfh))x-(3adfh- \cdots}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3b}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} + \frac{\int \frac{2deg+2cfg-\frac{3adfg}{b}+2ceh-\frac{3adeh}{b}-\frac{3acfh}{b}+\frac{3a^2dfh}{b^2}+\left(dfg+deh+cfh-\frac{3adfh}{b}\right)x \cdots}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3b}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{(2(bc-ad)(be-af)(bg-ah))\,\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+ \cdots}} \cdots\right)}{b^3}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{\left(2(bc-ad)(be-af)(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{\cdots}{\left(bc-ad- \cdots\right)} \cdots\right)}{b^3\sqrt{e+fx}}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+ \cdots}}{3b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}} \cdots}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+ \cdots}}{3b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}} \cdots}$$

Mathematica [C] time = 13.8408, size = 1250, normalized size = 2.19

$$2\sqrt{c+dx}\left(\frac{b^2fhc^3}{d^2(c+dx)}-\frac{b^2fhc^2}{d^2}-\frac{3abfhc^2}{d(c+dx)}+\frac{3abfhc}{d}+\frac{b^2fhxc}{d}-\frac{b^2egc}{c+dx}+\frac{3abfgc}{c+dx}+\frac{3abehc}{c+dx}-\frac{b^2e^2hc}{cf+dxf}-\frac{b^2fg^2c}{ch+dxh}+b^2fhx^2+3b^2eg-3a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(3*b^2*e*g - 3*a*b*f*g + (b^2*f*g^2)/h - 3*a*b*e*h + (b^2*
e^2*h)/f - (b^2*c^2*f*h)/d^2 + (3*a*b*c*f*h)/d + 2*b^2*f*g*x + 2*b^2*e*h*x
- 3*a*b*f*h*x + (b^2*c*f*h*x)/d + b^2*f*h*x^2 - (b^2*c*e*g)/(c + d*x) - (3*
a*b*d*e*g)/(c + d*x) + (3*a*b*c*f*g)/(c + d*x) + (3*a*b*c*e*h)/(c + d*x) +
(b^2*c^3*f*h)/(d^2*(c + d*x)) - (3*a*b*c^2*f*h)/(d*(c + d*x)) + (b^2*d*e^2*
g)/(c*f + d*f*x) - (b^2*c*e^2*h)/(c*f + d*f*x) + (b^2*d*e*g^2)/(c*h + d*h*x
) - (b^2*c*f*g^2)/(c*h + d*h*x) - (I*b*Sqrt[-c + (d*g)/h]*(3*a*d*f*h - b*(d
*f*g + d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt
[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c
 + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/d^2 + (I*b*Sqrt[-c + (d*g)/h]*(
-(b*f*g) - 2*b*e*h + 3*a*f*h)*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x)
)]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*g)/h]
/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/d + ((3*I)*b^2*e*g*Sqrt[
-c + (d*g)/h]*h*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g
+ h*x))/(h*(c + d*x))]*EllipticPi[((b*c - a*d)*h)/(b*(-(d*g) + c*h)), I*Arc
Sinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/(
d*g - c*h) + ((3*I)*a^2*f*Sqrt[-c + (d*g)/h]*h^2*Sqrt[c + d*x]*Sqrt[(d*(e +
 f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[((b*c -
a*d)*h)/(b*(-(d*g) + c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d
*e*h - c*f*h)/(d*f*g - c*f*h)])/(d*g - c*h) + ((3*I)*a*b*f*g*Sqrt[-c + (d*g
)/h]*h*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(
h*(c + d*x))]*EllipticPi[((b*c - a*d)*h)/(b*(-(d*g) + c*h)), I*ArcSinh[Sqrt
[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/(-(d*g) +
c*h) + ((3*I)*a*b*e*Sqrt[-c + (d*g)/h]*h^2*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))
/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[((b*c - a*d)*h
)/(b*(-(d*g) + c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h -
 c*f*h)/(d*f*g - c*f*h)])/(-(d*g) + c*h)))/(3*b^3*Sqrt[e + f*x]*Sqrt[g + h*
x])
```

Maple [B] time = 0.059, size = 3678, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x)
```

```
[Out] 2/3*(((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(
c*f-d*e))^(1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d
*g))^(1/2))*b^2*c*d^2*e*f*g*h+3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*
h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticPi(((d*x+c)*f/(c*f-d*e))
^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c*d^2*
e*f*h^2+3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e
```

```
)*d/(c*f-d*e))^(1/2)*EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/
(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c*d^2*f^2*g*h+3*((d*x+c)*f/(
c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*E
llipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h
/f/(c*h-d*g))^(1/2))*a^2*d^3*e*f*h^2-3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g
)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticF(((d*x+c)*f/(c*f
-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a^2*d^3*e*f*h^2+3*((d*x+c)*f/
(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*
EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*
d^3*e^2*h^2-2*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f
*x+e)*d/(c*f-d*e))^(1/2)*EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h
/f/(c*h-d*g))^(1/2))*b^2*c*d^2*e^2*h^2-3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x
+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticPi(((d*x+c)*f/(
c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a
^2*c*d^2*f^2*h^2-3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)
*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d
*e)*h/f/(c*h-d*g))^(1/2))*a*b*c^2*d*f^2*h^2+2*((d*x+c)*f/(c*f-d*e))^(1/2)*(
-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticF(((d*x+c)
*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*c^2*d*e*f*h^2+((d*
x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))
^(1/2)*EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2
))*b^2*c^2*d*f^2*g*h+3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(
1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c
*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c^2*d*f^2*h^2+x^2*b^2*c*d^2*f^2*h^2+x^2*b
^2*d^3*e*f*h^2+3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(
-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e
)*h/f/(c*h-d*g))^(1/2))*a^2*c*d^2*f^2*h^2-3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(
h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticPi(((d*x+c)*
f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2)
)*a*b*d^3*e^2*h^2+3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2
)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f
-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*d^3*e^2*g*h+((d*x+
c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(
1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))
*b^2*c*d^2*e^2*h^2+((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)
*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d
*e)*h/f/(c*h-d*g))^(1/2))*b^2*c*d^2*f^2*g^2-((d*x+c)*f/(c*f-d*e))^(1/2)*(-(
h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticE(((d*x+c)*f
/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*d^3*e^2*g*h+b^2*c*d^
2*e*f*g*h-((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e
)*d/(c*f-d*e))^(1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(
c*h-d*g))^(1/2))*b^2*d^3*e*f*g^2+x^2*b^2*d^3*f^2*g*h+x^3*b^2*d^3*f^2*h^2-((
d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e
))^(1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1
/2))*b^2*c^3*f^2*h^2+x*b^2*c*d^2*f^2*g*h+x*b^2*d^3*e*f*g*h+x*b^2*c*d^2*e*f*
h^2-3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/
(c*f-d*e))^(1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-
d*g))^(1/2))*a*b*c*d^2*e*f*h^2-3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c
*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticE(((d*x+c)*f/(c*f-d*e))
^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c*d^2*f^2*g*h+3*((d*x+c)*f/(c*f
-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*Elli
pticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*d^3*
e*f*g*h-3*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e
)*d/(c*f-d*e))^(1/2)*EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/
(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*d^3*e*f*g*h-3*((d*x+c)*f/(c*
f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*Ell
ipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f
/(c*h-d*g))^(1/2))*b^2*c*d^2*e*f*g*h-((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*
d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticF(((d*x+c)*f/(c*f-d
```

```
*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*c*d^2*f^2*g^2-((d*x+c)*f/(c
*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*El
lipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*d^
3*e^2*g*h+((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e
)*d/(c*f-d*e))^(1/2)*EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(
c*h-d*g))^(1/2))*b^2*d^3*e*f*g^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)
/b^3/h/f/d^2/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g
*x+c*e*g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(h*x+g)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

3.44 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=243

$$-\frac{2161804579\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{54432\sqrt{2x-5}}+\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3+\frac{1679}{756}\sqrt{2-3x}$$

```
[Out] (46134551*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/38880 + (26291*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/540 + (1679*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/756 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/9 + (2629157597*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipt
icE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(163296*Sqrt[5 - 2*x]) - (21
61804579*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]
], 1/3])/(54432*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.296547, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {162, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3+\frac{1679}{756}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2+\frac{26291}{540}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```


```
[Out] (46134551*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/38880 + (26291*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/540 + (1679*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/756 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/9 + (2629157597*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipt
icE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(163296*Sqrt[5 - 2*x]) - (21
61804579*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]
], 1/3])/(54432*Sqrt[-5 + 2*x])
```


Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
```

```
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
```

```
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 - \frac{1}{18}\int \frac{(7+5x)^2\left(-699-565x+3358x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,d$$

$$= \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 -$$

$$= \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) +$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) +$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) +$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) +$$

Mathematica [A] time = 0.367859, size = 130, normalized size = 0.53

$$\frac{-2161804579\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+6\sqrt{2-3x}\sqrt{4x+1}\left(1512000x^4+8614800x^3+21329\right.}{326592\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-455686385 + 51484034*x + 21329208*x^2 + 86
14800*x^3 + 1512000*x^4) + 2629157597*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcS
in[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 2161804579*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(326592*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.02, size = 155, normalized size = 0.6

$$-\frac{1}{7838208\,x^3-22861440\,x^2+6858432\,x+3265920}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(-108864000\,x^6+6485413737\,\sqrt{11}\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/326592*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(-108864000*x^6+6485413
737*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-3
3*x)^(1/2),1/2*I*2^(1/2))-5258315194*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(
```

```
4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-574905600*x^5-12
59114976*x^4-2963596608*x^3+34609891236*x^2-13052783142*x-5468236620)/(24*x
^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^3\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(125\,x^3+525\,x^2+735\,x+343\right)\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqr
t(2*x - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^3\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

3.45 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=205

$$-\frac{1679161\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{756\sqrt{2x-5}}+\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2+\frac{173}{60}\sqrt{2-3x}\sqrt{2x}$$

```
[Out] (73207*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1080 + (173*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/60 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/7 + (8198333*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE
[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (1679161
*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/(756*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.213009, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {162, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2+\frac{173}{60}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)+\frac{73207\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1080}-\frac{1679}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```


```
[Out] (73207*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1080 + (173*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/60 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/7 + (8198333*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE
[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (1679161
*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/(756*Sqrt[-5 + 2*x])
```


Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
```

```
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
```

```
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 - \frac{1}{14}\int \frac{(7+5x)\left(-543-175x+2422x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \cdots$$

$$= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1}{7}\sqrt{2}\cdots$$

$$= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1}{7}\sqrt{2}\cdots$$

$$= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1}{7}\sqrt{2}\cdots$$

$$= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1}{7}\sqrt{2}\cdots$$

Mathematica [A] time = 0.300078, size = 125, normalized size = 0.61

$$\frac{-6716644\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+12\sqrt{2-3x}\sqrt{4x+1}\left(10800x^3+46836x^2+102592x-71\cdots\right.}{18144\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-717955 + 102592*x + 46836*x^2 + 10800*x^3
) + 8198333*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3] - 6716644*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(18144*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.012, size = 150, normalized size = 0.7

$$-\frac{1}{217728\,x^3-635040\,x^2+190512\,x+90720}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(10074966\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\text{Elli}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/9072*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(10074966*11^(1/2)*(2-3*x
)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^
(1/2))-8198333*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE
(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-777600*x^5-3048192*x^4-5851944*x^3+553
32552*x^2-20307546*x-8615460)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^2\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(25x^2+70x+49\right)\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^2\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

3.46 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=162

$$-\frac{4543\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{36\sqrt{2x-5}}+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}+\frac{95}{18}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[Out] (95*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/18 + (Sqrt[2 - 3*x]*Sqrt[-5
 + 2*x]*(1 + 4*x)^(3/2))/4 + (1397*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin
[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(27*Sqrt[5 - 2*x]) - (4543*Sqrt[11/6]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5
 + 2*x])
```

Rubi [A] time = 0.0624533, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {154, 158, 114, 113, 121, 119}

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}+\frac{95}{18}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{4543\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{36\sqrt{2x-5}}+\frac{1397\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

```
[Out] (95*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/18 + (Sqrt[2 - 3*x]*Sqrt[-5
 + 2*x]*(1 + 4*x)^(3/2))/4 + (1397*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin
[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(27*Sqrt[5 - 2*x]) - (4543*Sqrt[11/6]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5
 + 2*x])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
```

```
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{1}{20}\int \frac{\left(\frac{1065}{2}-950x\right)\sqrt{1+4x}}{\sqrt{2-3x}\sqrt{-5+2x}}\,dx$$

$$= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{1}{180}\int \frac{-\frac{29535}{2}+ \cdots}{\sqrt{2-3x}\sqrt{-5 \cdots}}$$

$$= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{1397}{9}\int \frac{\sqrt{-5+2 \cdots}}{\sqrt{2-3x}\sqrt{1 \cdots}}$$

$$= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{\left(4543\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \cdots}{36\sqrt{- \cdots}}$$

$$= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{1397\sqrt{11}\sqrt{-5+2x}E \cdots}{27\sqrt{5 \cdots}}$$

Mathematica [A] time = 0.192639, size = 120, normalized size = 0.74

$$\frac{-4543\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+6\sqrt{2-3x}\sqrt{4x+1}\left(72x^2+218x-995\right)+5588\sqrt{66}\sqrt{5-2x}E\left(\cdots\right.}{216\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-995 + 218*x + 72*x^2) + 5588*Sqrt[66]*Sqrt
[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 4543*Sqrt[66]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(216*Sqrt[-
5 + 2*x])
```

Maple [A] time = 0.01, size = 145, normalized size = 0.9

$$-\frac{1}{5184\,x^3-15120\,x^2+4536\,x+2160}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\Big(13629\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\text{EllipticF}\Big(2/11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)*(2-3*x)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/216*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(13629*11^(1/2)*(2-3*x)^(1
/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2
))-11176*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*
(22-33*x)^(1/2),1/2*I*2^(1/2))-5184*x^4-13536*x^3+79044*x^2-27234*x-11940)/
(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5\,x+7)\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(5\,x+7)\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}\,(5x+7)}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)/sqrt(2*x - 5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

3.47 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=131

$$-\frac{11\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{3\sqrt{2x-5}}+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{55\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\right|-\frac{1}{2}\right)}{18\sqrt{5-2x}}$$

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3 + (55*Sqrt[11]*Sqrt[-5 + 2*x
]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(18*Sqrt[5 - 2*x]) -
 (11*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(3*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.0513602, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {101, 158, 114, 113, 121, 119}

$$\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{11\sqrt{\frac{22}{3}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)\middle|\frac{1}{3}\right)}{3\sqrt{2x-5}}+\frac{55\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)}{18\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3 + (55*Sqrt[11]*Sqrt[-5 + 2*x
]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(18*Sqrt[5 - 2*x]) -
 (11*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(3*Sqrt[-5 + 2*x])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) && !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) && !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{1}{3}\int \frac{-\frac{33}{2}+55x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{55}{6}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{121}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,d$$

$$= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{\left(11\sqrt{22}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{3\sqrt{-5+2x}} - \frac{\left(55\sqrt{-5+2x}\right)\int \frac{}{\sqrt{2-}}}{6\sqrt{5-2}}$$

$$= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{55\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{18\sqrt{5-2x}} - \frac{11\sqrt{\frac{22}{3}}\sqrt{5-2x}F\left(\sin\right.}{3\sqrt{-}}$$

Mathematica [A] time = 0.186648, size = 115, normalized size = 0.88

$$\frac{-44\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right) + 12\sqrt{2-3x}\sqrt{4x+1}(2x-5) + 55\sqrt{66}\sqrt{5-2x}E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+\right.\right.\right.}{36\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 55*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 44*Sqrt[66]*Sqrt[5 - 2*x]*El
```

```
lipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.007, size = 140, normalized size = 1.1

$$-\frac{1}{432\,x^3-1260\,x^2+378\,x+180}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(66\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\mathrm{EllipticF}\left(2/11\,\sqrt{22-3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/18*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(66*11^(1/2)*(2-3*x)^(1/2)*
(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-5
5*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*
x)^(1/2),1/2*I*2^(1/2))-144*x^3+420*x^2-126*x-60)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/sqrt(2*x - 5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

3.48 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx$

Optimal. Leaf size=151

$$-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{2\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}}+\frac{69\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin\ldots\right.}{25\sqrt{11}\sqrt{\ldots}}$$

```
[Out] (2*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(5*Sqrt[5 - 2*x]) - (41*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt
[3/11]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) + (69*Sqrt[5 - 2*x]*Ellipt
icPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt[11]*Sqrt[-
5 + 2*x])
```

Rubi [A] time = 0.121094, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {163, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{2\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}}+\frac{69\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-\ldots}}{\sqrt{11}}\right.\right.}{25\sqrt{11}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```


```
[Out] (2*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(5*Sqrt[5 - 2*x]) - (41*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt
[3/11]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) + (69*Sqrt[5 - 2*x]*Ellipt
icPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt[11]*Sqrt[-
5 + 2*x])
```


Rule 163

```
Int[(Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)])/(((a_.) + (b_.)*(x_
))*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[((b*e - a*f)*(b*g - a*h))/b
^2, Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + D
ist[1/b^2, Int[Simp[b*f*g + b*e*h - a*f*h + b*f*h*x, x]/(Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}dx = \frac{1}{25}\int \frac{109-60x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}dx - \frac{713}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}dx$$

$$= -\left(\frac{6}{5}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}dx\right) - \frac{41}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}dx + \frac{1426}{25}\text{Subst}\left(\int \frac{}{(3}\right.$$

$$= -\frac{\left(41\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}dx}{25\sqrt{-5+2x}} + \frac{\left(1426\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}}\right.}{25\sqrt{-5+2x}}$$

$$= \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}} - \frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{25\sqrt{-5+2x}} + \frac{69}{}$$

Mathematica [A] time = 0.403959, size = 97, normalized size = 0.64

$$\frac{\sqrt{5-2x}\left(41\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-110E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-69\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{25\sqrt{22x-55}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

```
[Out] (Sqrt[5 - 2*x]*(-110*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
41*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 69*EllipticPi[55/1
24, -ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(25*Sqrt[-55 + 22*x])
```

Maple [A] time = 0.013, size = 76, normalized size = 0.5

$$\frac{\sqrt{11}}{275}\left(41\,\text{EllipticF}\left(2/11\,\sqrt{22-33\,x},i/2\sqrt{2}\right)-110\,\text{EllipticE}\left(2/11\,\sqrt{22-33\,x},i/2\sqrt{2}\right)+69\,\text{EllipticPi}\left(2/11\,\sqrt{22-33}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)/(2*x-5)^(1/2),x)
```

```
[Out] 1/275*(41*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-110*EllipticE(2/11*
(22-33*x)^(1/2),1/2*I*2^(1/2))+69*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/
2*I*2^(1/2)))*(5-2*x)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{10\,x^2-11\,x-35},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 - 11*x - 35), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

3.49 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2} dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}-\frac{2\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\right.}{195\sqrt{5-2x}}$$

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*(7 + 5*x)) - (2*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(195*Sqr
t[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticPi[55/12
4, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(20150*Sqrt[11]*Sqrt[-5 + 2*x
])
```

Rubi [A] time = 0.214234, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {164, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{25\sqrt{2x-5}}-\frac{2\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}}-\frac{6101}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```


```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*(7 + 5*x)) - (2*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(195*Sqr
t[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticPi[55/12
4, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(20150*Sqrt[11]*Sqrt[-5 + 2*x
])
```


Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
```

```
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{-29+120x-24x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{\frac{768}{25}-\frac{24x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx + \frac{6101\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}}{1950}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} + \frac{2}{65}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{6}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{25\sqrt{-5+2x}} - \frac{\left(6101\sqrt{5-2x}\right)\mathrm{S}}{}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}} - \frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\right.}{25}$$

Mathematica [A] time = 0.623383, size = 132, normalized size = 0.7

$$\frac{3\sqrt{55-22x}\left(14508\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+18303\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\right.}{1994850\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

```
[Out] ((51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) + 3*Sqrt[55 - 22*
x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14508*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 18303*EllipticPi[55/124, -A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(1994850*Sqrt[-5 + 2*x])
```

Maple [B] time = 0.016, size = 320, normalized size = 1.7

$$-\frac{1}{\left(15958800\,x^3-46546500\,x^2+13963950\,x+6649500\right)(7+5\,x)}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(72540\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^2/(2*x-5)^(1/2),x)
```

```
[Out] -1/664950*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(72540*11^(1/2)*(2-3*x)
^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(
1/2))*x+34100*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(
2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-91515*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^
```

```
(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x
+101556*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(
22-33*x)^(1/2),1/2*I*2^(1/2))+47740*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4
*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-128121*11^(1/2)*(
2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55
/124,1/2*I*2^(1/2))-409200*x^3+1193500*x^2-358050*x-170500)/(24*x^3-70*x^2+
21*x+10)/(7+5*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^2\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{50x^3+15x^2-252x-245}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 + 15*x^2 - 252*
x - 245), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^2\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

3.50 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{6101\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{231725\sqrt{66}\sqrt{2x-5}}-\frac{361\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{481988(5x+7)}+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}+\frac{361\sqrt{11}}{}$$

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(78*(7 + 5*x)^2) - (361*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(481988*(7 + 5*x)) + (361*Sqrt[11]*Sq
rt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1204970*
Sqrt[5 - 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(231725*Sqrt[66]*Sqrt[-5 + 2*x]) - (6655867*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(747081400*Sqrt[11
]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.300811, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {164, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{361\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{481988(5x+7)}+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}-\frac{6101\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{231725\sqrt{66}\sqrt{2x-5}}+\frac{361\sqrt{11}\sqrt{2x-5}}{1204}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```


```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(78*(7 + 5*x)^2) - (361*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(481988*(7 + 5*x)) + (361*Sqrt[11]*Sq
rt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1204970*
Sqrt[5 - 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(231725*Sqrt[66]*Sqrt[-5 + 2*x]) - (6655867*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(747081400*Sqrt[11
]*Sqrt[-5 + 2*x])
```


Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
```

```
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
```

```
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{1}{156}\int \frac{-37+100x+24x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\int \frac{-272145+485280x+77976x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{8675784}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\int \frac{\frac{1880568}{25}+\frac{77976x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{8675784} + \frac{665\cdots}{\cdots}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{1083\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{1204970} - \frac{6101\cdots}{\cdots}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\left(6101\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}\cdots}}}{231725\sqrt{22}\sqrt{-5+2x}\cdots}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} + \frac{361\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\cdots}{\cdots}\right)\cdots\right)}{1204970\sqrt{5-2x}\cdots}$$

Mathematica [A] time = 0.56169, size = 137, normalized size = 0.61

$$\frac{-3\sqrt{55-22x}\left(-9834812\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+2462020E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-6655867\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\cdots}{\cdots}\right)\cdots\right)\cdots\right)}{24653686200\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

```
[Out] ((-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(-10957 + 5415*x))/(7 + 5*x
)^2 - 3*Sqrt[55 - 22*x]*(2462020*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11
]], -1/2] - 9834812*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 6
655867*EllipticPi[55/124, -ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(246
53686200*Sqrt[-5 + 2*x])
```

Maple [B] time = 0.016, size = 461, normalized size = 2.1

$$-\frac{1}{\left(591688468800\,x^3-1725758034000\,x^2+517727410200\,x+246536862000\right)(7+5\,x)^2}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^3/(2*x-5)^(1/2),x)
```

```
[Out] -1/24653686200*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(737610900*11^(1/2
)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),
1/2*I*2^(1/2))*x^2-184651500*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(
1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x^2-499190025*11^(1/2)*(
2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55
/124,1/2*I*2^(1/2))*x^2+2065310520*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*
x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-517024200*11^(1/
2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2)
,1/2*I*2^(1/2))*x-1397732070*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(
1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x+1445717364*11^
(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1
/2),1/2*I*2^(1/2))-361916940*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(
1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-978412449*11^(1/2)*(2-3*
x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124
,1/2*I*2^(1/2))+2215818000*x^4-10946406900*x^3+15016020250*x^2-2999896350*x
-1868168500)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{250\,x^4+425\,x^3-1155\,x^2-2989\,x-1715},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4 + 425*x^3 - 11
55*x^2 - 2989*x - 1715), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^3\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

3.51 $\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=205

$$-\frac{25260049\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{6048\sqrt{2x-5}}+\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2+\frac{121}{24}\sqrt{2-3x}\sqrt{2}$$

```
[Out] (110743*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/864 + (121*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 + (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2
*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/28 + (15629623*Sqrt[11]*Sqrt[-5 + 2*x]*Ellip
ticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (252
60049*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/(6048*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.212381, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {174, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2+\frac{121}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)+\frac{110743}{864}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{252}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (110743*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/864 + (121*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 + (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2
*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/28 + (15629623*Sqrt[11]*Sqrt[-5 + 2*x]*Ellip
ticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (252
60049*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/(6048*Sqrt[-5 + 2*x])
```

Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
```

```
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
```

```
a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 - \frac{1}{56}\int \frac{(7+5x)\left(-7223+2667x+16940x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{\int \frac{10251}{}}{}$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}\sqrt{2-3x}$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}\sqrt{2-3x}$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}\sqrt{2-3x}$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}\sqrt{2-3x}$$

Mathematica [A] time = 0.349892, size = 125, normalized size = 0.61

$$\frac{-25260049\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\tfrac{3}{11}}\sqrt{4x+1}\right),\tfrac{1}{3}\right)+30\sqrt{2-3x}\sqrt{4x+1}\left(10800x^3+64224x^2+188566x-1\right.}{36288\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (30*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-1041565 + 188566*x + 64224*x^2 + 10800*x^
3) + 31259246*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3] - 25260049*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sq
rt[1 + 4*x]], 1/3])/(36288*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.023, size = 150, normalized size = 0.7

$$-\frac{1}{870912\,x^3-2540160\,x^2+762048\,x+362880}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(75780147\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/36288*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(75780147*11^(1/2)*(2-3*
x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2
^(1/2))-62518492*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*Ellipti
cE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-3888000*x^5-21500640*x^4-57602160*x^
3+407101740*x^2-144920790*x-62493900)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^3\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(125x^3+525x^2+735x+343)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{8x^2-18x-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt
(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^3\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.52 $\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=167

$$-\frac{17533\sqrt{\frac{11}{6}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{72\sqrt{2x-5}}+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)+\frac{68}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{}$$

```
[Out] (68*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (Sqrt[2 - 3*x]*Sqrt[-5
+ 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/4 + (44569*Sqrt[11]*Sqrt[-5 + 2*x]*Elliptic
E[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(432*Sqrt[5 - 2*x]) - (17533*S
qrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
72*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.145422, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {174, 1615, 158, 114, 113, 121, 119}

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)+\frac{68}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{17533\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{72\sqrt{2x-5}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (68*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (Sqrt[2 - 3*x]*Sqrt[-5
+ 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/4 + (44569*Sqrt[11]*Sqrt[-5 + 2*x]*Elliptic
E[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(432*Sqrt[5 - 2*x]) - (17533*S
qrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
72*Sqrt[-5 + 2*x])
```

Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{1}{40}\int \frac{-5155+3605x+10880x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{\int \frac{-899460+2674140x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4}}}{4320}$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{44569}{144}\int \frac{\sqrt{-5}}{\sqrt{2-3x}}$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{\left(17533\sqrt{\frac{11}{2}}\sqrt{5-2}\right.}{72}$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{44569\sqrt{11}\sqrt{-5+2}}{43}$$

Mathematica [A] time = 0.280988, size = 120, normalized size = 0.72

$$\frac{-35066\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+120\sqrt{2-3x}\sqrt{4x+1}\left(18x^2+89x-335\right)+44569\sqrt{66}\sqrt{5-}}{864\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-335 + 89*x + 18*x^2) + 44569*Sqrt[66]*Sq
rt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 35066*Sqrt[6
6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(864*Sqr
t[-5 + 2*x])
```

Maple [A] time = 0.014, size = 145, normalized size = 0.9

$$-\frac{1}{10368\,x^3-30240\,x^2+9072\,x+4320}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(52599\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\text{EllipticF}\left(2/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/432*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(52599*11^(1/2)*(2-3*x)^(1
/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2
))-44569*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*
(22-33*x)^(1/2),1/2*I*2^(1/2))-12960*x^4-58680*x^3+270060*x^2-89820*x-40200
)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5\,x+7)^2\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(25\,x^2+70\,x+49)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{8\,x^2-18\,x-5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8
*x^2 - 18*x - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5\,x+7)^2\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.53 $\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=131

$$-\frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{12\sqrt{2x-5}}+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{241\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-}}{\sqrt{1}}\right.\right.}{36\sqrt{5-2x}}$$

```
[Out] (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/12 + (241*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(36*Sqrt[5 - 2*x
]) - (179*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(12*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.0510921, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {154, 158, 114, 113, 121, 119}

$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{12\sqrt{2x-5}}+\frac{241\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{36\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```


```
[Out] (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/12 + (241*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(36*Sqrt[5 - 2*x
]) - (179*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(12*Sqrt[-5 + 2*x])
```


Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{12}\int \frac{\frac{441}{2}-482x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{241}{12}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx-\frac{1969}{24}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1-}$$

$$= \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{\left(179\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{12\sqrt{-5+2x}}-\frac{\left(241\sqrt{-5+2x}\right)}{12\sqrt{}}$$

$$= \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{241\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{36\sqrt{5-2x}}-\frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}}{1}$$

Mathematica [A] time = 0.183481, size = 115, normalized size = 0.88

$$\frac{-179\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+30\sqrt{2-3x}\sqrt{4x+1}(2x-5)+241\sqrt{66}\sqrt{5-2x}E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x}\right.\right.}{72\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (30*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 241*Sqrt[66]*Sqrt[5 - 2*x]*Ell
ipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 179*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(72*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.012, size = 140, normalized size = 1.1

$$-\frac{1}{1728\,x^3-5040\,x^2+1512\,x+720}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(537\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+1}\mathrm{EllipticF}\left(2/11\,\sqrt{22}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)*(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/72*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(537*11^(1/2)*(2-3*x)^(1/2)
*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-
482*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-3
3*x)^(1/2),1/2*I*2^(1/2))-720*x^3+2100*x^2-630*x-300)/(24*x^3-70*x^2+21*x+1
0)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5\,x+7)\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{8\,x^2-18\,x-5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 - 18*x
 - 5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\,(5x+7)}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.54 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\sin^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{2x-5}}$$

```
[Out] (Sqrt[11/2]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[1 + 4*x]/Sqrt[11]], 3])/(2*
Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.0148583, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {114, 113}

$$\frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\sin^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[11/2]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[1 + 4*x]/Sqrt[11]], 3])/(2*
Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{\sqrt{5-2x}\int \frac{\sqrt{\frac{8}{11}-\frac{12x}{11}}}{\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{\sqrt{2}\sqrt{-5+2x}}\\ &= \frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\sin^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{-5+2x}}\end{aligned}$$

Mathematica [B] time = 0.340403, size = 111, normalized size = 2.36

$$-\frac{\frac{2(2x-5)(3x-2)}{\sqrt{2x+\frac{1}{2}}}+\sqrt{11}\sqrt{\frac{2x-5}{4x+1}}\sqrt{\frac{3x-2}{4x+1}}(4x+1)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{4x+1}}\right)\middle|3\right)}{2\sqrt{2-3x}\sqrt{4x-10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -((2*(-5 + 2*x)*(-2 + 3*x))/Sqrt[1/2 + 2*x] + Sqrt[11]*Sqrt[(-5 + 2*x)/(1 +
 4*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*EllipticE[ArcSin[Sqrt[11/3]/Sqr
t[1 + 4*x]], 3])/(2*Sqrt[2 - 3*x]*Sqrt[-10 + 4*x])
```

Maple [C] time = 0.011, size = 55, normalized size = 1.2

$$\frac{\sqrt{11}}{2}\left(\mathrm{EllipticF}\left(\frac{2}{11}\sqrt{22-33\,x},\frac{i}{2}\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{2}{11}\sqrt{22-33\,x},\frac{i}{2}\sqrt{2}\right)\right)\sqrt{5-2\,x}\frac{1}{\sqrt{2\,x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/2*(EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-EllipticE(2/11*(22-33*x)
^(1/2),1/2*I*2^(1/2)))*(5-2*x)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{8\,x^2-18\,x-5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.55 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{3\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{2x-5}}$$

```
[Out] -(Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3]
)/(5*Sqrt[-5 + 2*x]) - (3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2
 - 3*x])/Sqrt[11]], -1/2])/(5*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.0957722, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {175, 121, 119, 168, 538, 537}

$$-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{3\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```


```
[Out] -(Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3]
)/(5*Sqrt[-5 + 2*x]) - (3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2
 - 3*x])/Sqrt[11]], -1/2])/(5*Sqrt[11]*Sqrt[-5 + 2*x])
```


Rule 175

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\left(\frac{3}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\right)+\frac{31}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x}$$

$$= -\left(\frac{62}{5}\text{Subst}\left(\int \frac{1}{\left(31-5x^2\right)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)\right)-\frac{\left(3\sqrt{\frac{2}{11}}\sqrt{5-\right.}{}$$

$$= -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}}-\frac{\left(62\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{}{\left(31-5x^2\right)\sqrt{}}\right.}{5\sqrt{-5+2}}$$

$$= -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}}-\frac{3\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right.}{5\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] time = 0.400946, size = 70, normalized size = 0.68

$$\frac{3\sqrt{5-2x}\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{5\sqrt{22x-55}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (3*Sqrt[5 - 2*x]*(EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + Ell
ipticPi[55/124, -ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(5*Sqrt[-55 +
22*x])
```

Maple [A] time = 0.015, size = 56, normalized size = 0.5

$$\frac{3\sqrt{11}}{55}\left(\mathrm{EllipticF}\left(\frac{2}{11}\sqrt{22-33x},\frac{i}{2}\sqrt{2}\right)-\mathrm{EllipticPi}\left(\frac{2}{11}\sqrt{22-33x},\frac{55}{124},\frac{i}{2}\sqrt{2}\right)\right)\sqrt{5-2x}\frac{1}{\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 3/55*(EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-EllipticPi(2/11*(22-33*
x)^(1/2),55/124,1/2*I*2^(1/2)))*(5-2*x)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{40x^3-34x^2-151x-35},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 - 34*x^2 - 151*
x - 35), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.56 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{115\sqrt{2x-5}}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}+\frac{2\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}}$$

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*(7 + 5*x)) + (2*Sqrt[1
1]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(897
*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*S
qrt[1 + 4*x]], 1/3])/(115*Sqrt[-5 + 2*x]) - (3571*Sqrt[5 - 2*x]*EllipticPi[
55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(92690*Sqrt[11]*Sqrt[-5
+ 2*x])
```

Rubi [A] time = 0.209679, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {177, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{115\sqrt{2x-5}}+\frac{2\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}}-\frac{3571}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```


```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*(7 + 5*x)) + (2*Sqrt[1
1]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(897
*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*S
qrt[1 + 4*x]], 1/3])/(115*Sqrt[-5 + 2*x]) - (3571*Sqrt[5 - 2*x]*EllipticPi[
55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(92690*Sqrt[11]*Sqrt[-5
+ 2*x])
```


Rule 177

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*e - a*f)*(b*g - a*h))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
```

```
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{-479+336x+120x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{1794}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{\frac{168}{5}+24x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1794} + \frac{3571\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(\ldots}}{8970}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{2}{299}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{6}{115}\int \frac{1}{\sqrt{2-3x}\sqrt{-5\ldots}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{115\sqrt{-5+2x}} - \frac{(3571\sqrt{5\ldots}}{}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} + \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}} - \frac{2\sqrt{\frac{6}{11}}\sqrt{5\ldots}}{}$$

Mathematica [A] time = 0.653042, size = 132, normalized size = 0.7

$$\frac{3\sqrt{55-22x}\left(14508\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+10713\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\ldots}{9176310\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) + 3*Sqrt[55 - 22
*x]*(-6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14508*Elli
pticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 10713*EllipticPi[55/124,
-ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(9176310*Sqrt[-5 + 2*x])
```

Maple [B] time = 0.016, size = 320, normalized size = 1.7

$$-\frac{1}{\left(73410480\,x^3-214113900\,x^2+64234170\,x+30587700\right)(7+5\,x)}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(72540\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^2/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/3058770*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(72540*11^(1/2)*(2-3*x
)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^
(1/2))*x-34100*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE
(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-53565*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)
^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*
x+101556*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*
(22-33*x)^(1/2),1/2*I*2^(1/2))-47740*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(
4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-74991*11^(1/2)*(
2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55
/124,1/2*I*2^(1/2))+409200*x^3-1193500*x^2+358050*x+170500)/(24*x^3-70*x^2+
21*x+10)/(7+5*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{200x^4+110x^3-993x^2-1232x-245}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4 + 110*x^3 - 99
3*x^2 - 1232*x - 245), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.57 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{13243\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{1065935\sqrt{66}\sqrt{2x-5}}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{33257172(5x+7)}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}+$$

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1794*(7 + 5*x)^2) - (26825
*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(33257172*(7 + 5*x)) + (5365*S
qrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])
/(16628586*Sqrt[5 - 2*x]) - (13243*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11
]*Sqrt[1 + 4*x]], 1/3])/(1065935*Sqrt[66]*Sqrt[-5 + 2*x]) - (16369941*Sqrt[
5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(343
6574440*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.310402, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {177, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{33257172(5x+7)}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}-\frac{13243\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{1065935\sqrt{66}\sqrt{2x-5}}+\frac{5365\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1794*(7 + 5*x)^2) - (26825
*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(33257172*(7 + 5*x)) + (5365*S
qrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])
/(16628586*Sqrt[5 - 2*x]) - (13243*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11
]*Sqrt[1 + 4*x]], 1/3])/(1065935*Sqrt[66]*Sqrt[-5 + 2*x]) - (16369941*Sqrt[
5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(343
6574440*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 177

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*e - a*f)*(b*g - a*h))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
```

```
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
```

```
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{\int \frac{-1063+1372x-120x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx}{3588}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{\int \frac{-7905051+40\cdots}{\sqrt{2-3x}\sqrt{-5\cdots}}}{19\cdots}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{\int \frac{\frac{1369224}{5}+38\cdots}{\sqrt{2-3x}\sqrt{-5+\cdots}}}{199543\cdots}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{5365\int \frac{\sqrt{-\cdots}}{\sqrt{2-3\cdots}}}{55428\cdots}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{\left(5456647\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} + \frac{5365\sqrt{11}\sqrt{-\cdots}}{\cdots}$$

Mathematica [A] time = 0.420694, size = 144, normalized size = 0.64

$$\frac{-\sqrt{55-22x}(5x+7)^2\left(-64043148\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+36589300E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-49109823\Pi\left(\cdots\right.\right.}{113406956520\sqrt{2x-5}(5x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(56093 + 26825*x) - Sqrt[55
- 22*x]*(7 + 5*x)^2*(36589300*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] - 64043148*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 491
09823*EllipticPi[55/124, -ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(1134
06956520*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Maple [B] time = 0.018, size = 461, normalized size = 2.1

$$-\frac{1}{\left(2721766956480\,x^3-7938486956400\,x^2+2381546086920\,x+1134069565200\right)(7+5\,x)^2}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^3/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/113406956520*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1601078700*11^(1
/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2
),1/2*I*2^(1/2))*x^2-914732500*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)
^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x^2-1227745575*11^(1/2
)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2)
,55/124,1/2*I*2^(1/2))*x^2+4483020360*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*
(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-2561251000*11
^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(
1/2),1/2*I*2^(1/2))*x-3437687610*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+
1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x+3138114252
*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x
)^(1/2),1/2*I*2^(1/2))-1792875700*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x
+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-2406381327*11^(1/2)
*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),
55/124,1/2*I*2^(1/2))+10976790000*x^4-9062381900*x^3-57342304250*x^2+246577
61150*x+9563856500)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{1000\,x^5+1950\,x^4-4195\,x^3-13111\,x^2-9849\,x-1715},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5 + 1950*x^4 -
4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^3\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.58 $\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=293

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{h(de-cf)}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\sqrt{f}\right.\right.}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] (2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d
*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((
d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) - (
2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*
g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sq
rt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt
[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] time = 0.504431, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {175, 121, 120, 169, 538, 537}

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\right.}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```


```
[Out] (2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d
*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((
d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) - (
2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*
g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sq
rt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt
[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```


Rule 175

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
```

```
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{d\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b}$$

$$= -\frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b} + \frac{\left(d\sqrt{\frac{d(e+fx)}{de-cf}}\right.}{}$$

$$= -\frac{\left(2(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b\sqrt{e+fx}} + \cdots$$

$$= \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{\left(2(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\right.}{}$$

$$= \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}}{}$$

Mathematica [C] time = 1.71334, size = 202, normalized size = 0.69

$$-\frac{2i\sqrt{c+dx}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)-\Pi\left(\frac{b(cf-de)}{(bc-ad)f};i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)\right)}{b\sqrt{e+fx}\sqrt{g+hx}\sqrt{\frac{f(c+dx)}{d(e+fx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*Sqrt[c + d*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*(EllipticF[I*ArcSinh[
Sqrt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - Ellipt
icPi[(b*(-(d*e) + c*f))/((b*c - a*d)*f), I*ArcSinh[Sqrt[(f*(c + d*x))/(d*e
- c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(b*Sqrt[(f*(c + d*x))/(d*(e +
f*x))]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] time = 0.043, size = 382, normalized size = 1.3

$$2\frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{fb\left(dfhx^3+cfhx^2+dehx^2+dfgx^2+cehx+cfgx+degx+ceg\right)}\sqrt{\frac{f\left(dx+c\right)}{cf-de}}\sqrt{-\frac{\left(hx+g\right)d}{ch-dg}}\sqrt{-\frac{\left(fx+e\right)d}{cf-de}}\left(\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/b*((d*x+c)*f/(c*f-d*e))^(1/2)
*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*(EllipticF(((d*x
+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*c*f-EllipticF(((d*x
+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*d*e-EllipticPi(((d*
x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^
(1/2))*c*f+EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),
((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*d*e)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.59 $\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=449

$$\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{h(de-cf)}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\right.}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{}}$$

```
[Out] (2*d*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(b*f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g +
 h*x]) + (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*
Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e +
f*x]*Sqrt[g + h*x]) - (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/(
(b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
- c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] time = 0.671968, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {179, 121, 120, 169, 538, 537, 114, 113}

$$\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\text{s}\right.}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```


```
[Out] (2*d*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(b*f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g +
 h*x]) + (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*
Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e +
f*x]*Sqrt[g + h*x]) - (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/(
(b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
- c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```


Rule 179

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
```

```
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \left(\frac{d(bc-ad)}{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} + \frac{(bc-ad)^2}{b^2(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} + \frac{d\sqrt{c+dx}}{b\sqrt{e+fx}\sqrt{g\cdots}}\right.$$

$$= \frac{d\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(d(bc-ad))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\cdots}}{b^2}$$

$$= -\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^2} + \frac{\left(d(bc-ad)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} - \frac{\left(2(bc-ad)^2\sqrt{\frac{d(e+fx)}{de-cf}}\cdots\right.}{\cdots}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} + \frac{2(bc-ad)\sqrt{-de+cf}\cdots}{\cdots}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} + \frac{2(bc-ad)\sqrt{-de+cf}\cdots}{\cdots}$$

Mathematica [C] time = 8.34992, size = 1195, normalized size = 2.66

$$-\frac{2\left(b^2d^2\sqrt{\frac{fg}{h}-e}he^3 - abd^2f\sqrt{\frac{fg}{h}-e}he^2 - b^2cdf\sqrt{\frac{fg}{h}-e}he^2 - 2b^2d^2\sqrt{\frac{fg}{h}-e}h(e+fx)e^2 - b^2d^2fg\sqrt{\frac{fg}{h}-e}e^2 + b^2d^2\sqrt{\frac{fg}{h}\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(-(b^2*d^2*e^2*f*g*Sqrt[-e + (f*g)/h]) + b^2*c*d*e*f^2*g*Sqrt[-e + (f*g
)/h] + a*b*d^2*e*f^2*g*Sqrt[-e + (f*g)/h] - a*b*c*d*f^3*g*Sqrt[-e + (f*g)/h
] + b^2*d^2*e^3*Sqrt[-e + (f*g)/h]*h - b^2*c*d*e^2*f*Sqrt[-e + (f*g)/h]*h -
 a*b*d^2*e^2*f*Sqrt[-e + (f*g)/h]*h + a*b*c*d*e*f^2*Sqrt[-e + (f*g)/h]*h +
b^2*d^2*e*f*g*Sqrt[-e + (f*g)/h]*(e + f*x) - a*b*d^2*f^2*g*Sqrt[-e + (f*g)/
h]*(e + f*x) - 2*b^2*d^2*e^2*Sqrt[-e + (f*g)/h]*h*(e + f*x) + b^2*c*d*e*f*S
qrt[-e + (f*g)/h]*h*(e + f*x) + 2*a*b*d^2*e*f*Sqrt[-e + (f*g)/h]*h*(e + f*x
) - a*b*c*d*f^2*Sqrt[-e + (f*g)/h]*h*(e + f*x) + b^2*d^2*e*Sqrt[-e + (f*g)/
h]*h*(e + f*x)^2 - a*b*d^2*f*Sqrt[-e + (f*g)/h]*h*(e + f*x)^2 + I*b*d^2*(b*
e - a*f)*(f*g - e*h)*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt
[(f*(g + h*x))/(h*(e + f*x))]*EllipticE[I*ArcSinh[Sqrt[-e + (f*g)/h]/Sqrt[e
 + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] - I*b*f*(a*d^2*(-(f*g) + e*h)
 + b*(d^2*e*g - 2*c*d*e*h + c^2*f*h))*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e
+ f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*EllipticF[I*ArcSinh[Sqrt[-e
+ (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] + I*b^2*c^2*
```

```
f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/
(h*(e + f*x))]*EllipticPi[((b*e - a*f)*h)/(b*(-(f*g) + e*h)), I*ArcSinh[Sqr
t[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] - (2*I)
*a*b*c*d*f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g
 + h*x))/(h*(e + f*x))]*EllipticPi[((b*e - a*f)*h)/(b*(-(f*g) + e*h)), I*Ar
cSinh[Sqrt[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))
] + I*a^2*d^2*f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[
(f*(g + h*x))/(h*(e + f*x))]*EllipticPi[((b*e - a*f)*h)/(b*(-(f*g) + e*h)),
 I*ArcSinh[Sqrt[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) +
e*h))]))/(b^2*f^2*(-(b*e) + a*f)*Sqrt[-e + (f*g)/h]*h*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x])
```

Maple [B] time = 0.024, size = 968, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] -2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/h/b^2*((d*x+c)*f/(c*f-d*e))^
(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*(EllipticF(
((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*c*d*f*h-Elli
pticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*d^2*e*
h-2*EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*
b*c^2*f*h+2*EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))
^(1/2))*b*c*d*e*h+EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h
-d*g))^(1/2))*b*c*d*f*g-EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/
f/(c*h-d*g))^(1/2))*b*d^2*e*g+EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d
*e)*h/f/(c*h-d*g))^(1/2))*b*c^2*f*h-EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),(
(c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c*d*e*h-EllipticE(((d*x+c)*f/(c*f-d*e))^(
1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c*d*f*g+EllipticE(((d*x+c)*f/(c*f-d
*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*d^2*e*g-EllipticPi(((d*x+c)*f
/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))
*a*c*d*f*h+EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),
((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*d^2*e*h+EllipticPi(((d*x+c)*f/(c*f-d*e))
^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c^2*f*h-
EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*
h/f/(c*h-d*g))^(1/2))*b*c*d*e*h)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c
*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.60 $\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=203

$$\frac{392989907\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{2016\sqrt{66}\sqrt{2x-5}}-\frac{25}{84}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{305}{24}\sqrt{2-3x}\sqrt{2x-\ldots}$$

```
[Out] (-120355*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/288 - (305*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 - (25*Sqrt[2 - 3*x]*Sqrt[-5 +
 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/84 - (5109835*Sqrt[11]*Sqrt[-5 + 2*x]*Elli
pticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(756*Sqrt[5 - 2*x]) + (392
989907*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(201
6*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.215818, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {167, 1600, 1615, 158, 114, 113, 121, 119}

$$-\frac{25}{84}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{305}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{120355}{288}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```


```
[Out] (-120355*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/288 - (305*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 - (25*Sqrt[2 - 3*x]*Sqrt[-5 +
 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/84 - (5109835*Sqrt[11]*Sqrt[-5 + 2*x]*Elli
pticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(756*Sqrt[5 - 2*x]) + (392
989907*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(201
6*Sqrt[66]*Sqrt[-5 + 2*x])
```


Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
```

```
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
```

```
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{1}{168}\int \frac{(7+5x)\left(48949+134855x+1\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}$$

$$= -\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)-\frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)-\frac{25}{84}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)-\frac{25}{84}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)-\frac{25}{84}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)-\frac{25}{84}$$

Mathematica [A] time = 0.496743, size = 125, normalized size = 0.62

$$\frac{392989907\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)-1650\sqrt{2-3x}\sqrt{4x+1}\left(1200x^3+10608x^2+50078x-2\right.}{133056\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-1650*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-210245 + 50078*x + 10608*x^2 + 1200*x^
3) - 449665480*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 +
4*x]], 1/3] + 392989907*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*
Sqrt[1 + 4*x]], 1/3])/(133056*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.024, size = 150, normalized size = 0.7

$$\frac{1}{3193344\,x^3-9313920\,x^2+2794176\,x+1330560}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4\,x+1}\left(1178969721\,\sqrt{11}\sqrt{2-3\,x}\sqrt{5-2\,x}\sqrt{4\,x+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^4/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/133056*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1178969721*11^(1/2)*(2-
3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I
*2^(1/2))-899330960*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*Elli
pticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-23760000*x^5-200138400*x^4-900068
400*x^3+4611000900*x^2-1569263850*x-693808500)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^4}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(625\,x^4+3500\,x^3+7350\,x^2+6860\,x+2401\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{24\,x^3-70\,x^2+21\,x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-(625*x^4 + 3500*x^3 + 7350*x^2 + 6860*x + 2401)*sqrt(4*x + 1)*sqr
t(2*x - 5)*sqrt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^4}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**4/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**4/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^4}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.61 $\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=165

$$\frac{2474201\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{216\sqrt{66}\sqrt{2x-5}}-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{2135}{108}\sqrt{2-3x}\sqrt{2x-5}\sqrt{}$$

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/108 - (5*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/12 - (487585*Sqrt[11]*Sqrt[-5 + 2*x]
*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1296*Sqrt[5 - 2*x])
+ (2474201*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/
(216*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.15253, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {167, 1615, 158, 114, 113, 121, 119}

$$-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{2135}{108}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{2474201\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{216\sqrt{66}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```



```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/108 - (5*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/12 - (487585*Sqrt[11]*Sqrt[-5 + 2*x]
*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1296*Sqrt[5 - 2*x])
+ (2474201*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/
(216*Sqrt[66]*Sqrt[-5 + 2*x])
```


Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1}{120}\int \frac{34985+104825x+85400x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{\int \frac{108}{\sqrt{2-}}}{}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{4875}{432}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{(2474}{}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{4875}{}$$

Mathematica [A] time = 0.346811, size = 120, normalized size = 0.73

$$\frac{4948402\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right) - 6600\sqrt{2-3x}\sqrt{4x+1}\left(18x^2+151x-490\right) - 5363435\sqrt{66}}{28512\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-6600*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-490 + 151*x + 18*x^2) - 5363435*Sqrt[6
6]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 4948402
*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
28512*Sqrt[-5 + 2*x])
```

Maple [A] time = 0.017, size = 145, normalized size = 0.9

$$\frac{1}{342144x^3-997920x^2+299376x+142560}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\left(7422603\sqrt{11}\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/14256*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(7422603*11^(1/2)*(2-3*x)
^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(
1/2))-5363435*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(
2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-712800*x^4-5682600*x^3+22014300*x^2-708
8400*x-3234000)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^3}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(125\,x^{3}+525\,x^{2}+735\,x+343\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{24\,x^{3}-70\,x^{2}+21\,x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-(125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(5x+7)^3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**3/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(5\,x+7)^3}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.62 $\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=129

$$\frac{24353\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{36\sqrt{66}\sqrt{2x-5}}-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{2135\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\right.}{108\sqrt{5-2x}}$$

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/36 - (2135*Sqrt[11]*Sqrt[-
5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(108*Sqrt[5 -
 2*x]) + (24353*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(36*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.0627385, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {167, 24, 158, 114, 113, 121, 119}

$$-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{24353\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{36\sqrt{66}\sqrt{2x-5}}-\frac{2135\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{108\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```



```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/36 - (2135*Sqrt[11]*Sqrt[-
5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(108*Sqrt[5 -
 2*x]) + (24353*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(36*Sqrt[66]*Sqrt[-5 + 2*x])
```


Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{72}\int \frac{21021+74795x+42700x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$$

$$= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{\int \frac{75075+213500x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1800}$$

$$= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{2135}{36}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx+\frac{24353}{72}\int \frac{}{\sqrt{2-3x}\,\sqrt{}}$$

$$= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{\left(24353\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{36\sqrt{22}\sqrt{-5+2x}}+\frac{\left(2135\sqrt{\right.}}{}$$

$$= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{2135\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)}{108\sqrt{5-2x}}+\frac{2435}{}$$

Mathematica [A] time = 0.238652, size = 115, normalized size = 0.89

$$\frac{24353\sqrt{66}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+1650\sqrt{2-3x}\sqrt{4x+1}(5-2x)-23485\sqrt{66}\sqrt{5-2x}E\left(\sin^{-1}\left(\right.\right.}{2376\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (1650*Sqrt[2 - 3*x]*(5 - 2*x)*Sqrt[1 + 4*x] - 23485*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 24353*Sqrt[66]*Sqrt[5 -
2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2376*Sqrt[-5 + 2*x]
)
```

Maple [A] time = 0.017, size = 140, normalized size = 1.1

$$\frac{1}{57024x^3-166320x^2+49896x+23760}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\left(73059\sqrt{11}\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/2376*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(73059*11^(1/2)*(2-3*x)^(1
/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2
))-46970*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*
(22-33*x)^(1/2),1/2*I*2^(1/2))-39600*x^3+115500*x^2-34650*x-16500)/(24*x^3-
70*x^2+21*x+10)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^2}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(25x^2+70x+49\right)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{24x^3-70x^2+21x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-(25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(
24*x^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^2}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**2/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^2}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.63 $\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=98

$$\frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}}$$

```
[Out] (-5*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(6*Sqrt[5 - 2*x]) + (13*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqr
t[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x]
```

Rubi [A] time = 0.0368844, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {158, 114, 113, 121, 119}

$$\frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-5*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(6*Sqrt[5 - 2*x]) + (13*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqr
t[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{2}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx + \frac{39}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{\left(39\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{\sqrt{22}\sqrt{-5+2x}} + \frac{\left(5\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{2\sqrt{5-2x}}$$
$$= -\frac{5\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}} + \frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right.}{\sqrt{-5+2x}}$$

Mathematica [A] time = 0.451645, size = 187, normalized size = 1.91

$$\frac{-124\sqrt{22}\sqrt{\frac{2x-5}{4x+1}}\sqrt{\frac{3x-2}{4x+1}}(4x+1)^2\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{4x+1}}\right),3\right)+220\left(6x^2-19x+10\right)\sqrt{4x+1}+55\sqrt{22}\sqrt{\frac{2x-5}{4x+1}}\sqrt{\frac{3x-2}{4x+1}}(4x+}{132\sqrt{2-3x}\sqrt{2x-5}(4x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (220*Sqrt[1 + 4*x]*(10 - 19*x + 6*x^2) + 55*Sqrt[22]*Sqrt[(-5 + 2*x)/(1 + 4
*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*EllipticE[ArcSin[Sqrt[11/3]/Sqr
t[1 + 4*x]], 3] - 124*Sqrt[22]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*Sqrt[(-2 + 3*x)/(
1 + 4*x)]*(1 + 4*x)^2*EllipticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(132*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*(1 + 4*x))
```

Maple [A] time = 0.015, size = 57, normalized size = 0.6

$$-\frac{\sqrt{11}}{66}\left(117\,\text{EllipticF}\left(2/11\,\sqrt{22-33\,x},i/2\sqrt{2}\right)-55\,\text{EllipticE}\left(2/11\,\sqrt{22-33\,x},i/2\sqrt{2}\right)\right)\sqrt{5-2\,x}\frac{1}{\sqrt{2\,x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/66*(117*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-55*EllipticE(2/11*
(22-33*x)^(1/2),1/2*I*2^(1/2)))*(5-2*x)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x+7}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{24\,x^{3}-70\,x^{2}+21\,x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3 - 70
*x^2 + 21*x + 10), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x+7}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x+7}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.64 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}$$

```
[Out] (Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/Sqrt[-5 + 2*x]
```

Rubi [A] time = 0.0139663, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {121, 119}

$$\frac{\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/Sqrt[-5 + 2*x]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{\left(\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{\sqrt{-5+2x}}\\ &= \frac{\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{\sqrt{-5+2x}}\end{aligned}$$

Mathematica [A] time = 0.10906, size = 79, normalized size = 1.65

$$-\frac{\sqrt{\frac{3x-2}{4x+1}}(4x+1)\sqrt{\frac{4x-10}{44x+11}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{4x+1}}\right),3\right)}{\sqrt{2-3x}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -((Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*Sqrt[(-10 + 4*x)/(11 + 44*x)]*Ellip
ticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]))
```

Maple [C] time = 0.013, size = 36, normalized size = 0.8

$$-\frac{\sqrt{11}}{11}\text{EllipticF}\left(\frac{2}{11}\sqrt{22-33x},\frac{i}{2}\sqrt{2}\right)\sqrt{5-2x}\frac{1}{\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/11*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/
(2*x-5)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{24x^3-70x^2+21x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*
x + 10), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.65 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$

Optimal. Leaf size=51

$$-\frac{3\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{2x-5}}$$

```
[Out] (-3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(31*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.0700453, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {168, 538, 537}

$$-\frac{3\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (-3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(31*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{\sqrt{-5+2x}}$$

$$= -\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124}; \sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle| -\frac{1}{2}\right)}{31\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] time = 0.468092, size = 99, normalized size = 1.94

$$-\frac{3(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{11}}{2\sqrt{2-3x}}\right), -2\right) + \Pi\left(\frac{124}{55}; -\sin^{-1}\left(\frac{\sqrt{11}}{2\sqrt{2-3x}}\right)\middle| -2\right)\right)}{31\sqrt{4x+1}\sqrt{11x-\frac{55}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (-3*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(EllipticF[ArcSin[Sqrt
[11]/(2*Sqrt[2 - 3*x])], -2] + EllipticPi[124/55, -ArcSin[Sqrt[11]/(2*Sqrt[
2 - 3*x])], -2]))/(31*Sqrt[1 + 4*x]*Sqrt[-55/2 + 11*x])
```

Maple [A] time = 0.016, size = 37, normalized size = 0.7

$$-\frac{3\sqrt{11}}{341}\text{EllipticPi}\left(\frac{2}{11}\sqrt{22-33\,x}, \frac{55}{124}, \frac{i}{2}\sqrt{2}\right)\sqrt{5-2\,x}\frac{1}{\sqrt{2\,x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -3/341*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*
11^(1/2)/(2*x-5)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4 - 182*x^3 - 3
85*x^2 + 197*x + 70), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.66 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{713\sqrt{2x-5}}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}+\frac{10\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\right.}{27807\sqrt{5-2x}}$$

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)) + (10*Sq
rt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/
(27807*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3])/(713*Sqrt[-5 + 2*x]) - (8953*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(574678*Sqrt[11]*S
qrt[-5 + 2*x])
```

Rubi [A] time = 0.220147, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {172, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{713\sqrt{2x-5}}+\frac{10\sqrt{11}\sqrt{2x-5}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27807\sqrt{5-2x}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)) + (10*Sq
rt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/
(27807*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3])/(713*Sqrt[-5 + 2*x]) - (8953*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(574678*Sqrt[11]*S
qrt[-5 + 2*x])
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b^2*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*
(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
```

```
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{\int \frac{7777-1680x-600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{55614}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{\int \frac{-168-120x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{55614} + \frac{8953\int \frac{}{\sqrt{2-3x}}}{}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} - \frac{10\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{9269} - \frac{6}{713}\int \frac{}{\sqrt{2-3x}}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,d}{713\sqrt{-5+2x}}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{10\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right.}{27807\sqrt{5-2x}}$$

Mathematica [A] time = 0.634306, size = 132, normalized size = 0.7

$$\frac{3\sqrt{55-22x}\left(14508\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)+26859\Pi\left(\frac{55}{124};-\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\right.\right.}{56893122\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) + 3*Sqrt[55 - 22
*x]*(-6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14508*Elli
pticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 26859*EllipticPi[55/124,
-ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(56893122*Sqrt[-5 + 2*x])
```

Maple [B] time = 0.019, size = 320, normalized size = 1.7

$$-\frac{1}{\left(455144976x^3-1327506180x^2+398251854x+189643740\right)(7+5x)}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\left(72540\sqrt{11}\sqrt{2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^2/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/18964374*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(72540*11^(1/2)*(2-3*
x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2
^(1/2))*x-34100*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*Elliptic
E(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-134295*11^(1/2)*(2-3*x)^(1/2)*(5-2*
x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2)
)*x+101556*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/1
1*(22-33*x)^(1/2),1/2*I*2^(1/2))-47740*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)
*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-188013*11^(1/2
)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2)
,55/124,1/2*I*2^(1/2))+409200*x^3-1193500*x^2+358050*x+170500)/(24*x^3-70*x
^2+21*x+10)/(7+5*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{600\,x^5-70\,x^4-3199\,x^3-1710\,x^2+1729\,x+490}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5 - 70*x^4 - 31
99*x^3 - 1710*x^2 + 1729*x + 490), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.67 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3} dx$

Optimal. Leaf size=225

$$-\frac{24007\sqrt{5-2x}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{6608797\sqrt{66}\sqrt{2x-5}}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1030972332(5x+7)}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}+$$

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(55614*(7 + 5*x)^2) - (223
825*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1030972332*(7 + 5*x)) + (4
4765*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
-1/2])/(515486166*Sqrt[5 - 2*x]) - (24007*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sq
rt[3/11]*Sqrt[1 + 4*x]], 1/3])/(6608797*Sqrt[66]*Sqrt[-5 + 2*x]) - (4849330
5*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2
])/(21306761528*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.309706, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {172, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1030972332(5x+7)}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}-\frac{24007\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{6608797\sqrt{66}\sqrt{2x-5}}+\frac{44765\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```


```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(55614*(7 + 5*x)^2) - (223
825*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1030972332*(7 + 5*x)) + (4
4765*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
-1/2])/(515486166*Sqrt[5 - 2*x]) - (24007*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sq
rt[3/11]*Sqrt[1 + 4*x]], 1/3])/(6608797*Sqrt[66]*Sqrt[-5 + 2*x]) - (4849330
5*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2
])/(21306761528*Sqrt[11]*Sqrt[-5 + 2*x])
```


Rule 172

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b^2*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*
(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
```

```
1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
```

```
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} + \frac{\int \frac{16079-6860x+600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx}{111228}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} + \frac{\int 123\cdots}{\cdots}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} + \frac{\int \frac{-3\cdots}{\sqrt{2}\cdots}}{\cdots}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} - \frac{4476\cdots}{\cdots}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} - \frac{(161\cdots}{\cdots}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} + \frac{4476\cdots}{\cdots}$$

Mathematica [A] time = 0.438753, size = 147, normalized size = 0.65

$$\frac{-\sqrt{55-22x}(5x+7)^2\left(61059460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-3\left(38699284\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+48493305\Pi\left(\frac{5}{12}\cdots\right.\right.\right.}{703123130424\sqrt{2x-5}(5x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(81209 + 44765*x) - Sqrt[55
- 22*x]*(7 + 5*x)^2*(61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] - 3*(38699284*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
48493305*EllipticPi[55/124, -ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])))/(
703123130424*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Maple [B] time = 0.02, size = 461, normalized size = 2.1

$$-\frac{1}{\left(16874955130176\,x^3-49218619129680\,x^2+14765585738904\,x+7031231304240\right)(7+5\,x)^2}\sqrt{2-3\,x}\sqrt{2\,x-5}\sqrt{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^3/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/703123130424*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(2902446300*11^(1
/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2
),1/2*I*2^(1/2))*x^2-1526486500*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1
)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x^2-3636997875*11^(1/
2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2
),55/124,1/2*I*2^(1/2))*x^2+8126849640*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)
*(4*x+1)^(1/2)*EllipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))*x-4274162200*1
1^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^
(1/2),1/2*I*2^(1/2))*x-10183594050*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*
x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2),55/124,1/2*I*2^(1/2))*x+56887947
48*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(22-33
*x)^(1/2),1/2*I*2^(1/2))-2991913540*11^(1/2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4
*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-7128515835*11^(1/
2)*(2-3*x)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(22-33*x)^(1/2
),55/124,1/2*I*2^(1/2))+18317838000*x^4-20196304700*x^3-80894833250*x^2+367
09314950*x+13846134500)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{3000\,x^6+3850\,x^5-16485\,x^4-30943\,x^3-3325\,x^2+14553\,x+3430},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^6 + 3850*x^5 -
 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)^3\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.68 $\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=137

$$\frac{2i\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{g+hx}\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

```
[Out] (2*Sqrt[-(f*g) + e*h]*i*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h
*x])
```

Rubi [A] time = 0.0612999, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 114, 113}

$$\frac{2i\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{g+hx}\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(f*g) + e*h]*i*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h
*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
```

```
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\int \frac{68c+68dx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = 68\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\left(68\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\right)\int \frac{\sqrt{\frac{cf}{-de+cf}+\frac{dfx}{-de+cf}}}{\sqrt{e+fx}\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}\,dx}{\sqrt{\frac{f(c+dx)}{-de+cf}}\sqrt{g+hx}}$$

$$= \frac{136\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$

Mathematica [C] time = 0.586411, size = 180, normalized size = 1.31

$$-\frac{2ii\sqrt{c+dx}\sqrt{g+hx}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)\right)}{h\sqrt{e+fx}\sqrt{\frac{f(c+dx)}{d(e+fx)}}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*i*Sqrt[c + d*x]*Sqrt[g + h*x]*(EllipticE[I*ArcSinh[Sqrt[(f*(c + d*x
))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - EllipticF[I*ArcSinh[Sq
rt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(h*Sqrt[
(f*(c + d*x))/(d*(e + f*x))]*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
```

Maple [B] time = 0.022, size = 552, normalized size = 4.

$$2\frac{i\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{dfh\left(dfhx^3+cfhx^2+dehx^2+dfgx^2+cehx+cfgx+degx+ceg\right)}\left(\text{EllipticF}\left(\sqrt{\frac{(dx+c)f}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)c^2fh-\text{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*i*(EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))
*c^2*f*h-EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1
/2))*c*d*e*h-EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g)
)^(1/2))*c*d*f*g+EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-
d*g))^(1/2))*d^2*e*g-EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(
c*h-d*g))^(1/2))*c^2*f*h+EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h
/f/(c*h-d*g))^(1/2))*c*d*e*h+EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*
e)*h/f/(c*h-d*g))^(1/2))*c*d*f*g-EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*
f-d*e)*h/f/(c*h-d*g))^(1/2))*d^2*e*g)/d*(-(f*x+e)*d/(c*f-d*e))^(1/2)*(-(h*x
+g)*d/(c*h-d*g))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)/h/f*(d*x+c)^(1/2)*(f*x+e
```

```
)^(1/2)*(h*x+g)^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dix + ci}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}i}{fhx^2 + eg + \left(fg + eh\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*i/(f*h*x^2 + e*g + (f*g
+ e*h)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$i \int \frac{\sqrt{c + dx}}{\sqrt{e + fx}\sqrt{g + hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] i*Integral(sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dix + ci}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.69 $\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=284

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{\cdots}}{\sqrt{cf\cdots}}\cdots\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] (2*b*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ellip
ticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f
*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
 - (2*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*S
qrt[g + h*x])
```

Rubi [A] time = 0.168916, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {158, 114, 113, 121, 120}

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de\cdots}{f(\cdots}\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ellip
ticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f
*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
 - (2*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*S
qrt[g + h*x])
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{h} + \frac{(-bg+ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{h}$$

$$= \frac{\left((-bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{h\sqrt{e+fx}} + \frac{\left(b\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \cdots}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}\cdots}$$

$$= \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{\left((-bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\cdots\right)\cdots}{\cdots}$$

$$= \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(bg-ah)\cdots}{\cdots}$$

Mathematica [C] time = 1.91082, size = 319, normalized size = 1.12

$$-\frac{2\left(idh(c+dx)^{3/2}(be-af)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{de}{f}-c}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)-bd^2(e+fx)(g+hx)\sqrt{\frac{de}{f}-c}-ib\cdots\right)}{d^2fh\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\sqrt{\frac{de}{f}-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(-(b*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*b*(d*e - c*f)*h*(c
 + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c +
d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*
h)/(d*e*h - c*f*h)] + I*d*(b*e - a*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/
(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-
c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*Sqrt[-
c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [B] time = 0.033, size = 559, normalized size = 2.

$$2\frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{d^2fh\left(dfhx^3+cfhx^2+dehx^2+dfgx^2+cehx+cfgx+degx+ceg\right)}\left(\mathrm{EllipticF}\left(\sqrt{\frac{(dx+c)f}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)acdfh-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a
*c*d*f*h-EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1
/2))*a*d^2*e*h-EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*
g))^(1/2))*b*c*d*f*g+EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(
c*h-d*g))^(1/2))*b*d^2*e*g-EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)
*h/f/(c*h-d*g))^(1/2))*b*c^2*f*h+EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*
f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c*d*e*h+EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2
),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c*d*f*g-EllipticE(((d*x+c)*f/(c*f-d*e)
)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*d^2*e*g)*(-(f*x+e)*d/(c*f-d*e))^
(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*((d*x+c)*f/(c*f-d*e))^(1/2)/h/f/d^2*(d*x
+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{dfhx^3+ceg+\left(dfg+\left(de+cf\right)h\right)x^2+\left(ceh+\left(de+cf\right)g\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(d*f*h*x^3 + c
*e*g + (d*f*g + (d*e + c*f)*h)*x^2 + (c*e*h + (d*e + c*f)*g)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.70 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)}$$

```
[Out] (-2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c
 - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] time = 0.37394, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {169, 538, 537}

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```


```
[Out] (-2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c
 - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```


Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{e+fx}}$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}\,dx, x, \sqrt{c+\right.}{\sqrt{e+fx}\sqrt{g+hx}}$$

$$= -\frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] time = 1.36514, size = 225, normalized size = 1.36

$$\frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{de}{f}-c}}{\sqrt{c+dx}}\right), \frac{dfg-cfh}{deh-cfh}\right)-\Pi\left(\frac{(bc-ad)f}{b(cf-de)}; i\sinh^{-1}\left(\frac{\sqrt{\frac{de}{f}-c}}{\sqrt{c+dx}}\right)|\frac{dfg-cfh}{deh-cfh}\right)\right)}{\sqrt{e+fx}\sqrt{g+hx}(ad-bc)\sqrt{\frac{de}{f}-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g -
c*f*h)/(d*e*h - c*f*h)] - EllipticPi[((b*c - a*d)*f)/(b*(-(d*e) + c*f)), I*
ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]
))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] time = 0.026, size = 223, normalized size = 1.4

$$2\,\frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}\left(cf-de\right)}{(ad-bc)\,f\left(dfhx^3+cfhx^2+dehx^2+dfgx^2+cehx+cfgx+degx+ceg\right)}\sqrt{\frac{(dx+c)\,f}{cf-de}}\sqrt{-\frac{\left(hx+g\right)d}{ch-dg}}\sqrt{-\frac{\left(fx+e\right)d}{cf-de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f*((d*x+c)*f/(c*f-d*e))^(1/2)*(
-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)*EllipticPi(((d*x+c
)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/
2))*(c*f-d*e)/(a*d-b*c)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*
f*g*x+d*e*g*x+c*e*g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.71 $\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=393

$$\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{\sqrt{c+dx}(bc-ad)(de-cf)(dg-ch)}-\frac{2d\sqrt{h}\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{\sqrt{g+hx}(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}}-\frac{2b\sqrt{cf-de}\ldots}{}$$

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*Sq
rt[c + d*x]) - (2*d*Sqrt[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h
*x))/(f*g - e*h)]*EllipticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*
h]], -((d*(f*g - e*h))/((d*e - c*f)*h))])/((b*c - a*d)*(d*e - c*f)*(d*g - c
*h)*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*b*Sqrt[-(d*e) +
c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*Ellipt
icPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^2*Sqrt[f]*
Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] time = 0.623636, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {179, 104, 21, 114, 113, 169, 538, 537}

$$\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{\sqrt{c+dx}(bc-ad)(de-cf)(dg-ch)}-\frac{2d\sqrt{h}\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{\sqrt{g+hx}(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}}-\frac{2b\sqrt{cf-de}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```


```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*Sq
rt[c + d*x]) - (2*d*Sqrt[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h
*x))/(f*g - e*h)]*EllipticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*
h]], -((d*(f*g - e*h))/((d*e - c*f)*h))])/((b*c - a*d)*(d*e - c*f)*(d*g - c
*h)*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*b*Sqrt[-(d*e) +
c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*Ellipt
icPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^2*Sqrt[f]*
Sqrt[e + f*x]*Sqrt[g + h*x])
```


Rule 179

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx = \int \left(-\frac{d}{(bc-ad)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}} + \frac{b}{(bc-ad)(a+bx)\sqrt{c+dx}\sqrt{e}}\right.$$

$$= \frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{bc-ad} - \frac{d\int \frac{1}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{bc-ad}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\right.}{bc-ad}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(dfh)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}dx}{(bc-ad)(de-cf)(dg-ch)} - \frac{\left(2b\sqrt{\right.}{}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{\left(2b\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int\right.}{(bc-ad}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2d\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}}{(bc-ad)(de-cf)(dg-c}$$

Mathematica [C] time = 4.38364, size = 321, normalized size = 0.82

$$\frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left((adf-2bcf+bde)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{dg}{h}-c}}{\sqrt{c+dx}}\right),\frac{deh-cfh}{dfg-cfh}\right)+f(bc-ad)E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{dg}{h}-c}}{\sqrt{c+dx}}\right.\right.\right.}{\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2(cf-de)\sqrt{\frac{dg}{h}-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*((b*c - a*d)*f*EllipticE[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*
x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] + (b*d*e - 2*b*c*f + a*d*f)*EllipticF
[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*
h)] + b*(-(d*e) + c*f)*EllipticPi[((b*c - a*d)*h)/(b*(-(d*g) + c*h)), I*Arc
Sinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/
((b*c - a*d)^2*(-(d*e) + c*f)*Sqrt[-c + (d*g)/h]*Sqrt[e + f*x]*Sqrt[g + h*x
])
```

Maple [B] time = 0.08, size = 2842, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)*(-EllipticF(((d*x+c)*f/(c*f-d*e
))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*c*d^2*e*f*h*((d*x+c)*f/(c*f-d*e
))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)+Elliptic
F(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c^2*d*e*f*
h*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f
-d*e))^(1/2)-EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g)
)^(1/2))*b*c*d^2*e*f*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(
1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)+EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c
*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*c*d^2*e*f*h*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(
h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)-EllipticE(((d*x+c)*f
/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c^2*d*e*f*h*((d*x+c)*f
/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)
+EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c
*d^2*e*f*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+
e)*d/(c*f-d*e))^(1/2)+2*EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b
/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c^2*d*e*f*h*((d*x+c)*f/(c*f
-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)-2*El
lipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/
f/(c*h-d*g))^(1/2))*b*c*d^2*e*f*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(
c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)+x*b*c*d^2*f^2*g+EllipticE(((d*
x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c^3*f^2*h*((d*x+
c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(
1/2)-EllipticPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-
d*e)*h/f/(c*h-d*g))^(1/2))*b*c^3*f^2*h*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g
)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)+EllipticPi(((d*x+c)*f/(c*
f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b*d
^3*e^2*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)
*d/(c*f-d*e))^(1/2)-x^2*a*d^3*f^2*h-a*d^3*e*f*g+x*b*c*d^2*e*f*h+b*c*d^2*e*f
*g-EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b
*c^3*f^2*h*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+
e)*d/(c*f-d*e))^(1/2)-x*a*d^3*e*f*h+x^2*b*c*d^2*f^2*h-x*a*d^3*f^2*g-Ellipti
cPi(((d*x+c)*f/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*
h-d*g))^(1/2))*b*c*d^2*e^2*h*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d
*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)+EllipticF(((d*x+c)*f/(c*f-d*e))^(1/
2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*c^2*d*f^2*h*((d*x+c)*f/(c*f-d*e))^(1/
2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)-EllipticF(((d*
x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*c*d^2*f^2*g*((d*
x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))
^(1/2)+EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2
))*a*d^3*e*f*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(
f*x+e)*d/(c*f-d*e))^(1/2)+EllipticF(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*
h/f/(c*h-d*g))^(1/2))*b*c^2*d*f^2*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d
/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)-EllipticE(((d*x+c)*f/(c*f-d*
e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*c^2*d*f^2*h*((d*x+c)*f/(c*f-d*
e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)+Ellipti
cE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*c*d^2*f^2
*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*
f-d*e))^(1/2)-EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*f-d*e)*h/f/(c*h-d*g
))^(1/2))*a*d^3*e*f*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1
/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)-EllipticE(((d*x+c)*f/(c*f-d*e))^(1/2),((c*
f-d*e)*h/f/(c*h-d*g))^(1/2))*b*c^2*d*f^2*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h
*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)*d/(c*f-d*e))^(1/2)+EllipticPi(((d*x+c)*f
/(c*f-d*e))^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))
*b*c^2*d*f^2*g*((d*x+c)*f/(c*f-d*e))^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(
f*x+e)*d/(c*f-d*e))^(1/2))/f/(a*d-b*c)^2/(c*h-d*g)/(c*f-d*e)/(d*f*h*x^3+c*f
*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\,(c+dx)^{\frac{3}{2}}\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.72 $\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=875

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)b^2}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2d\sqrt{h}\sqrt{eh-fg}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{}}{\sqrt{eh}}\right.\right.}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+d}{de-c}}}$$

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)*(d*g - c*h)*
(c + d*x)^(3/2)) + (2*b*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(d*
e - c*f)*(d*g - c*h)*Sqrt[c + d*x]) - (4*d^2*(d*f*g + d*e*h - 2*c*f*h)*Sqrt
[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)^2*(d*g - c*h)^2*Sqrt[c
+ d*x]) + (4*d*Sqrt[f]*(d*f*g + d*e*h - 2*c*f*h)*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) +
c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2
)*(d*g - c*h)^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*b*d*Sqr
t[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/((b*c - a*d)^2*(d*e - c*f)*(d*g - c*h)*Sqrt[-((f*(c + d*
x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*Sqrt[f]*(2*d*f*g + d*e*h - 3*c*f*h)*S
qrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[Ar
cSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
 c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*Sqrt[e + f*x]*Sqrt
[g + h*x]) - (2*b^2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)),
ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g
 - c*h))])/((b*c - a*d)^3*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] time = 1.33919, antiderivative size = 875, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {179, 104, 152, 158, 114, 113, 121, 120, 21, 169, 538, 537}

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)b^2}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2d\sqrt{h}\sqrt{eh-fg}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{}}{\sqrt{eh}}\right.\right.}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+d}{de-c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)*(d*g - c*h)*
(c + d*x)^(3/2)) + (2*b*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(d*
e - c*f)*(d*g - c*h)*Sqrt[c + d*x]) - (4*d^2*(d*f*g + d*e*h - 2*c*f*h)*Sqrt
[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)^2*(d*g - c*h)^2*Sqrt[c
+ d*x]) + (4*d*Sqrt[f]*(d*f*g + d*e*h - 2*c*f*h)*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) +
c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2
)*(d*g - c*h)^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*b*d*Sqr
t[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/((b*c - a*d)^2*(d*e - c*f)*(d*g - c*h)*Sqrt[-((f*(c + d*
x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*Sqrt[f]*(2*d*f*g + d*e*h - 3*c*f*h)*S
qrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[Ar
cSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
```

```
 c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*Sqrt[e + f*x]*Sqrt
[g + h*x]) - (2*b^2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)),
ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g
 - c*h))])/((b*c - a*d)^3*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 179

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
```

```
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \left(-\frac{d}{(bc-ad)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}} - \frac{bd}{(bc-ad)^2(c+dx)^{3/2}\sqrt{e+fx}\right.$$

$$= \frac{b^2\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2} - \frac{(bd)\int \frac{1}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{}{(c+dx)^5}}{b}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c}}$$

Mathematica [C] time = 17.4941, size = 12191, normalized size = 13.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.257, size = 17330, normalized size = 19.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)^{\frac{5}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)^{\frac{5}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.73 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rubi [A] time = 0.159653, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {168, 538, 537}

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx, x, \sqrt{1-fx}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f}\right)f}}}\,dx, x, \sqrt{1-fx}\right)}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

Mathematica [C] time = 0.868142, size = 203, normalized size = 2.74

$$\frac{2i(c+dx)\sqrt{\frac{d(fx-1)}{f(c+dx)}}\sqrt{\frac{dfx+d}{cf+dfx}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cf+d}{f}}}{\sqrt{c+dx}}\right),\frac{cf-d}{cf+d}\right)-\Pi\left(\frac{bcf-adf}{bd+bcf};i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{cf-d}{d+cf}\right)\right)}{\sqrt{1-f^2x^2}\sqrt{-\frac{cf+d}{f}}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Maple [B] time = 0.097, size = 184, normalized size = 2.5

$$-2\,\frac{\left(cf-d\right)\sqrt{fx+1}\sqrt{-fx+1}\sqrt{dx+c}}{f\left(ad-bc\right)\left(df^2x^3+cf^2x^2-dx-c\right)}\text{EllipticPi}\left(\sqrt{\frac{(dx+c)\,f}{cf-d}},-\frac{b\left(cf-d\right)}{f\left(ad-bc\right)},\sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{\left(fx+1\right)d}{cf-d}}\sqrt{-\frac{\left(fx-1\right)d}{cf+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)*f/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)*d/(c*f-d))^(1/2)*(-(f*x-1)*d/(c*f+d))^(1/2)*
((d*x+c)*f/(c*f-d))^(1/2)*(f*x+1)^(1/2)*(-f*x+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d
-b*c)/(d*f^2*x^3+c*f^2*x^2-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+1}\sqrt{-fx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{-fx+1}\sqrt{fx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f*x+1)**(1/2)/(f*x+1)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f*x + 1)*sqrt(f*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+1}\sqrt{-fx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

3.74 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rubi [A] time = 0.175884, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {932, 168, 538, 537}

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```


```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```


Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx &= \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx,x,\sqrt{1-fx}\right)\right) \\
&= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f}\right)f}}}\,dx,x,\sqrt{1-fx}\right)}{\sqrt{c+dx}} \\
&= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}
\end{aligned}$$

Mathematica [C] time = 0.102766, size = 203, normalized size = 2.74

$$\frac{2i(c+dx)\sqrt{\frac{d(fx-1)}{f(c+dx)}}\sqrt{\frac{dfx+d}{cf+dfx}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cf+d}{f}}}{\sqrt{c+dx}}\right),\frac{cf-d}{cf+d}\right)-\Pi\left(\frac{bcf-adf}{bd+bcf};i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{cf-d}{d+cf}\right)\right)}{\sqrt{1-f^2x^2}\sqrt{-\frac{cf+d}{f}}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Maple [B] time = 0.025, size = 181, normalized size = 2.5

$$-2\,\frac{\left(cf-d\right)\sqrt{-f^2x^2+1}\sqrt{dx+c}}{\left(ad-bc\right)f\left(df^2x^3+cf^2x^2-dx-c\right)}\text{EllipticPi}\left(\sqrt{\frac{(dx+c)\,f}{cf-d}},-\frac{\left(cf-d\right)b}{\left(ad-bc\right)f},\sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{\left(fx+1\right)d}{cf-d}}\sqrt{-\frac{\left(fx-1\right)}{cf+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)*f/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)*d/(c*f-d))^(1/2)*(-(f*x-1)*d/(c*f+d))^(1/2)*
((d*x+c)*f/(c*f-d))^(1/2)*(-f^2*x^2+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d-b*c)/(d*f
^2*x^3+c*f^2*x^2-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-f^2x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(fx-1\right)\left(fx+1\right)}\,(a+bx)\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(f*x - 1)*(f*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-f^2x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

3.75 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rubi [A] time = 0.160048, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {168, 538, 537}

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\,dx, x, \sqrt{1-f^2x}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f^2}\right)f^2}}}\,dx, x, \sqrt{1-f^2x}\right)}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{\left(b+af^2\right)\sqrt{c+dx}}$$

Mathematica [C] time = 0.812583, size = 218, normalized size = 2.53

$$\frac{2i(c+dx)\sqrt{\frac{d\left(f^2x-1\right)}{f^2(c+dx)}}\sqrt{\frac{d\left(f^2x+1\right)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right),\frac{cf^2-d}{cf^2+d}\right)-\Pi\left(\frac{(bc-ad)f^2}{b\left(cf^2+d\right)};i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{cf^2-d}{cf^2+d}\right)\right)}{\sqrt{1-f^4x^2}\sqrt{-c-\frac{d}{f^2}}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x)
)/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-
d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*A
rcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c
) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

Maple [B] time = 0.096, size = 212, normalized size = 2.5

$$-2\,\frac{\left(cf^2-d\right)\sqrt{f^2x+1}\sqrt{-f^2x+1}\sqrt{dx+c}}{f^2\,(ad-bc)\left(df^4x^3+cf^4x^2-dx-c\right)}\text{EllipticPi}\left(\sqrt{\frac{f^2\,(dx+c)}{cf^2-d}},-\frac{b\left(cf^2-d\right)}{f^2\,(ad-bc)},\sqrt{\frac{cf^2-d}{cf^2+d}}\right)\sqrt{-\frac{\left(f^2x+1\right)d}{cf^2-d}}\sqrt{-\frac{\left(f^2x\right.}{cf^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x)
```

```
[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*d
-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)*d/(c*f^2-d))^(1/2)*(-(f^2*x-
1)*d/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(f^2*x+1)^(1/2)*(-f^2*x
+1)^(1/2)*(d*x+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{f^2x+1}\sqrt{-f^2x+1}(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(1/(sqrt(f^2*x + 1)*sqrt(-f^2*x + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{-f^2x+1}\sqrt{f^2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x+1)**(1/2)/(f**2*x+1)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f**2*x + 1)*sqrt(f**2*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{f^2x+1}\sqrt{-f^2x+1}(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(f^2*x + 1)*sqrt(-f^2*x + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

3.76 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rubi [A] time = 0.174857, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {932, 168, 538, 537}

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx=\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx$$
$$=-\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\,dx,x,\sqrt{1-f^2x}\right)\right)$$
$$=-\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f^2}\right)f^2}}}\,dx,x,\sqrt{1-f^2x}\right)}{\sqrt{c+dx}}$$
$$=-\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{\left(b+af^2\right)\sqrt{c+dx}}$$

Mathematica [C] time = 0.107044, size = 218, normalized size = 2.53

$$\frac{2i(c+dx)\sqrt{\frac{d\left(f^2x-1\right)}{f^2(c+dx)}}\sqrt{\frac{d\left(f^2x+1\right)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right),\frac{cf^2-d}{cf^2+d}\right)-\Pi\left(\frac{(bc-ad)f^2}{b\left(cf^2+d\right)};i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{cf^2-d}{cf^2+d}\right)\right)}{\sqrt{1-f^4x^2}\sqrt{-c-\frac{d}{f^2}}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x)
)/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-
d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*A
rcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c
) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

Maple [B] time = 0.024, size = 205, normalized size = 2.4

$$-2\,\frac{\left(cf^2-d\right)\sqrt{-f^4x^2+1}\sqrt{dx+c}}{(ad-bc)\,f^2\left(df^4x^3+cf^4x^2-dx-c\right)}\text{EllipticPi}\left(\sqrt{\frac{f^2\,(dx+c)}{cf^2-d}},-\frac{\left(cf^2-d\right)b}{(ad-bc)\,f^2},\sqrt{\frac{cf^2-d}{cf^2+d}}\right)\sqrt{-\frac{\left(f^2x+1\right)d}{cf^2-d}}\sqrt{-\frac{\left(f^2\right.}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x)
```

```
[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*d
-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)*d/(c*f^2-d))^(1/2)*(-(f^2*x-
1)*d/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(-f^4*x^2+1)^(1/2)*(d*x
+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-f^4x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(f^2x-1\right)\left(f^2x+1\right)}\,(a+bx)\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**4*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(f**2*x - 1)*(f**2*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-f^4x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

3.77 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx$

Optimal. Leaf size=471

$$-\frac{245264762213\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{99532800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{25}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}-\frac{427\sqrt{2-3}}{}$$

```
[Out] (-1450582567*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3686400*Sqrt[-5 +
2*x]) - (70489981*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
/1658880 - (83363*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2
))/34560 - (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))
/2400 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(7/2))/25 + (
1450582567*Sqrt[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[Ar
cSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(2457600*Sqrt[(2
 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (245264762213*Sqrt[11/23]*Sqrt[7 + 5*x]
*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(9953280
0*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (57691792727443*(2 - 3*x)*Sqr
t[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcS
in[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(497664000*Sqrt[429
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.665089, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {161, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{25}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}-\frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}{2400}-\frac{83363\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x}{34560}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```


```
[Out] (-1450582567*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3686400*Sqrt[-5 +
2*x]) - (70489981*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
/1658880 - (83363*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2
))/34560 - (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))
/2400 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(7/2))/25 + (
1450582567*Sqrt[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[Ar
cSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(2457600*Sqrt[(2
 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (245264762213*Sqrt[11/23]*Sqrt[7 + 5*x]
*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(9953280
0*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (57691792727443*(2 - 3*x)*Sqr
t[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcS
in[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(497664000*Sqrt[429
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
```

```
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
```

```
a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} + \frac{1}{50}\int \frac{(7+5x)^{5/2}\left(-3-1190x+\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}$$

$$= -\frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{2400}$$

$$= -\frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1658880} - \frac{83363\sqrt{2-3x}\sqrt{-5+2x}}{34560}$$

$$= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{1658880}$$

$$= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{1658880}$$

$$= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{1658880}$$

$$= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}{1658880}$$

Mathematica [A] time = 4.38093, size = 350, normalized size = 0.74

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(-62507925572\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+78331458618\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```

```
[Out] -(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(78331458618*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x
^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62] - 62507925572*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x
^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(6*(111823466541
5 + 5225923788019*x + 2861488598626*x^2 - 795166559320*x^3 - 849459145920*x
^4 - 288728294400*x^5 + 71414784000*x^6 + 39813120000*x^7) - 60033082963*Sq
rt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(
2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x
)]], 39/62])))/(398131200*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 +
3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.074, size = 895, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/284663808000*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(624
33731183120*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x
+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-684857410441904*11^(1/2)*((7+
5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x
+1))^(1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),12
4/55,1/39*31^(1/2)*78^(1/2))-896111886589920*11^(1/2)*((7+5*x)/(4*x+1))^(1/
2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*El
lipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/
2))+31216865591560*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-342428705220952*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)
/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)
,124/55,1/39*31^(1/2)*78^(1/2))-448055943294960*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*E
llipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1
/2))+3902108198945*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-42803588152619*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4
*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/
55,1/39*31^(1/2)*78^(1/2))-56006992911870*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*
3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE
(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+170
798284800000*x^7+306369423360000*x^6-1238644382976000*x^5-3644179735996800*
x^4-3411264539482800*x^3+18436555308411240*x^2+15642366908265240*x-16765465
556439600)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^{\frac{5}{2}} \sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25x^2+70x+49\right)\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

3.78 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx$

Optimal. Leaf size=429

$$-\frac{982275517\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{4147200\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}-\frac{427\sqrt{2-3x}\sqrt{}}{}$$

```
[Out] (-1471781*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(51200*Sqrt[-5 + 2*x])
 - (267029*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/69120
- (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/1440 + (
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/20 + (1471781*S
qrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(102400*Sqrt[(2 - 3*x)/(5 - 2*
x)]*Sqrt[7 + 5*x]) - (982275517*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[
Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(4147200*Sqrt[-5 + 2*x]*Sq
rt[(7 + 5*x)/(5 - 2*x)]) - (145131624827*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)
]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[
7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(20736000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[
1 + 4*x])
```

Rubi [A] time = 0.541804, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {161, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}-\frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}{1440}-\frac{267029\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{}}{69120}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```


```
[Out] (-1471781*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(51200*Sqrt[-5 + 2*x])
 - (267029*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/69120
- (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/1440 + (
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/20 + (1471781*S
qrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(102400*Sqrt[(2 - 3*x)/(5 - 2*
x)]*Sqrt[7 + 5*x]) - (982275517*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[
Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(4147200*Sqrt[-5 + 2*x]*Sq
rt[(7 + 5*x)/(5 - 2*x)]) - (145131624827*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)
]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[
7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(20736000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[
1 + 4*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && !LtQ[m,
-1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
```

```
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} + \frac{1}{40}\int \frac{(7+5x)^{3/2}\left(-3-1190\right.}{\sqrt{2-3x}\sqrt{-5+2x}}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} + \frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}$$

$$= -\frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{1440}$$

$$= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{69120}$$

$$= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{69120}$$

$$= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{69120}$$

$$= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{69120}$$

Mathematica [A] time = 4.15613, size = 345, normalized size = 0.8

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(-5426733148\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+7391284182\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

```
[Out] -(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7391284182*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^
2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5
 + 2*x)/(-2 + 3*x)]], 39/62] - 5426733148*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2
)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5
+ 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(3497259535 +
 16491468251*x + 9107809874*x^2 - 4479491480*x^3 - 3503236800*x^4 + 4032000
0*x^5 + 414720000*x^6) - 4681665317*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-
2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin
[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(514252800*Sqrt[2 - 3*
x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.018, size = 890, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/11860992000*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(2017
97192080*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5
*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1722852836656*11^(1/2)*((7+5*x)/
(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^
(1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,
1/39*31^(1/2)*78^(1/2))-2727622291680*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1
/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE
(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+100
898596040*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1
))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*
x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-861426418328*11^(1/2)*((7+5*x)/(4
*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1
/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39
*31^(1/2)*78^(1/2))-1363811145840*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*
31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+126123245
05*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2),1/39*31^(1/2)*78^(1/2))-107678302291*11^(1/2)*((7+5*x)/(4*x+1))^(1
/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*Ellip
ticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*7
8^(1/2))-170476393230*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2
*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1
/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1779148800000*x^6+17297
2800000*x^5-15028885872000*x^4-19217018449200*x^3+57824907614760*x^2+501207
```

```
55215960*x-50630167988400)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

3.79 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx$

Optimal. Leaf size=391

$$-\frac{1368371\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{43200\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{1}{9}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}(2-3x)^{3/2}+\frac{23}{240}\sqrt{2x-5}\sqrt{4x+1}$$

```
[Out] (-13027*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4800*Sqrt[-5 + 2*x]) +
(23*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/240 - ((2 - 3
*x)^(3/2)*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/9 + (13027*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(3200*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1368371*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(43200*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) - (65750101*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(216000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.428624, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {161, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$-\frac{1}{9}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}(2-3x)^{3/2}+\frac{23}{240}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x}-\frac{13027\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x}}{4800\sqrt{2x-5}}-\frac{1368}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```


```
[Out] (-13027*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4800*Sqrt[-5 + 2*x]) +
(23*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/240 - ((2 - 3
*x)^(3/2)*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/9 + (13027*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(3200*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1368371*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(43200*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) - (65750101*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(216000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{1}{18}\int \frac{\sqrt{2-3x}\left(617+1042x-13\right.}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5}}$$

$$= \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+}$$

$$= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} -$$

$$= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} -$$

$$= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} -$$

$$= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} -$$

Mathematica [A] time = 4.06636, size = 340, normalized size = 0.87

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(-4532324\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+7269066\sqrt{682}\sqrt{\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

```
[Out] -(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7269066*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/
(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
2*x)/(-2 + 3*x)]], 39/62] - 4532324*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)
/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(3848705 + 17658613
*x + 7278862*x^2 - 7723240*x^3 - 2184000*x^4 + 1152000*x^5) - 2120971*Sqrt[
682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 -
 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]]
, 39/62])))/(5356800*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]
*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.02, size = 885, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2),x)
```

```
[Out] 1/123552000*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2)*(454813
040*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/
2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(
4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-780517328*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2
*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(
1/2)*78^(1/2))-2682519840*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/
2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+227406520*11^(1
/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3
*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/
2),1/39*31^(1/2)*78^(1/2))-390258664*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1
/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2)
)-1341259920*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7
+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+28425815*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/
2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*7
8^(1/2))-48782333*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-167657490*11^(1/2)*
((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/
(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/3
9*31^(1/2)*78^(1/2))+4942080000*x^5-9369360000*x^4-33132699600*x^3+49668641
880*x^2+55468893480*x-48037189200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

3.80 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx$

Optimal. Leaf size=351

$$-\frac{20057\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{427\sqrt{2-3x}\sqrt{4x+1}}{600\sqrt{2x-5}}$$

```
[Out] (-427*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(600*Sqrt[-5 + 2*x]) + (Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + (427*Sqrt[143/
3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(400*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (20057*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]
/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1800*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (1008833*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(9000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.323878, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {161, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{427\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{600\sqrt{2x-5}}-\frac{20057\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```


```
[Out] (-427*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(600*Sqrt[-5 + 2*x]) + (Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + (427*Sqrt[143/
3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(400*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (20057*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]
/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1800*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (1008833*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(9000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && !LtQ[m,
-1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
```

```
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{20}\int \frac{-3-1190x+854x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{\int \frac{2}{\sqrt{2-3x}}}{}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{32543\int}{}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{427\sqrt{\frac{14}{3}}}{}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{427\sqrt{\frac{14}{3}}}{}$$

Mathematica [A] time = 2.93339, size = 347, normalized size = 0.99

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{117924\sqrt{682}(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-238266\sqrt{682}(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}E\left(\sin^{-1}\right.}{\left(\frac{5x+7}{3x-2}\right)^{3/}}\right.}{66960}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) + (-238266*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE
[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 117924*Sqrt[682]
*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSi
n[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 7*Sqrt[(7 + 5*x)/(-2 +
 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) + 13947*Sqrt[682]*(2 - 3*x)
^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Ellip
ticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/((2
 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(669600*Sqrt[2 -
 3*x])
```

Maple [B] time = 0.03, size = 880, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(1/2),x)
```

```
[Out] 1/5148000*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2)*(15321680
*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x
+1))^(1/2),1/39*31^(1/2)*78^(1/2))+11975824*11^(1/2)*((7+5*x)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*Ell
ipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)
*78^(1/2))-29309280*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x
-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+7660840*11^(1/2)*((7+
5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x
+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*
31^(1/2)*78^(1/2))+5987912*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2
)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/
2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-14654640
*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2),1/39*31^(1/2)*78^(1/2))+957605*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1
/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+74848
9*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)
*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-1831830*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*Ell
ipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2
))-313513200*x^3+61776000*x^4+190149960*x^2+709583160*x-476876400)/(120*x^4
-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/sqrt(5*x + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

3.81 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{296\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{75\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{6\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{25\sqrt{2x-5}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}-\frac{3\sqrt{429}\sqrt{}}{}$$

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) + (6*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(25*Sqrt[-5 + 2*x]) - (3*Sqrt[429]*S
qrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1
 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(25*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5
*x]) + (296*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(75*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
- (26474*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*E
llipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39
])/(375*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.318856, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {160, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{6\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{25\sqrt{2x-5}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}+\frac{296\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{75\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{3\sqrt{429}\sqrt{2-3x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```


```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) + (6*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(25*Sqrt[-5 + 2*x]) - (3*Sqrt[429]*S
qrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1
 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(25*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5
*x]) + (296*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(75*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
- (26474*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*E
llipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39
])/(375*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
```

```
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{1}{5}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{\int \frac{-12384-20496x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+}}}{1200}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{427}{75}\int \frac{\sqrt{2-}}{\sqrt{-5+2x}\sqrt{1+}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{3\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E}{25\sqrt{\frac{2-}{5-}}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{3\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E}{25\sqrt{\frac{2-}{5-}}}$$

Mathematica [A] time = 2.79307, size = 330, normalized size = 0.95

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(262\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-558\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15\right.\right.}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

```
[Out] -(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-558*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2
- 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x
)/(-2 + 3*x)]], 39/62] + 262*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2
]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 +
3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-415 - 1569*x + 394*x^2 +
 120*x^3) - 427*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35
- 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62])))/(4650*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7
+ 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.041, size = 875, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(3/2),x)
```

```
[Out] -1/107250*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2)*(157520*1
1^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((
-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2),1/39*31^(1/2)*78^(1/2))+157136*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*Ellipti
cPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^
(1/2))-205920*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4
*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*
((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+78760*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/
2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)
*78^(1/2))+78568*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)
/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2
)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-102960*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4
*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/3
9*31^(1/2)*78^(1/2))+9845*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*1
1^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+9821*11^(1/2)*((7+5
*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+
1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,
1/39*31^(1/2)*78^(1/2))-12870*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(
1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/
2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-514800*x^3-2745
60*x^2+5173740*x-3174600)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{25x^2+70x+49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 +
 70*x + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

3.82 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{496\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1725\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{35812\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2085525\sqrt{2x-5}}+\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x}}{417105\sqrt{5x+7}}$$

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^(3/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(417105*Sqrt[7 + 5*x]) - (35
812*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2085525*Sqrt[-5 + 2*x]) + (
17906*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[
(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(53475*Sqrt[(2 - 3*x)
/(5 - 2*x)]*Sqrt[7 + 5*x]) - (496*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTa
n[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1725*Sqrt[-5 + 2*x]*Sqr
t[(7 + 5*x)/(5 - 2*x)]) + (496*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((
1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/
Sqrt[2 - 3*x]], -23/39])/(125*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.429386, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {160, 1604, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$-\frac{35812\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2085525\sqrt{2x-5}}+\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{417105\sqrt{5x+7}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}-\frac{496\sqrt{\frac{11}{23}}\sqrt{5x+7}F}{1725\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```


```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^(3/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(417105*Sqrt[7 + 5*x]) - (35
812*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2085525*Sqrt[-5 + 2*x]) + (
17906*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[
(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(53475*Sqrt[(2 - 3*x)
/(5 - 2*x)]*Sqrt[7 + 5*x]) - (496*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTa
n[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1725*Sqrt[-5 + 2*x]*Sqr
t[(7 + 5*x)/(5 - 2*x)]) + (496*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((
1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/
Sqrt[2 - 3*x]], -23/39])/(125*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{1}{15}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} + \frac{\int \frac{40642-72631}{\sqrt{2-3x}\sqrt{-5+2}}}{41}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3}}{208552}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3}}{208552}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3}}{208552}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3}}{208552}$$

Mathematica [A] time = 2.97431, size = 366, normalized size = 0.94

$$\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{30\sqrt{6-9x}(44765x+34864)}{(5x+7)^2} - \frac{2\left(-37053\sqrt{682}(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+80577\sqrt{682}(3\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*((30*Sqrt[6 - 9*x]*(34864 + 447
65*x))/(7 + 5*x)^2 - (2*(80577*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 +
 3*x)]], 39/62] - 37053*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*
x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]]
, 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-241731*(-35 - 151*x - 34*x^2 + 40*x
^3) + 48438*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11
*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 +
2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[3]*(2 - 3*x)^(3/2)*((7 + 5*x)/(-2 + 3*x)
)^(3/2)*(5 + 18*x - 8*x^2))))/(6256575*Sqrt[3])
```

Maple [B] time = 0.04, size = 1149, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(5/2),x)
```

```
[Out] -2/114703875*(39393200*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3-23614560*11^(1/2)*
((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/
(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),12
4/55,1/39*31^(1/2)*78^(1/2))*x^3-24389200*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*
3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF
(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3
+74847080*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1
))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+
5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-44867664*11^(1/2)*((7+5*x)/(4*x
+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2
)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39
*31^(1/2)*78^(1/2))-46339480*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1
/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^
(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+30037315*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2
+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(
1/2),1/39*31^(1/2)*78^(1/2))-18006102*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1
/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(
1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2
))-18596765*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x
+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+
5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+3446905*11^(1/2)*((7+5*x)/(4*x+
1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)
*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^
(1/2))-2066274*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(
4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-2134055*11^(1/2)*((7+5
*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+
1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))+523292550*x^3-1135538635*x^2-779434810*x+869257400)*(4*x+1)
^(1/2)*(2*x-5)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*
```

```
x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{125x^3+525x^2+735x+343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3
+ 525*x^2 + 735*x + 343), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

3.83 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{48884\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{9593415\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{2852696\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{11598438735\sqrt{2x-5}}+\frac{1426348\sqrt{2-3x}\sqrt{2x-5}}{2319687747\sqrt{5x+}}$$

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(25*(7 + 5*x)^(5/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2085525*(7 + 5*x)^(3/2)) +
(1426348*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5
*x]) - (2852696*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(11598438735*Sqr
t[-5 + 2*x]) + (1426348*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]
*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(29
7395865*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (48884*Sqrt[11/23]*Sqrt[
7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/
(9593415*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.394505, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {160, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$-\frac{2852696\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{11598438735\sqrt{2x-5}}+\frac{1426348\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2319687747\sqrt{5x+7}}+\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2085525(5x+7)^{3/2}}-\frac{2\sqrt{2-3x}\sqrt{}}{25(5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```



```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(25*(7 + 5*x)^(5/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2085525*(7 + 5*x)^(3/2)) +
(1426348*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5
*x]) - (2852696*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(11598438735*Sqr
t[-5 + 2*x]) + (1426348*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]
*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(29
7395865*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (48884*Sqrt[11/23]*Sqrt[
7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/
(9593415*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```


Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
```

```
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{1}{25}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{\int \frac{-254100+3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}}{20855}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}}{23196877}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}}{23196877}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}}{23196877}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}}{23196877}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}}{23196877}$$

Mathematica [A] time = 2.25788, size = 251, normalized size = 0.76

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(-236555\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^3\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+31\sqrt{\frac{5x+7}{3x-2}}\left(5010538\right.\right.}{11598438735\sqrt{2-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(137502130
+ 880765228*x + 1137407943*x^2 - 729949210*x^3 + 50105384*x^4) + 713174*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
```

```
cE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 236555*Sqrt[68
2]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[A
rcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(11598438735*Sqrt[
2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.042, size = 973, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(7/2),x)
```

```
[Out] -2/11598438735*(285269600*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*1
1^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4+17811200*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*
x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),
1/39*31^(1/2)*78^(1/2))*x^4+941389680*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1
/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/3
1*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+587
76960*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(
1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+976335206*11^(1/2)*((7+5*x)/(4*x+1)
)^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x
^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*7
8^(1/2))+60958832*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1
/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+329486388*11^(1/2)*((7+
5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x
+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*
31^(1/2)*78^(1/2))+20571936*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/
2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/
2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+34945526*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*
x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),
1/39*31^(1/2)*78^(1/2))+2181872*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13
^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(
1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-3514495404*x^
4+19294337060*x^3-26198770563*x^2-3855274122*x+9191461480)*(4*x+1)^(1/2)*(2
*x-5)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{625\,x^4+3500\,x^3+7350\,x^2+6860\,x+2401},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4
+ 3500*x^3 + 7350*x^2 + 6860*x + 2401), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

3.84 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{1212290288\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1867348636335\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{65687975672\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2257624501329015\sqrt{2x-5}}+\frac{3284398783}{45152}$$

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(35*(7 + 5*x)^(7/2)) + (255
8*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(695175*(7 + 5*x)^(5/2)) + (2
3758016*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(57992193675*(7 + 5*x)^
(3/2)) + (32843987836*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(45152490
0265803*Sqrt[7 + 5*x]) - (65687975672*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 +
5*x])/(2257624501329015*Sqrt[-5 + 2*x]) + (32843987836*Sqrt[11/39]*Sqrt[2 -
 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x]
)/Sqrt[-5 + 2*x]], -23/39])/(57887807726385*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1212290288*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1867348636335*Sqrt[-5 + 2*x]*Sqrt
[(7 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.51458, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {160, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$-\frac{65687975672\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2257624501329015\sqrt{2x-5}}+\frac{32843987836\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{451524900265803\sqrt{5x+7}}+\frac{23758016\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+}}{57992193675(5x+7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```


```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(35*(7 + 5*x)^(7/2)) + (255
8*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(695175*(7 + 5*x)^(5/2)) + (2
3758016*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(57992193675*(7 + 5*x)^
(3/2)) + (32843987836*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(45152490
0265803*Sqrt[7 + 5*x]) - (65687975672*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 +
5*x])/(2257624501329015*Sqrt[-5 + 2*x]) + (32843987836*Sqrt[11/39]*Sqrt[2 -
 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x]
)/Sqrt[-5 + 2*x]], -23/39])/(57887807726385*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1212290288*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1867348636335*Sqrt[-5 + 2*x]*Sqrt
[(7 + 5*x)/(5 - 2*x)])
```


Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{1}{35}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2}}\,dx$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{\int \frac{-548842+1382\cdots}{\sqrt{2-3x}\sqrt{-5+2x}\cdots}}{486\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-\cdots}}{5799219\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-\cdots}}{5799219\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-\cdots}}{5799219\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-\cdots}}{5799219\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-\cdots}}{5799219\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-\cdots}}{5799219\cdots}$$

Mathematica [A] time = 2.59452, size = 259, normalized size = 0.7

$$\frac{2\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{242\left(19017205\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+203578437\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x-5\right)-67859479\sqrt{\cdots}\right.}{\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x-5\right)}\cdots\right.}{225762450132901\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

```
[Out] (2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-(((-2 + 3*x)*(1539551542327
0 + 113490310442229*x + 54668919175710*x^2 + 10263746198750*x^3))/(7 + 5*x)
^4) + (242*(203578437*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 6785
9479*Sqrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[A
rcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 19017205*Sqrt[682]
*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/
39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5
- 18*x + 8*x^2))))/(2257624501329015*Sqrt[2 - 3*x])
```

Maple [B] time = 0.044, size = 1160, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(9/2),x)
```

```
[Out] -2/2257624501329015*(4737011092000*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)
*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*3
1^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5+328439
87836000*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/
(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5+22263952132400*11^(1/2)*((7+5*x)
/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))
^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/
2)*78^(1/2))*x^4+154366742829200*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*1
3^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^
(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4+38097411
707410*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^
(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4
*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+264147772171030*11^(1/2)*((7+5*x)/
(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^
(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2
)*78^(1/2))*x^3+28168636458578*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^
(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*3
1^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1953067736
66774*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(
1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)
/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+8240030794534*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/
2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)
*78^(1/2))+57132116840722*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)
*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+812397402278*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+
3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2
),1/39*31^(1/2)*78^(1/2))+5632743913874*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1
/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+58109
51702460*x^5-173342585590346*x^4+2153615020704860*x^3-4639703191080657*x^2+
51366440607272*x+1423213141652020)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(2-3*x)^(1/2
)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{3125\,x^5+21875\,x^4+61250\,x^3+85750\,x^2+60025\,x+16807},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3125*x^5
 + 21875*x^4 + 61250*x^3 + 85750*x^2 + 60025*x + 16807), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

3.85 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=429

$$\frac{861015607\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\mathrm{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{331776\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{8}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}+\frac{1445}{576}\sqrt{2-3x}\sqrt{2x-$$

```
[Out] (2466927*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4096*Sqrt[-5 + 2*x]) +
 (1561915*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/27648 +
 (1445*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/576 + (S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/8 - (2466927*Sqr
t[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23
]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8192*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (861015607*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(331776*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)]) + (331574321009*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqr
t[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5
*x])/Sqrt[2 - 3*x]], -23/39])/(1658880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*
x])
```

Rubi [A] time = 0.525891, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {162, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{8}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}+\frac{1445}{576}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}+\frac{1561915\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x}}{27648}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```


```
[Out] (2466927*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4096*Sqrt[-5 + 2*x]) +
 (1561915*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/27648 +
 (1445*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/576 + (S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/8 - (2466927*Sqr
t[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23
]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8192*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (861015607*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(331776*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)]) + (331574321009*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqr
t[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5
*x])/Sqrt[2 - 3*x]], -23/39])/(1658880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*
x])
```


Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
```

```
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
```

```
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} - \frac{1}{16}\int \frac{(7+5x)^{3/2}\left(-621-370x+2890x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$$

$$= \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}$$

$$= \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} + \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5$$

$$= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} +$$

$$= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} +$$

$$= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} +$$

$$= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} +$$

Mathematica [A] time = 3.66124, size = 345, normalized size = 0.8

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(10666876180\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-123889073\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-12388907394*Sqrt[682]*Sqrt[(-5 - 18*x + 8*
x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(
-5 + 2*x)/(-2 + 3*x)]], 39/62] + 10666876180*Sqrt[682]*Sqrt[(-5 - 18*x + 8*
x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(
-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-57523418
05 - 26349657233*x - 12645389558*x^2 + 3088122056*x^3 + 1004819520*x^4 + 43
9372800*x^5 + 82944000*x^6) + 10695945839*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4
*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62,
ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(41140224*Sqrt[2
 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.031, size = 890, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/948879360*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(42417
0712240*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))
^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*
x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-3936108068752*11^(1/2)*((7+5*x)/(
4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(
1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1
/39*31^(1/2)*78^(1/2))-4571906470560*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(
1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+2120
85356120*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x
)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1968054034376*11^(1/2)*((7+5*x)/(4
*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1
/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39
*31^(1/2)*78^(1/2))-2285953235280*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*
31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+265106695
15*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2),1/39*31^(1/2)*78^(1/2))-246006754297*11^(1/2)*((7+5*x)/(4*x+1))^(1
/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*Ellip
ticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*7
8^(1/2))-285744154410*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2
*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1
/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-355829760000*x^6-188490
9312000*x^5-4310675740800*x^4-13248043620240*x^3+85680578188920*x^2+7846498
```

```
6845960*x-85333953104400)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(25x^2+70x+49\right)\sqrt{5x+7}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sq
rt(2*x - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

3.86 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}} dx$

Optimal. Leaf size=391

$$\frac{2824441\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{17280\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}+\frac{977}{288}\sqrt{2-3x}\sqrt{2x-$$

```
[Out] (66377*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1920*Sqrt[-5 + 2*x]) + (
977*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/288 + (Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/6 - (66377*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1280*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) + (2824441*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(17280*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) + (963142751*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*
x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
 - 3*x]], -23/39])/(86400*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.411787, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {162, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}+\frac{977}{288}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{66377\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{1920\sqrt{2x-5}}+\frac{28}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```


```
[Out] (66377*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1920*Sqrt[-5 + 2*x]) + (
977*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/288 + (Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/6 - (66377*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1280*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) + (2824441*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(17280*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) + (963142751*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*
x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
 - 3*x]], -23/39])/(86400*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} - \frac{1}{12}\int \frac{\sqrt{7+5x}\left(-465+20x+1954x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$$

$$= \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2}$$

Mathematica [A] time = 3.59569, size = 340, normalized size = 0.87

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(31389484\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-37038366\sqrt{682}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-37038366*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2
)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5
+ 2*x)/(-2 + 3*x)]], 39/62] + 31389484*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(
2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2
*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-17232355 - 791
87903*x - 38640362*x^2 + 10641080*x^3 + 4555200*x^4 + 1152000*x^5) + 310691
21*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x
^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2
+ 3*x)]], 39/62])))/(2142720*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2
 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.02, size = 885, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/49420800*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(124073
2240*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1
/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/
(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-11433436528*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*
x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*3
1^(1/2)*78^(1/2))-13668351840*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(
1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31
^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+620366120*1
1^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((
-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))
^(1/2),1/39*31^(1/2)*78^(1/2))-5716718264*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*
3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*Ellipti
cPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^
(1/2))-6834175920*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2
)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+77545765*11^(1/2)*((7+5*x
)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1)
)^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))-714589783*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11
^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-854271990*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2
+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/
2),1/39*31^(1/2)*78^(1/2))-4942080000*x^5-19541808000*x^4-45650233200*x^3+2
59737071880*x^2+236349193080*x-254967913200)/(120*x^4-182*x^3-385*x^2+197*x
+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

3.87 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=351

$$\frac{8959\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{720\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{509\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+}}{240\sqrt{2x-5}}$$

```
[Out] (509*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(240*Sqrt[-5 + 2*x]) + (Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/4 - (509*Sqrt[143/3]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(160*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (8959*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(S
qrt[2]*Sqrt[2 - 3*x])], -39/23])/(720*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*
x)]) + (2198489*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3
*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]],
 -23/39])/(3600*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.316346, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {161, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{509\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{240\sqrt{2x-5}}+\frac{8959\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{720\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{509}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```


```
[Out] (509*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(240*Sqrt[-5 + 2*x]) + (Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/4 - (509*Sqrt[143/3]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(160*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (8959*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(S
qrt[2]*Sqrt[2 - 3*x])], -39/23])/(720*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*
x)]) + (2198489*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3
*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]],
 -23/39])/(3600*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
```

```
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{8}\int \frac{309-410x-1018x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{\int \frac{-320516+2\cdots}{\sqrt{2-3x}\sqrt{-5+2x}\cdots}}{192\cdots}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{70919\int \frac{\cdots}{\sqrt{-5+2x}\cdots}}{14\cdots}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{509\sqrt{\frac{143}{3}}\sqrt{2-\cdots}}{\cdots}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{509\sqrt{\frac{143}{3}}\sqrt{2-\cdots}}{\cdots}$$

Mathematica [A] time = 3.87506, size = 347, normalized size = 0.99

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(66960(2-3x) - \frac{3\left(76756\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(15x^2+11x-14)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+94674\sqrt{682}(2-3x)(5x+7\cdots\right.}{\cdots}\right.}{267840\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) - (3*(94674*Sq
rt[682]*(2 - 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE
[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 76756*Sqrt[682]*
Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSi
n[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3
*x)]*(284022*(-35 - 151*x - 34*x^2 + 40*x^3) + 70919*Sqrt[682]*(2 - 3*x)^2*
Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Elliptic
Pi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/((2 -
 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(267840*Sqrt[2 - 3
*x])
```

Maple [B] time = 0.019, size = 880, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/2059200*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2)*(2*x-5)^(1/2)*(3622960
*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x
+1))^(1/2),1/39*31^(1/2)*78^(1/2))-26098192*11^(1/2)*((7+5*x)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*Ell
ipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)
*78^(1/2))-34937760*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x
-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1811480*11^(1/2)*((7+
5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x
+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*
31^(1/2)*78^(1/2))-13049096*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/
2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1
/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-1746888
0*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)
*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))+226435*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3
^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(
1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1631
137*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/
2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x
+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-2183610*11^(1/2)*((7+5*x)/(4*x+1)
)^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*E
llipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1
/2))-168339600*x^3-61776000*x^4+661123320*x^2+623542920*x-647446800)/(120*x
^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*sqrt(5*x + 7)/sqrt(2*x - 5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

3.88 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx$

Optimal. Leaf size=365

$$\frac{7\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{41\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{20\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x}}{5\sqrt{2x}}$$

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(10*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) + (7*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(10*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
+ (41*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(20*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (943*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/S
qrt[-5 + 2*x]], 39/62])/(100*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rubi [A] time = 0.216143, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {173, 176, 424, 170, 418, 165, 536, 539}

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}+\frac{7\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{41\sqrt{\frac{11}{62}}\sqrt{2-3x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{20\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```


```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(10*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) + (7*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(10*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
+ (41*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(20*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (943*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/S
qrt[-5 + 2*x]], 39/62])/(100*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```


Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 536

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> -Dist[f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{41}{20}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx + \frac{77}{20}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\ldots}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\left(1599\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\operatorname{Subst}\left(\int \frac{1}{(5-2x^2)\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\ldots}}\ldots\right)}{10\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{7\sqrt{\frac{11}{23}}\sqrt{7+\ldots}}{\ldots}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{7\sqrt{\frac{11}{23}}\sqrt{7+\ldots}}{\ldots}$$

Mathematica [A] time = 1.22962, size = 318, normalized size = 0.87

$$\frac{\sqrt{2-3x}\left(1984\sqrt{682}\sqrt{\frac{5-2x}{5x+7}}\sqrt{\frac{4x+1}{5x+7}}\left(15x^2+11x-14\right)\operatorname{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{155-62x}{55x+77}}\right),\frac{23}{62}\right)-3410\sqrt{682}\sqrt{\frac{5-2x}{5x+7}}\sqrt{\frac{4x+1}{5x+7}}\left(15x^2\ldots\right.\right.}{34\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*(-3410*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7
 + 5*x)]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*
x)]], 23/62] + 1984*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7 +
 5*x)]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)
]], 23/62] + Sqrt[(-2 + 3*x)/(7 + 5*x)]*(17050*(10 + 21*x - 70*x^2 + 24*x^3
) - 1599*Sqrt[682]*Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^2*Sqrt[(-10 + 19*x -
 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x
)]], 23/62])))/(34100*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*((-2 + 3*x)/(7 + 5*x))^(
3/2)*(7 + 5*x)^(3/2))
```

Maple [A] time = 0.021, size = 875, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] 1/42900*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2)*(2*x-5)^(1/2)*(15088*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+
3*x)/(4*x+1))^(1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))
^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+68640*11^(1/2)*((7+5*x)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*Ell
```

```
ipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2
))-20240*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5
*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+7544*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*E
llipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/
2)*78^(1/2))+34320*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-10120*11^(1/2)*((7+5*x)/
(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^
(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))+943*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+4290*11^(1/2)*((7+5
*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+
1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))-1265*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*
x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+514800*x^3-909480*x^2-14
24280*x+1201200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{5x+7}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{10x^2-11x-35}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 -
 11*x - 35), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*sqrt(5*x + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{5x+7}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

3.89 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{4\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{195\sqrt{2x-5}}+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}+\frac{2\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{69\sqrt{\frac{2}{341}}\cdots}{}$$

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*Sqrt[7 + 5*x]) - (4*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(195*Sqrt[-5 + 2*x]) + (2*Sqrt[11/39
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) - (69*Sqrt[2/341]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[-((5 - 2*x)/(1 +
 4*x))]*(1 + 4*x)*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt
[1 + 4*x]], 39/62])/(25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.194712, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {164, 1586, 1595, 165, 537, 176, 424}

$$-\frac{4\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{195\sqrt{2x-5}}+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}+\frac{2\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{69\sqrt{\frac{2}{341}}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*Sqrt[7 + 5*x]) - (4*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(195*Sqrt[-5 + 2*x]) + (2*Sqrt[11/39
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) - (69*Sqrt[2/341]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[-((5 - 2*x)/(1 +
 4*x))]*(1 + 4*x)*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt
[1 + 4*x]], 39/62])/(25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1595

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b
*B*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*Sqrt[a + b*x]), x] + (
-Dist[(B*(b*g - a*h))/(2*f*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[g + h*x]), x], x] + Dist[(B*(b*e - a*f)*(b*g - a*h))/(2*d*f*h), Int
[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{-25+130x-48x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{(5-24x)\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{3}{5}\int \frac{\sqrt{1+4x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{\left(46\sqrt{\frac{3}{403}}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{\frac{-5+2x}{1+4x}}(1+4\right.}{5}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{195\sqrt{-5+2x}} + \frac{2\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\right.\right.}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5}}$$

Mathematica [A] time = 2.29731, size = 326, normalized size = 1.17

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\left(23\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-62\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2\right.\right.}{6045\sqrt{2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/
(-2 + 3*x)]], 39/62] + 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(
-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x
)]], 39/62] - 2*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-961*(-5 - 18*x + 8*x^2) + 39*S
qrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/
(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*
x)]], 39/62])))/(6045*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)
]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.026, size = 870, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(3/2)/(2*x-5)^(1/2),x)
```

```
[Out] 2/10725*(2-3*x)^(1/2)*(4*x+1)^(1/2)*(7+5*x)^(1/2)*(2*x-5)^(1/2)*(880*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*
x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1
/2),1/39*31^(1/2)*78^(1/2))+1104*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*1
3^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticPi(1/3
1*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-
880*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/
```

```
2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(
4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+440*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*
3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*Ellipti
cF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+5
52*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-440*11^(1/2)*((7+5*x)/(4*x+1))^(
1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*El
lipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/
2))+55*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^
(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4
*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+69*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(
1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2
))-55*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(
1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-7590*x^2+24035*x-12650)/(120*x^4-182*x^
3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^{\frac{3}{2}}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{50x^3+15x^2-252x-245}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 +
 15*x^2 - 252*x - 245), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^{\frac{3}{2}}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

3.90 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{44\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{2691\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{3740\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{3253419\sqrt{2x-5}}-\frac{9350\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{3253419\sqrt{5x+7}}+$$

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(117*(7 + 5*x)^(3/2)) - (935
0*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(3253419*Sqrt[7 + 5*x]) + (37
40*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3253419*Sqrt[-5 + 2*x]) - (1
870*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(S
qrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(83421*Sqrt[(2 - 3*x)/(
5 - 2*x)]*Sqrt[7 + 5*x]) + (44*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[S
qrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(2691*Sqrt[-5 + 2*x]*Sqrt[(
7 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.319986, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {164, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{3740\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{3253419\sqrt{2x-5}}-\frac{9350\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{3253419\sqrt{5x+7}}+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}+\frac{44\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan\right.}{2691\sqrt{2x-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```


```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(117*(7 + 5*x)^(3/2)) - (935
0*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(3253419*Sqrt[7 + 5*x]) + (37
40*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3253419*Sqrt[-5 + 2*x]) - (1
870*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(S
qrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(83421*Sqrt[(2 - 3*x)/(
5 - 2*x)]*Sqrt[7 + 5*x]) + (44*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[S
qrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(2691*Sqrt[-5 + 2*x]*Sqrt[(
7 + 5*x)/(5 - 2*x)])
```


Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
```

```
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{1}{117}\int \frac{-33+110x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} - \frac{\int \frac{-66308-170170x+22440}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7}}}{3253419}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-3x}\sqrt{1+4x}}{3253419\sqrt{-5+}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-3x}\sqrt{1+4x}}{3253419\sqrt{-5+}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-3x}\sqrt{1+4x}}{3253419\sqrt{-5+}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-3x}\sqrt{1+4x}}{3253419\sqrt{-5+}}$$

Mathematica [A] time = 1.81928, size = 246, normalized size = 0.85

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(506\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+31\sqrt{\frac{5x+7}{3x-2}}\left(58928x^3-\right.\right.}{3253419\sqrt{2-3x}(5x+7)^{3/2}\sqrt{\frac{5x+7}{3x-2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-23755 - 1
22348*x - 94580*x^2 + 58928*x^3) - 935*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqr
t[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
2*x)/(-2 + 3*x)]], 39/62] + 506*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[(-5 -
 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-
2 + 3*x)]], 39/62]))/(3253419*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7 + 5*x)/
(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.028, size = 786, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(5/2)/(2*x-5)^(1/2),x)
```

```
[Out] -2/3253419*(20240*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*
```

```
((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3-74800*11^(1/2)*((7+5*x)
/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))
^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/
2)*78^(1/2))*x^3+38456*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*
11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-142120*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4
*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1
/39*31^(1/2)*78^(1/2))+15433*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1
/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1
/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-57035*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)
/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),
1/39*31^(1/2)*78^(1/2))+1771*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1
/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2
)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-6545*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4
*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*
31^(1/2)*78^(1/2))-1312518*x^3+3086255*x^2+1200968*x-1783420)*(2*x-5)^(1/2)
*(4*x+1)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{250\,x^4+425\,x^3-1155\,x^2-2989\,x-1715},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4
+ 425*x^3 - 1155*x^2 - 2989*x - 1715), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

3.91 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx$

Optimal. Leaf size=330

$$\frac{111628\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{74828637\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{8185936\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{90467822133\sqrt{2x-5}}-\frac{20464840\sqrt{2-3x}\sqrt{2x-5}}{90467822133\sqrt{5x+}}$$

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(195*(7 + 5*x)^(5/2)) - (364
6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(16267095*(7 + 5*x)^(3/2)) -
(20464840*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(90467822133*Sqrt[7 +
 5*x]) + (8185936*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(90467822133*S
qrt[-5 + 2*x]) - (4092968*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x
)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(
2319687747*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (111628*Sqrt[11/23]*S
qrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/2
3])/(74828637*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.398223, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {164, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{8185936\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{90467822133\sqrt{2x-5}}-\frac{20464840\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{90467822133\sqrt{5x+7}}-\frac{3646\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{16267095(5x+7)^{3/2}}+\frac{2\sqrt{2-3x}\sqrt{2}}{195(5x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```


```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(195*(7 + 5*x)^(5/2)) - (364
6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(16267095*(7 + 5*x)^(3/2)) -
(20464840*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(90467822133*Sqrt[7 +
 5*x]) + (8185936*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(90467822133*S
qrt[-5 + 2*x]) - (4092968*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x
)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(
2319687747*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (111628*Sqrt[11/23]*S
qrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/2
3])/(74828637*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```


Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
```

```
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{1}{195}\int \frac{-41+90x+48x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{\int \frac{-489390+1112210x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3}}}{16267095}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+}}{90467822133\sqrt{7-}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+}}{90467822133\sqrt{7-}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+}}{90467822133\sqrt{7-}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+}}{90467822133\sqrt{7-}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+}}{90467822133\sqrt{7-}}$$

Mathematica [A] time = 1.78881, size = 251, normalized size = 0.76

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(958111\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^3\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+31\sqrt{\frac{5x+7}{3x-2}}\left(37005125\right.\right.}{90467822133\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-374624540
 - 2271416114*x - 2953846743*x^2 + 643813106*x^3 + 370051256*x^4) - 2046484
```

```
*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ell
ipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 958111*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
cF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(90467822133*S
qrt[2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2)
)
```

Maple [B] time = 0.031, size = 973, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(7/2)/(2*x-5)^(1/2),x)
```

```
[Out] 2/90467822133*(818593600*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*
((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11
^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4-126500000*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*
x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),
1/39*31^(1/2)*78^(1/2))*x^4+2701358880*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(
1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/
31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3-41
7450000*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))
^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(
4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+2801636596*11^(1/2)*((7+5*x)/(4*x
+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2
)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2
)*78^(1/2))-432946250*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2
*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*1
1^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+945475608*11^(1/2)*
((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/
(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1
/39*31^(1/2)*78^(1/2))-146107500*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*1
3^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*3
1^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+100277716*
11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*(
(-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2),1/39*31^(1/2)*78^(1/2))-15496250*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(
1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/
31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+584375
7936*x^4+10390893586*x^3-65568669813*x^2-3127552098*x+26993559920)*(2*x-5)^
(1/2)*(4*x+1)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x
)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^{\frac{7}{2}}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{1250\,x^5+3875\,x^4-2800\,x^3-23030\,x^2-29498\,x-12005},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1250*x^5
 + 3875*x^4 - 2800*x^3 - 23030*x^2 - 29498*x - 12005), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

3.92 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx$

Optimal. Leaf size=370

$$\frac{258506776\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1618368818157\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{16377776536\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{1956607901151813\sqrt{2x-5}}-\frac{40944441340}{19566079}$$

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(273*(7 + 5*x)^(7/2)) + (98*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1807455*(7 + 5*x)^(5/2)) - (32
17468*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(50259901185*(7 + 5*x)^(3
/2)) - (40944441340*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1956607901
151813*Sqrt[7 + 5*x]) + (16377776536*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5
*x])/(1956607901151813*Sqrt[-5 + 2*x]) - (8188888268*Sqrt[11/39]*Sqrt[2 - 3
*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/
Sqrt[-5 + 2*x]], -23/39])/(50169433362867*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (258506776*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1618368818157*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.521292, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {164, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{16377776536\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{1956607901151813\sqrt{2x-5}}-\frac{40944441340\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1956607901151813\sqrt{5x+7}}-\frac{3217468\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{50259901185(5x+7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```


```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(273*(7 + 5*x)^(7/2)) + (98*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1807455*(7 + 5*x)^(5/2)) - (32
17468*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(50259901185*(7 + 5*x)^(3
/2)) - (40944441340*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1956607901
151813*Sqrt[7 + 5*x]) + (16377776536*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5
*x])/(1956607901151813*Sqrt[-5 + 2*x]) - (8188888268*Sqrt[11/39]*Sqrt[2 - 3
*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/
Sqrt[-5 + 2*x]], -23/39])/(50169433362867*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (258506776*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1618368818157*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)])
```


Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} - \frac{1}{273}\int \frac{-49+70x+96x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2}}\,dx$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{\int \frac{-958104+2280510x+49392\ldots}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\ldots}}{37956555}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+\ldots}}{50259901185(7+5\ldots}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+\ldots}}{50259901185(7+5\ldots}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+\ldots}}{50259901185(7+5\ldots}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+\ldots}}{50259901185(7+5\ldots}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+\ldots}}{50259901185(7+5\ldots}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+\ldots}}{50259901185(7+5\ldots}$$

Mathematica [A] time = 2.19595, size = 258, normalized size = 0.7

$$\frac{2\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{(3x-2)\left(2559027583750x^3+12313608173580x^2+19165803061167x+2552362046246\right)}{(5x+7)^4} - \frac{22\left(71545594\sqrt{682}(3x-2)\sqrt{\ldots}\right.}{\ldots}\right.}{1956607901151813\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(((-2 + 3*x)*(2552362046246 +
 19165803061167*x + 12313608173580*x^2 + 2559027583750*x^3))/(7 + 5*x)^4 -
(22*(558333291*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 186111097*S
qrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[
Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 71545594*Sqrt[682]*(-2 +
 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
 + 8*x^2))))/(1956607901151813*Sqrt[2 - 3*x])
```

Maple [B] time = 0.031, size = 1160, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^(9/2)/(2*x-5)^(1/2),x)
```

```
[Out] -2/1956607901151813*(175178212000*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31
^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5-8188888
268000*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^
(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4
*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5+823337596400*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/
2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*7
8^(1/2))*x^4-38487774859600*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/
2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)
*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4+1408870770010
*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))
^(1/2),1/39*31^(1/2)*78^(1/2))*x^3-65859133895390*11^(1/2)*((7+5*x)/(4*x+1)
)^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*E
llipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1
/2))*x^3+1041697237658*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*
11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-48695224085662*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2
+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))
^(1/2),1/39*31^(1/2)*78^(1/2))+304722499774*11^(1/2)*((7+5*x)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*Ellip
ticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))
-14244571142186*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/
(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*
((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+30043063358*11^(1/2)*((7+5*
x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1
))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(
1/2)*78^(1/2))-1404394337962*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1
/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2
)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-33052545587580*x
^5-83732628367442*x^4+43651554581534*x^3+1041927172311711*x^2-1311200489909
80*x-367706794166900)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(2-3*x)^(1/2)/(120*x^4-18
2*x^3-385*x^2+197*x+70)/(7+5*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^{\frac{9}{2}}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{6250x^6+28125x^5+13125x^4-134750x^3-308700x^2-266511x-84035}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(6250*x^6
 + 28125*x^5 + 13125*x^4 - 134750*x^3 - 308700*x^2 - 266511*x - 84035), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^{\frac{9}{2}}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

3.93 $\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=391

$$\frac{5241511\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{13824\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}+\frac{6955\sqrt{2-3x}\sqrt{2x-5}}{1152}$$

```
[Out] (102487*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1536*Sqrt[-5 + 2*x]) +
(6955*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/1152 + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/24 - (102487*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1024*Sqrt[(2 - 3*x)/(5 - 2*x)
]*Sqrt[7 + 5*x]) + (5241511*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(13824*Sqrt[-5 + 2*x]*Sqrt[(7
+ 5*x)/(5 - 2*x)]) + (295576909*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-(
(1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])
/Sqrt[2 - 3*x]], -23/39])/(13824*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.420862, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {174, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}+\frac{6955\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}{1152}+\frac{102487\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{1536\sqrt{2x-5}}+\frac{52}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```



```
[Out] (102487*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1536*Sqrt[-5 + 2*x]) +
(6955*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/1152 + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/24 - (102487*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1024*Sqrt[(2 - 3*x)/(5 - 2*x)
]*Sqrt[7 + 5*x]) + (5241511*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(13824*Sqrt[-5 + 2*x]*Sqrt[(7
+ 5*x)/(5 - 2*x)]) + (295576909*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-(
(1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])
/Sqrt[2 - 3*x]], -23/39])/(13824*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} - \frac{1}{48}\int \frac{\sqrt{7+5x}\left(-6189+3136x+13910x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{\int \frac{6899\cdots}{\sqrt{2\cdots}}}{\cdots}$$

$$= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{\cdots}$$

$$= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{\cdots}$$

$$= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{\cdots}$$

$$= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{\cdots}$$

Mathematica [A] time = 2.44502, size = 340, normalized size = 0.87

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(46704724\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-57187746\sqrt{682}\sqrt{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-57187746*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2
)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5
+ 2*x)/(-2 + 3*x)]], 39/62] + 46704724*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(
2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2
*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-27447805 - 124
999073*x - 56065622*x^2 + 20626760*x^3 + 6542400*x^4 + 1152000*x^5) + 47673
695*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*
x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2
 + 3*x)]], 39/62])))/(1714176*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-
2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.036, size = 885, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/7907328*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1939599
20*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4
*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-3508783952*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2
*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(
1/2)*78^(1/2))-4220824608*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/
2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+96979960*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*
x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2
),1/39*31^(1/2)*78^(1/2))-1754391976*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1
/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2)
)-2110412304*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7
+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+12122495*11^(1/2)*((7+5*x)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/
2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*7
8^(1/2))-219298997*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2
)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-263801538*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)
/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/
39*31^(1/2)*78^(1/2))-988416000*x^5-5613379200*x^4-17697760080*x^3+77122472
856*x^2+75329218536*x-78013375440)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{5}{2}}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(25\,x^{2}+70\,x+49\right)\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{8\,x^{2}-18\,x-5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.94 $\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=351

$$\frac{17515\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{785\sqrt{2-3x}\sqrt{4x+1}\sqrt{}}{192\sqrt{2x-5}}$$

```
[Out] (785*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/16 - (785*Sqrt[143
/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*S
qrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt
[7 + 5*x]) + (17515*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (3730013*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(2880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.321651, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {174, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{785\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{192\sqrt{2x-5}}+\frac{17515\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```


```
[Out] (785*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/16 - (785*Sqrt[143
/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*S
qrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt
[7 + 5*x]) + (17515*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (3730013*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(2880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
```

```
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{1}{32}\int \frac{-4121+4074x+7850x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{\int \frac{2888740-240\ldots}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1\ldots}}}{7680\ldots}$$

$$= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{120323\int \frac{\ldots}{\sqrt{-5+2x}\ldots}}{115\ldots}$$

$$= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{785\sqrt{\frac{143}{3}}\sqrt{2-3x\ldots}}{\ldots}$$

$$= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{785\sqrt{\frac{143}{3}}\sqrt{2-3x\ldots}}{\ldots}$$

Mathematica [A] time = 3.14792, size = 349, normalized size = 0.99

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{(2-3x)\left(\frac{998820\sqrt{682}(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)}{(2-3x)^2} - \frac{1314090\sqrt{682}(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}E\left(\sin^{-1}\left(\sqrt{\ldots}\right.\right.}{(2-3x)^2}\ldots\right)}{\left(\frac{5x+7}{3x-2}\right)^{\ldots}}\ldots\right)}{642816\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(200880 + ((2 - 3
*x)*((-1314090*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*El
lipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^
2 + (998820*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellip
ticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^2 +
 Sqrt[(7 + 5*x)/(-2 + 3*x)]*((3942270*(-35 - 151*x - 34*x^2 + 40*x^3))/(-2
+ 3*x)^3 + (1082907*Sqrt[682]*((1 + 4*x)/(-2 + 3*x))^(3/2)*Sqrt[(-35 - 11*x
 + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])/(1 + 4*x))))/(((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18
```

```
*x - 8*x^2))))/642816
```

Maple [B] time = 0.024, size = 880, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/1647360*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1963280*
11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*(
(-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))+44278864*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)
*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*Elli
pticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*
78^(1/2))+53882400*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+981640*11^(1/2)*((7+5*
x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1
))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31
^(1/2)*78^(1/2))+22139432*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2
)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+26941200*
11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*(
(-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1)
)^(1/2),1/39*31^(1/2)*78^(1/2))+122705*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(
1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/
31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+276742
9*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)
*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+3367650*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*Ell
ipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2
))+310424400*x^3+61776000*x^4-912139800*x^2-1016644200*x+978978000)/(120*x^
4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{3}{2}}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{8x^2-18x-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2
- 18*x - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{3}{2}}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.95 $\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=365

$$-\frac{39\sqrt{\frac{11}{23}}\sqrt{5x+7}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{179\sqrt{\frac{11}{62}}\sqrt{2-3x}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{16\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x}}{4\sqrt{2x}}$$

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
5*x]) - (39*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) +
 (179*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(16*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (4117*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/
Sqrt[-5 + 2*x]], 39/62])/(80*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rubi [A] time = 0.201979, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {173, 176, 424, 170, 418, 165, 536, 539}

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{179\sqrt{\frac{11}{62}}\sqrt{2-3x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{16\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}-\frac{\sqrt{4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```


```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
5*x]) - (39*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) +
 (179*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(16*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (4117*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/
Sqrt[-5 + 2*x]], 39/62])/(80*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```


Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 536

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> -Dist[f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{179}{16}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx - \frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\cdots}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\left(6981\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(5-2x^2\right)\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\frac{22x^2}{23}}\cdots}\right.}{8\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{39\sqrt{\frac{11}{23}}\sqrt{7+5\cdots}}{8\cdots}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{39\sqrt{\frac{11}{23}}\sqrt{7+5\cdots}}{8\cdots}$$

Mathematica [A] time = 1.36944, size = 347, normalized size = 0.95

$$-\frac{-1265\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5x+7}{4x+1}}\left(8x^2-18x-5\right)\mathrm{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{5x+7}{4x+1}}\right),\frac{39}{62}\right)+6820\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5x+7}{4x+1}}\left(8x^2-18x-5\right)\cdots}{27280\sqrt{2}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 + 4*x)]*(-5 -
 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]], 39/
62] - 1265*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 + 4*x)]*(
-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]],
 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83*x - 53*x^2 + 3
0*x^3) + 4117*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*Sqrt[(-35 -
11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22/39]*Sqrt[(7 +
5*x)/(1 + 4*x)]], 39/62]))/(27280*Sqrt[2 - 3*x]*Sqrt[-10 + 4*x]*Sqrt[(-5 +
2*x)/(1 + 4*x)]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
```

Maple [A] time = 0.023, size = 875, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(1/2)*(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/34320*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(20240*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+
3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2),1/39*31^(1/2)*78^(1/2))+65872*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2
)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticPi(
```

```
1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2
))+68640*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5
*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+10120*11^(1/2)*((7+5*x)/(4*x+1))
^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*
EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(
1/2))+32936*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x
+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7
+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+34320*11^(1/2)*((7+5*x)
/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))
^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(
1/2)*78^(1/2))+1265*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x
-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2
)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+4117*11^(1/2)*((7+5*x)/(4
*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1
/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*3
1^(1/2)*78^(1/2))+4290*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+514800*x^3-909480*x^2-
1424280*x+1201200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x+7}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{8x^2-18x-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 -
18*x - 5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}\sqrt{5x+7}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(1/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5\,x+7}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.96 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$

Optimal. Leaf size=101

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$

```
[Out] (62*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*Ellipt
icPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(5
*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.0371325, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {165, 537}

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (62*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*Ellipt
icPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(5
*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{\left(62(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}}$$
$$= \frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [A] time = 0.546155, size = 170, normalized size = 1.68

$$\frac{\sqrt{\frac{4x+1}{5x+7}}(5x+7)^{3/2}\left(117\sqrt{\frac{-6x^2+19x-10}{(5x+7)^2}}\Pi\left(-\frac{55}{62};\sin^{-1}\left(\sqrt{\frac{155-62x}{55x+77}}\right)|\frac{23}{62}\right)-62\sqrt{\frac{5-2x}{5x+7}}\sqrt{\frac{3x-2}{5x+7}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{155-62x}{55x+77}}\right),\frac{23}{62}\right)\right)}{5\sqrt{682}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^(3/2)*(-62*Sqrt[(5 - 2*x)/(7 + 5*x)]*S
qrt[(-2 + 3*x)/(7 + 5*x)]*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)]],
 23/62] + 117*Sqrt[(-10 + 19*x - 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, Arc
Sin[Sqrt[(155 - 62*x)/(77 + 55*x)]], 23/62]))/(5*Sqrt[682]*Sqrt[2 - 3*x]*Sq
rt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Maple [B] time = 0.025, size = 172, normalized size = 1.7

$$\frac{\sqrt{13}\sqrt{3}\sqrt{11}}{128700\,x^3-227370\,x^2-356070\,x+300300}\left(55\,\text{EllipticF}\left(1/31\,\sqrt{31}\sqrt{11}\sqrt{\frac{7+5\,x}{4\,x+1}},1/39\,\sqrt{31}\sqrt{78}\right)+69\,\text{EllipticPi}\left(1/3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/4290*(55*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31
^(1/2)*78^(1/2))+69*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/
2),124/55,1/39*31^(1/2)*78^(1/2)))*((-2+3*x)/(4*x+1))^(1/2)*((2*x-5)/(4*x+1
))^(1/2)*13^(1/2)*3^(1/2)*((7+5*x)/(4*x+1))^(1/2)*11^(1/2)*(4*x+1)^(3/2)*(2
*x-5)^(1/2)*(7+5*x)^(1/2)*(2-3*x)^(1/2)/(30*x^3-53*x^2-83*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{40\,x^{3}-34\,x^{2}-151\,x-35},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 -
 34*x^2 - 151*x - 35), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.97 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2\sqrt{\frac{11}{39}}\sqrt{5-2x}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{22}}\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{62}{39}\right)}{23\sqrt{2x-5}}$$

```
[Out] (2*Sqrt[11/39]*Sqrt[5 - 2*x]*EllipticE[ArcSin[(Sqrt[39/22]*Sqrt[1 + 4*x])/S
qrt[7 + 5*x]], 62/39])/(23*Sqrt[-5 + 2*x])
```

Rubi [B] time = 0.130884, antiderivative size = 195, normalized size of antiderivative = 3.25, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {176, 422, 418, 492, 411}

$$-\frac{62\sqrt{2x-5}\sqrt{4x+1}}{897\sqrt{2-3x}\sqrt{5x+7}}-\frac{\sqrt{\frac{22}{31}}\sqrt{4x+1}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{39\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}}+\frac{2\sqrt{682}\sqrt{4x+1}E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-62*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (2*S
qrt[682]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7
 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) - (Sqrt[
22/31]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 +
 5*x]], 39/62])/(39*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{\left(\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{39\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= \frac{\left(\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{23x^2}{22}}\sqrt{1+\frac{31x^2}{11}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{39\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}} + \frac{\left(31\sqrt{2}\sqrt{2-3x}\right.}{}$$

$$= -\frac{62\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} - \frac{\sqrt{\frac{22}{31}}\sqrt{1+4x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)\middle|\frac{39}{62}\right)}{39\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} - \frac{\left(62\sqrt{2}\sqrt{2-\right.}{}$$

$$= -\frac{62\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{2\sqrt{682}\sqrt{1+4x}E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)\middle|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} - \frac{\sqrt{\frac{22}{31}}\sqrt{1+}}{}$$

Mathematica [B] time = 1.72862, size = 237, normalized size = 3.95

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\left(-23\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-1922\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x\right.\right.}{27807\sqrt{2-3x}\sqrt{5x+7}\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x-5\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-1922*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
+ 8*x^2) + 62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x +
 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]
- 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*
EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(27807*
Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.025, size = 330, normalized size = 5.5

$$\frac{2}{107640 x^4 - 163254 x^3 - 345345 x^2 + 176709 x + 62790}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{7+5x}\left(16\sqrt{11}\sqrt{\frac{7+5x}{4x+1}}\sqrt{3}\sqrt{13}\sqrt{\frac{2}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(3/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/897*(2-3*x)^(1/2)*(7+5*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(16*11^(1/2)*
((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/
(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)
,1/39*31^(1/2)*78^(1/2))+8*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2
)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2
)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+11^(1/2)*((7+5*x
)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1)
)^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))+138*x^2-437*x+230)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{200x^4+110x^3-993x^2-1232x-245}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4
+ 110*x^3 - 993*x^2 - 1232*x - 245), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.98 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{716\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{61893\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{39332\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{74828637\sqrt{2x-5}}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{74828637\sqrt{5x+7}}$$

```
[Out] (-10*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2691*(7 + 5*x)^(3/2)) - (
98330*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(74828637*Sqrt[7 + 5*x])
+ (39332*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(74828637*Sqrt[-5 + 2*x
]) - (19666*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[A
rcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1918683*Sqrt[(
2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (716*Sqrt[11/23]*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(61893*Sqrt[-5 +
 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.296052, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {177, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{39332\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{74828637\sqrt{2x-5}}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{74828637\sqrt{5x+7}}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}+\frac{716\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\text{ta}\right.}{61893\sqrt{2x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```


```
[Out] (-10*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2691*(7 + 5*x)^(3/2)) - (
98330*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(74828637*Sqrt[7 + 5*x])
+ (39332*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(74828637*Sqrt[-5 + 2*x
]) - (19666*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[A
rcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1918683*Sqrt[(
2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (716*Sqrt[11/23]*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(61893*Sqrt[-5 +
 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```


Rule 177

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*e - a*f)*(b*g - a*h))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
```

```
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{\int \frac{-771+854x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{2691}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} - \frac{\int \frac{-2381456-17\ldots}{\sqrt{2-3x}\sqrt{-5+\ldots}}}{74\ldots}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39332\sqrt{2-3\ldots}}{748286\ldots}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39332\sqrt{2-3\ldots}}{748286\ldots}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39332\sqrt{2-3\ldots}}{748286\ldots}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39332\sqrt{2-3\ldots}}{748286\ldots}$$

Mathematica [A] time = 1.80488, size = 248, normalized size = 0.86

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(31\left(92\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+\sqrt{\frac{5x+7}{3x-2}}\left(285680x^3-2\ldots\right.\right.\right.}{74828637\sqrt{2-3x}(5x+7)^{3/2}\sqrt{\frac{5x+\ldots}{3x-\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-9833*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sq
rt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]], 39/62] + 31*(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-389005 - 1578
968*x - 20372*x^2 + 285680*x^3) + 92*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[
(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])))/(74828637*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7
+ 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.029, size = 786, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(5/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/74828637*(101200*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+786640*11^(1/2)*((7+5*
x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1
))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(
1/2)*78^(1/2))*x^3+192280*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/
2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1494616*11^(1/2
)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x
)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/
2),1/39*31^(1/2)*78^(1/2))+77165*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*1
3^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*3
1^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+599813*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2
+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(
1/2),1/39*31^(1/2)*78^(1/2))+8855*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31
^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+68831*11^(1
/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3
*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)
,1/39*31^(1/2)*78^(1/2))+3447930*x^3-2253977*x^2-21690932*x+14440780)*(4*x+
1)^(1/2)*(2*x-5)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+
5*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{1000\,x^5+1950\,x^4-4195\,x^3-13111\,x^2-9849\,x-1715}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5
 + 1950*x^4 - 4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

3.99 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=721

$$\frac{\sqrt{g+hx}(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e+fx}\sqrt{bg-ah}}{\sqrt{a+bx}\sqrt{fg-eh}}\right),-\frac{(bc-ad)(fg-eh)}{(bg-ah)(de-cf)}\right)}{f^2h\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{(e+fx)\sqrt{bg-\ldots}}{\ldots}$$

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]) - (Sqrt[d*g -
 c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*
h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt[d*g -
 c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))]
)/(f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqrt[g + h*
x]) + ((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqr
t[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/(
(d*e - c*f)*(b*g - a*h)))])/(f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c +
 d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[b*g
 - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*h)*(a + b*x))/
((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x
))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e
- a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g -
 a*h))/((b*e - a*f)*(d*g - c*h))])/(f^2*Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*S
qrt[c + d*x])
```

Rubi [A] time = 0.674845, antiderivative size = 721, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {173, 176, 424, 170, 419, 165, 537}

$$\frac{(e+fx)\sqrt{bg-ah}\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\Pi\left(\frac{f(bg-ah)}{(be-af)h};\sin^{-1}\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|\frac{(de-cf)(bg-\ldots}{(be-af)(dg-\ldots}\right)}{f^2h^2\sqrt{a+bx}\sqrt{c+dx}\sqrt{be-af}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```


```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]) - (Sqrt[d*g -
 c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*
h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt[d*g -
 c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))]
)/(f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqrt[g + h*
x]) + ((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqr
t[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/(
(d*e - c*f)*(b*g - a*h)))])/(f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c +
 d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[b*g
 - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*h)*(a + b*x))/
((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x
))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e
- a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g -
 a*h))/((b*e - a*f)*(d*g - c*h))])/(f^2*Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*S
qrt[c + d*x])
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{((de-cf)(fg-eh))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx}{2fh} + \frac{((de-cf)(bfg+$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} + \frac{\left((adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+f\right.}{f^2h\sqrt{a+bx}}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)\right.}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

Mathematica [B] time = 15.1257, size = 6667, normalized size = 9.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]

[Out] Result too large to show
```

Maple [B] time = 0.151, size = 18077, normalized size = 25.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")

[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.100 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sqrt{c+dx}E\left(\tan^{-1}\left(\frac{\sqrt{af-be}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|\frac{(ad-bc)(fg-eh)}{(af-be)(dg-ch)}\right)}{\sqrt{a+bx}\sqrt{af-be}\sqrt{bg-ah}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}}$$

```
[Out] (-2*Sqrt[c + d*x]*EllipticE[ArcTan[(Sqrt[-(b*e) + a*f]*Sqrt[g + h*x])/(Sqrt
[b*g - a*h]*Sqrt[e + f*x])], ((-(b*c) + a*d)*(f*g - e*h))/((-(b*e) + a*f)*(
d*g - c*h))])/(Sqrt[-(b*e) + a*f]*Sqrt[b*g - a*h]*Sqrt[a + b*x]*Sqrt[((b*g
- a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])
```

Rubi [A] time = 0.0925571, antiderivative size = 208, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {176, 424}

$$-\frac{2\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*
h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g
- e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h
)))])/((b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]*Sqrt[g + h*x])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{\left(2\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}}{\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(be-af)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$= -\frac{2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

Mathematica [A] time = 4.70662, size = 206, normalized size = 1.28

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}E\left(\sin^{-1}\left(\sqrt{\frac{(af-be)(g+hx)}{(fg-eh)(a+bx)}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(a+bx)^{3/2}(eh-fg)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{-\frac{(e+fx)(g+hx)(be-af)(bg-ah)}{(a+bx)^2(fg-eh)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*EllipticE[ArcSin[Sqrt[((-(b*e)
 + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h))/((b
*e - a*f)*(d*g - c*h))])/((-(f*g) + e*h)*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*
(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*
x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2))])
```

Maple [B] time = 0.118, size = 4590, normalized size = 28.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(x^2*a*d*e*f*h^2-x^2*a*d*f^2*g*h+x*a*c*e*f*h^2-x^2*b*d*e^2*h^2-b*c*e^2*g*
h+b*c*e*f*g^2-x*a*d*f^2*g^2-x*b*c*e^2*h^2-x*a*c*f^2*g*h+EllipticF(((a*f-b*e
)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e)
)^(1/2))*a*c*e^2*h^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)
*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/
2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*
g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*c*f^2*h^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)
/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x
+a)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e)
)^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b*d*f^2*g^2*((
a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x
+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticE(((a*f-b*e)
*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))
^(1/2))*x^2*a*c*f^2*h^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f
*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^
(1/2)-EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h
-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b*d*f^2*g^2*((a*f-b*e)*(h*x+g)/(a*h-b
```

```
*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(
b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x
+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*d*e^2*g*h*((a
*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+
e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*
(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^
(1/2))*b*c*e^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(
d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)
+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)
/(c*h-d*g)/(a*f-b*e))^(1/2))*a*d*e^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+
e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a
*h-b*g)/(f*x+e))^(1/2)+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2
),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b*c*e^2*g*h*((a*f-b*e)*(
h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)
*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+a*c*e*f*g*h+x^2*b*d*e*f*g*h+x*
a*d*e*f*g*h+x*b*c*e*f*g*h-a*c*f^2*g^2-2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b
*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b*d*e
*f*g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-
d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticF((
(a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(
a*f-b*e))^(1/2))*x^2*a*d*f^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2
)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/
(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-
d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b*c*f^2*g*h*((a*f-b*e)*(h*x+
g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e
*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-
b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*
d*f^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c
*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+Elliptic
E(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g
)/(a*f-b*e))^(1/2))*x^2*b*c*f^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(
1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*
g)/(f*x+e))^(1/2)+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),(
(c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*c*e*f*h^2*((a*f-b*e)*(h
*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*
((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticF(((a*f-b*e)*(h*x+g)/
(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x
*b*d*e*f*g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)
/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-x*b*d
*e^2*g*h+x*b*d*e*f*g^2-2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1
/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*c*e*f*h^2*((a*f-b*
e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(
1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticE(((a*f-b*e)*(h*
x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/
2))*x*b*c*e*f*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d
*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-
2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g
)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*d*e*f*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f
*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)
/(a*h-b*g)/(f*x+e))^(1/2)-2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))
^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b*c*e*f*g*h*((a*f
-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e)
)^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticE(((a*f-b*e)*
(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^
(1/2))*x*a*d*e*f*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)
*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/
2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*
g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b*d*e^2*g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*
x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/
```

```
(a*h-b*g)/(f*x+e))^(1/2)-EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1
/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*c*e^2*h^2*((a*f-b*e)
*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/
2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticE(((a*f-b*e)*(h*x+g)
/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*
b*d*e^2*g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/
(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2))/(h*x+
g)^(1/2)/(f*x+e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*h-b*g)/(e*h-f*g)/(a*f
-b*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{b^2fhx^4+a^2eg+\left(b^2fg+\left(b^2e+2\,abf\right)h\right)x^3+\left(\left(b^2e+2\,abf\right)g+\left(2\,abe+a^2f\right)h\right)x^2+\left(a^2eh+\left(2\,abe+a^2f\right)g\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*f*h*x
^4 + a^2*e*g + (b^2*f*g + (b^2*e + 2*a*b*f)*h)*x^3 + ((b^2*e + 2*a*b*f)*g +
 (2*a*b*e + a^2*f)*h)*x^2 + (a^2*e*h + (2*a*b*e + a^2*f)*g)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.101 $\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=351

$$\frac{29047\sqrt{\frac{23}{11}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{2135\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x}}{192\sqrt{2x-5}}$$

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) - (2
5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/48 + (2135*Sqrt
[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/2
3]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (29047*Sqrt[23/11]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)
/(5 - 2*x)]) - (3431855*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(576*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.318894, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {167, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$-\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{2135\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{192\sqrt{2x-5}}+\frac{29047\sqrt{\frac{23}{11}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```



```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) - (2
5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/48 + (2135*Sqrt
[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/2
3]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (29047*Sqrt[23/11]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)
/(5 - 2*x)]) - (3431855*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(576*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```


Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
```

```
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}+\frac{1}{96}\int \frac{28003+89810x+64050x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,d$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}-\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}-\frac{\int \frac{8779}{\sqrt{2-3x}\sqrt{-}}}{}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}-\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}-\frac{553525\int}{}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}-\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}+\frac{2135\sqrt{\frac{143}{3}}}{}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}-\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}+\frac{2135\sqrt{\frac{143}{3}}}{}$$

Mathematica [A] time = 2.43074, size = 347, normalized size = 0.99

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{17113116\sqrt{682}(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-13104630\sqrt{682}(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}E\left(\sin\right.}{(2-3x)\left(\right.}\right.}{23569}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(1227600*(-2 + 3*x) + (-1310463
0*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elli
pticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 17113116*Sq
rt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elliptic
F[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 385*Sqrt[(7 + 5
*x)/(-2 + 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) - 47445*Sqrt[682]*
(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)
^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/
62]))/((2 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(235699
2*Sqrt[2 - 3*x])
```

Maple [B] time = 0.037, size = 880, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/329472*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(11088208*
11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*(
(-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))-40739440*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)
*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*Elli
pticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*
78^(1/2))-29309280*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+5544104*11^(1/2)*((7+5
*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+
1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*3
1^(1/2)*78^(1/2))-20369720*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2
)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/
2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-14654640
*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1
))^(1/2),1/39*31^(1/2)*78^(1/2))+693013*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1
/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-25462
15*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((-2+3*x)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+
1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-1831830*11^(1/2)*((7+5*x)/(4*x+1))
^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*El
lipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/
2))-188588400*x^3-20592000*x^4+454413960*x^2+574362360*x-524924400)/(120*x^
4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{5}{2}}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(25\,x^2+70\,x+49\right)\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{24\,x^3-70\,x^2+21\,x+10}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sq
rt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{5}{2}}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.102 $\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=469

$$\frac{65\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{48\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}-\frac{5\sqrt{2-3x}\sqrt{}}{12\sqrt{}}$$

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(12*Sqrt[-5 + 2*x]) + (5*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*S
qrt[7 + 5*x]) + (65*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 -
2*x)]) - (895*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[
7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(48*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x]) + (23*Sqrt[31/22]*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x
)]*(7 + 5*x)*EllipticPi[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 +
 5*x]], 39/62])/(6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]) - (4117*Sqrt[2 - 3*x]*Elli
pticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(
48*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.280348, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {166, 173, 176, 424, 170, 418, 165, 536, 539, 537}

$$-\frac{5\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{12\sqrt{2x-5}}+\frac{65\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{48\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```


```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(12*Sqrt[-5 + 2*x]) + (5*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*S
qrt[7 + 5*x]) + (65*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 -
2*x)]) - (895*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[
7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(48*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x]) + (23*Sqrt[31/22]*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x
)]*(7 + 5*x)*EllipticPi[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 +
 5*x]], 39/62])/(6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]) - (4117*Sqrt[2 - 3*x]*Elli
pticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(
48*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
```


Rule 166

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[(Sqrt[a + b*x]
*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 536

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> -Dist[f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
```

```
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\left(\frac{5}{3}\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx\right)+\frac{31}{3}\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{895}{48}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx+\frac{715}{16}\int \frac{}{\sqrt{2-}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+}}{\sqrt{7+5x}}\right.\right.}{6\sqrt{2-3x}\sqrt{-5+2x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}+$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}+$$

Mathematica [A] time = 1.2229, size = 347, normalized size = 0.74

$$\frac{\sqrt{2x-5}\left(-6969\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5x+7}{4x+1}}\left(8x^2-18x-5\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{5x+7}{4x+1}}\right),\frac{39}{62}\right)+6820\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5x+7}{4x+1}}\left(8x^2\right.\right.}{1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(
1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(
```

```
1 + 4*x)]], 39/62] - 6969*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*
x)/(1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*
x)/(1 + 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83
*x - 53*x^2 + 30*x^3) + 9821*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)
^2*Sqrt[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22
/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]], 39/62])))/(16368*Sqrt[4 - 6*x]*((-5 + 2*x)
/(1 + 4*x))^(3/2)*(1 + 4*x)^(3/2)*Sqrt[7 + 5*x])
```

Maple [A] time = 0.023, size = 875, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/20592*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(71024*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+
3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2),1/39*31^(1/2)*78^(1/2))-157136*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticPi
(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/
2))-68640*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1
))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+
5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+35512*11^(1/2)*((7+5*x)/(4*x+1)
)^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x
*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^
(1/2))-78568*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-34320*11^(1/2)*((7+5*x
)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1)
)^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))+4439*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*
x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/
2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-9821*11^(1/2)*((7+5*x)/(
4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(
1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124/55,1/39*
31^(1/2)*78^(1/2))-4290*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-514800*x^3+909480*x^2
+1424280*x-1201200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x+7)^{\frac{3}{2}}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{24\,x^{3}-70\,x^{2}+21\,x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^
3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.103 $\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=100

$$\frac{23\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\sqrt{2-3x}\sqrt{2x-5}}$$

```
[Out] (23*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x)]*(7 + 5*x)*EllipticP
i[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 39/62])/(2*Sqr
t[682]*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rubi [A] time = 0.0383713, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {165, 537}

$$\frac{23\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\sqrt{2-3x}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (23*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x)]*(7 + 5*x)*EllipticP
i[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 39/62])/(2*Sqr
t[682]*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\left(23\sqrt{2}\sqrt{\frac{2-3x}{7+5x}}\sqrt{-\frac{-5+2x}{7+5x}}(7+5x)\right)\mathrm{Subst}\left(\int \frac{1}{(4-5x^2)\sqrt{1-\frac{31x^2}{11}}\sqrt{1-\frac{39x^2}{22}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{7+5x}}\right)}{11\sqrt{2-3x}\sqrt{-5+2x}}$$
$$= \frac{23\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}}$$

Mathematica [A] time = 0.158327, size = 95, normalized size = 0.95

$$-\frac{62\sqrt{4x+1}\sqrt{\frac{5-2x}{5x+7}}\Pi\left(-\frac{55}{69};\sin^{-1}\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{5x+7}}\right)|-\frac{39}{23}\right)}{3\sqrt{253}\sqrt{2x-5}\sqrt{\frac{4x+1}{5x+7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-62*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticPi[-55/69, ArcSin[(Sqr
t[23/11]*Sqrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(3*Sqrt[253]*Sqrt[-5 + 2*x
]*Sqrt[(1 + 4*x)/(7 + 5*x)])
```

Maple [B] time = 0.021, size = 170, normalized size = 1.7

$$\frac{23\sqrt{13}\sqrt{3}\sqrt{11}}{25740x^3-45474x^2-71214x+60060}\left(\mathrm{EllipticF}\left(\frac{\sqrt{31}\sqrt{11}}{31}\sqrt{\frac{7+5x}{4x+1}},\frac{\sqrt{31}\sqrt{78}}{39}\right)-\mathrm{EllipticPi}\left(\frac{\sqrt{31}\sqrt{11}}{31}\sqrt{\frac{7+5x}{4x+1}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(1/2)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 23/858*(EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))-EllipticPi(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),124
/55,1/39*31^(1/2)*78^(1/2)))*((-2+3*x)/(4*x+1))^(1/2)*((2*x-5)/(4*x+1))^(1/
2)*13^(1/2)*3^(1/2)*((7+5*x)/(4*x+1))^(1/2)*11^(1/2)*(4*x+1)^(3/2)*(2*x-5)^
(1/2)*(2-3*x)^(1/2)*(7+5*x)^(1/2)/(30*x^3-53*x^2-83*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{24x^3-70x^2+21x+10}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3
- 70*x^2 + 21*x + 10), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.104 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$

Optimal. Leaf size=71

$$\frac{2\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

```
[Out] (2*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -
39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.0428364, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {170, 418}

$$\frac{2\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)\middle|-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (2*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -
39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx &= \frac{\left(\sqrt{\frac{2}{253}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}\\ &= \frac{2\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)\middle|-\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}\end{aligned}$$

Mathematica [A] time = 0.139292, size = 90, normalized size = 1.27

$$-\frac{2\sqrt{4x+1}\sqrt{\frac{5-2x}{5x+7}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{5x+7}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{4x+1}{5x+7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (-2*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticF[ArcSin[(Sqrt[23/11]*S
qrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(1 +
4*x)/(7 + 5*x)])
```

Maple [A] time = 0.022, size = 134, normalized size = 1.9

$$\frac{2\sqrt{13}\sqrt{3}\sqrt{11}}{12870x^3-22737x^2-35607x+30030}\text{EllipticF}\left(\frac{\sqrt{31}\sqrt{11}}{31}\sqrt{\frac{7+5x}{4x+1}},\frac{\sqrt{31}\sqrt{78}}{39}\right)\sqrt{\frac{-2+3x}{4x+1}}\sqrt{\frac{2x-5}{4x+1}}\sqrt{\frac{7+5x}{4x+1}}(4x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/429*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2
)*78^(1/2))*((-2+3*x)/(4*x+1))^(1/2)*((2*x-5)/(4*x+1))^(1/2)*13^(1/2)*3^(1/
2)*((7+5*x)/(4*x+1))^(1/2)*11^(1/2)*(4*x+1)^(3/2)*(2*x-5)^(1/2)*(2-3*x)^(1/
2)*(7+5*x)^(1/2)/(30*x^3-53*x^2-83*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4
 - 182*x^3 - 385*x^2 + 197*x + 70), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

3.105 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{2\sqrt{\frac{3}{143}}(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{31\sqrt{2x-5}\sqrt{4x+1}}+\frac{10\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5-2x}{5x+7}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{22}}\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{62}{39}\right)}{713\sqrt{2x-5}\sqrt{\frac{2-3x}{5x+7}}}$$

```
[Out] (10*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticE[ArcSin[(S
qrt[39/22]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 62/39])/(713*Sqrt[-5 + 2*x]*Sqrt[
(2 - 3*x)/(7 + 5*x)]) + (2*Sqrt[3/143]*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*
Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/S
qrt[2 - 3*x]], -23/39])/(31*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] time = 0.182889, antiderivative size = 270, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {171, 170, 418, 176, 422, 492, 411}

$$-\frac{10\sqrt{2x-5}\sqrt{4x+1}}{897\sqrt{2-3x}\sqrt{5x+7}}+\frac{6\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{5\sqrt{\frac{22}{31}}\sqrt{4x+1}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{1209\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}}+\frac{10\sqrt{\frac{22}{31}}\sqrt{4x+}}{897}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-10*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (10*
Sqrt[22/31]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqr
t[7 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) + (6*
Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/
23])/(31*Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (5*Sqrt[22/3
1]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x
]], 39/62])/(1209*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])
```

Rule 171

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> -Dist[d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{3}{31}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx + \frac{5}{31}\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7}$$

$$= \frac{\left(5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{1209\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}} + \frac{\left(3\sqrt{\frac{2}{253}}\sqrt{-\frac{-5+2x}{2-3x}}\right.}{}$$

$$= \frac{6\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{\left(5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int\right.}{1209\sqrt{1+4x}}$$

$$= -\frac{10\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{6\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} - \frac{5\sqrt{\frac{22}{31}}\sqrt{1}}{}$$

$$= -\frac{10\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{10\sqrt{\frac{22}{31}}\sqrt{1+4x}E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} + \frac{6\sqrt{7}}{}$$

Mathematica [A] time = 1.57502, size = 237, normalized size = 1.22

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(-23\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+1705\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x\right.\right.}{305877\sqrt{2-3x}\sqrt{5x+7}\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x-5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*
x + 8*x^2) - 55*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x
 + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62
] - 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2
)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(3058
77*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^
2))
```

Maple [B] time = 0.03, size = 599, normalized size = 3.1

$$\frac{2}{36705240x^4-55669614x^3-117762645x^2+60257769x+21411390}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{7+5x}\left(1104\sqrt{11}\sqrt{\frac{7}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/305877*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1104*11^(
1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+
```

```
3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^
(1/2),1/39*31^(1/2)*78^(1/2))+880*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/3
1*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+552*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2
+3*x)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(
1/2),1/39*31^(1/2)*78^(1/2))+440*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*1
3^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*3
1^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+69*11^(1/2
)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x
)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1
/39*31^(1/2)*78^(1/2))+55*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*1
1^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+7590*x^2-24035*x+12
650)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{600x^5-70x^4-3199x^3-1710x^2+1729x+490}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5
 - 70*x^4 - 3199*x^3 - 1710*x^2 + 1729*x + 490), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

3.106 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=288

$$\frac{103964\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1918683\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{358120\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2319687747\sqrt{2x-5}}-\frac{895300\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x}}{2319687747\sqrt{5x+7}}$$

```
[Out] (-50*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) -
(895300*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5*
x]) + (358120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2319687747*Sqrt[-
5 + 2*x]) - (179060*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*Ell
ipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(594791
73*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (103964*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1918683*Sqrt[25
3]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rubi [A] time = 0.302918, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {172, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{358120\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2319687747\sqrt{2x-5}}-\frac{895300\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2319687747\sqrt{5x+7}}-\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}+\frac{103964\sqrt{5x+7}}{1918683}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```


```
[Out] (-50*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) -
(895300*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5*
x]) + (358120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2319687747*Sqrt[-
5 + 2*x]) - (179060*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*Ell
ipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(594791
73*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (103964*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1918683*Sqrt[25
3]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```


Rule 172

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b^2*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*
(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
```

```
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}+\frac{\int \frac{11928-4270x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{83421}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}+$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}+$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}+$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}+$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}+$$

Mathematica [A] time = 1.71931, size = 246, normalized size = 0.85

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(-28819\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+1705\sqrt{\frac{5x+7}{3x-2}}\left(608\right.\right.}{25516565217\sqrt{2-3x}(5x+7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-671560
- 2797991*x - 294854*x^2 + 608600*x^3) - 984830*Sqrt[682]*(-2 + 3*x)*(7 + 5
*x)^2*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 28819*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2
*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62]))/(25516565217*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*
Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] time = 0.031, size = 786, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/25516565217*(50128960*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^
(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+78786400*11^(1/2)
*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)
/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/
39*31^(1/2)*78^(1/2))*x^3+95245024*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)
*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticF(1/
31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+149694
160*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/
2)*((-2+3*x)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(
4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+38223332*11^(1/2)*((7+5*x)/(4*x+1))^(
1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*El
lipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/
2))+60074630*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((7
+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+4386284*11^(1/2)*((7+5*x)/(4*x
+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2
)*EllipticF(1/31*31^(1/2)*11^(1/2)*((7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78
^(1/2))+6893810*11^(1/2)*((7+5*x)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/
(4*x+1))^(1/2)*((-2+3*x)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((
7+5*x)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+496006500*x^3-665223020*x^2-2
040625895*x+1509107050)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(2-3*x)^(1/2)/(120*x^4-
182*x^3-385*x^2+197*x+70)/(7+5*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{3000x^6+3850x^5-16485x^4-30943x^3-3325x^2+14553x+3430}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^
6 + 3850*x^5 - 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

3.107 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=968

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)b}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}+\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af}{(de-cf}}}{df^2h\sqrt{bg-}}$$

```
[Out] (b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(d*h*Sqrt[e + f*x]) - (b*Sqrt
[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*
g - c*h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt
[d*g - c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g -
e*h))])/(d*f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqr
t[g + h*x]) + (b*(d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
- a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g
 - e*h))/((d*e - c*f)*(b*g - a*h)))])/(d*f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e
*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
 + (b*Sqrt[b*g - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*
h)*(a + b*x))/((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g -
 c*h)*(e + f*x))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), Arc
Sin[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e
 - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))])/(d*f^2*Sqrt[b*e - a*f]*h^2
*Sqrt[a + b*x]*Sqrt[c + d*x]) - (2*Sqrt[b*c - a*d]*Sqrt[-(d*g) + c*h]*(a +
b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h
)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c -
a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a
 + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*h*Sqrt[
c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 0.882066, antiderivative size = 968, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {166, 173, 176, 424, 170, 419, 165, 537}

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)b}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}+\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af}{(de-cf}}}{df^2h\sqrt{bg-ah}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(d*h*Sqrt[e + f*x]) - (b*Sqrt
[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*
g - c*h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt
[d*g - c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g -
e*h))])/(d*f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqr
t[g + h*x]) + (b*(d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
- a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g
 - e*h))/((d*e - c*f)*(b*g - a*h)))])/(d*f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e
*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
 + (b*Sqrt[b*g - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*
```

```
h)*(a + b*x))/((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g -
 c*h)*(e + f*x))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), Arc
Sin[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e
 - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))])/(d*f^2*Sqrt[b*e - a*f]*h^2
*Sqrt[a + b*x]*Sqrt[c + d*x]) - (2*Sqrt[b*c - a*d]*Sqrt[-(d*g) + c*h]*(a +
b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h
)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c -
a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a
 + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*h*Sqrt[
c + d*x]*Sqrt[e + f*x])
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[(Sqrt[a + b*x]
*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{d}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{(b(de-cf)(fg-eh))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx}{2dfh} + \frac{(b(de-c}{}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{2\sqrt{bc-ad}\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}{dh\sqrt{c+dx}\sqrt{e}}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\sin^{-1}\left(\frac{\sqrt{fg-}}{\sqrt{dg-}}\right.\right.}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

Mathematica [B] time = 14.2736, size = 6638, normalized size = 6.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.087, size = 16526, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.108 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=228

$$\frac{2(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

```
[Out] (2*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi
[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])
/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d
)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 0.149761, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {165, 537}

$$\frac{2(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi
[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])
/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d
)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{\left(2(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{\left(h-bx^2\right)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\sqrt{1+\frac{(be-af)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{g\cdots}}{\sqrt{a\cdots}}\cdots\right)}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$= \frac{2\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)|\frac{(be-\cdots}{(bc-\cdots}\cdots\right)}{\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [B] time = 5.57101, size = 584, normalized size = 2.56

$$-\frac{2(c+dx)^{3/2}\sqrt{\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}}\left(\frac{ad(g+hx)\sqrt{\frac{(e+fx)(dg-ch)}{(c+dx)(fg-eh)}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(g+hx)(cf-de)}{(c+dx)(fg-eh)}}\right),\frac{(bc-ad)(eh-fg)}{(bg-ah)(de-cf)}\right)}{(c+dx)(dg-ch)\sqrt{\frac{(g+hx)(cf-de)}{(c+dx)(fg-eh)}}}+\frac{bc(g+hx)\sqrt{\frac{(e+fx)(dg-ch)}{(c+dx)(fg-eh)}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(g\cdots}{(c\cdots}}\cdots\right)\cdots\right)}{(c+dx)(ch-dg)\sqrt{\frac{(g+hx)(c\cdots}{(c+dx)(f\cdots}}}\cdots\right)}{d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]*(c + d*x)^(3/2)*(
(a*d*Sqrt[((d*g - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*Ellipt
icF[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c
 - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/((d*g - c*h)*(c + d*x)*
Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*c*Sqrt[((d*g
 - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*EllipticF[ArcSin[Sqrt
[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g)
 + e*h))/((d*e - c*f)*(b*g - a*h))])/((-(d*g) + c*h)*(c + d*x)*Sqrt[((-(d*e
) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*(f*g - e*h)*Sqrt[-(((d*e
 - c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(c + d*x)^2))]*Elli
pticPi[(d*(-(f*g) + e*h))/((d*e - c*f)*h), ArcSin[Sqrt[((-(d*e) + c*f)*(g +
 h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)
*(b*g - a*h))])/((d*e - c*f)*h)))/(d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])
```

Maple [B] time = 0.053, size = 2465, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*
g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a^2*f^3*h^2-EllipticF(((a*f-b*e)*(h*x+g)
/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*
x^2*a*b*e*f^2*h^2-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c
*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*f^3*g*h+EllipticF(((a
*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*
f-b*e))^(1/2))*x^2*b^2*e*f^2*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f
*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*
e))^(1/2))*x^2*a*b*e*f^2*h^2-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e
))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^
```

```
(1/2))*x^2*a*b*f^3*g*h-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/
2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))
*x^2*b^2*e*f^2*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),(
a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2
*b^2*f^3*g^2+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-
d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a^2*e*f^2*h^2-2*EllipticF(((a*
f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f
-b*e))^(1/2))*x*a*b*e^2*f*h^2-2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x
+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*e*f^2*g*h
+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*
g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b^2*e^2*f*g*h+2*EllipticPi(((a*f-b*e)*(h*x
+g)/(a*h-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(
c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*e^2*f*h^2-2*EllipticPi(((a*f-b*e)*(h*x+g)/
(a*h-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-
d*g)/(a*f-b*e))^(1/2))*x*a*b*e*f^2*g*h-2*EllipticPi(((a*f-b*e)*(h*x+g)/(a*h
-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)
/(a*f-b*e))^(1/2))*x*b^2*e^2*f*g*h+2*EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*
f-b*e))^(1/2))*x*b^2*e*f^2*g^2+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+
e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a^2*e^2*f*h^2-El
lipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c
*h-d*g)/(a*f-b*e))^(1/2))*a*b*e^3*h^2-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*b*e^2*f
*g*h+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-
b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b^2*e^3*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/
(a*h-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-
d*g)/(a*f-b*e))^(1/2))*a*b*e^3*h^2-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/
(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-
b*e))^(1/2))*a*b*e^2*f*g*h-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))
^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1
/2))*b^2*e^3*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),(a*
h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b^2*e
^2*f*g^2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c
*h-d*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)/h/f/(h*x
+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)/(e*h-f*g)/(a*f-b*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.109 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sqrt{e+fx}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{g+hx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{fg-eh}}\right),\frac{(bg-ah)(de-cf)}{(be-af)(dg-ch)}\right)}{\sqrt{c+dx}\sqrt{be-af}\sqrt{fg-eh}}$$

```
[Out] (-2*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x))]*Sqrt[e + f*x]*Ell
ipticF[ArcTan[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x
])], ((d*e - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[b*e - a*f]
*Sqrt[f*g - e*h]*Sqrt[c + d*x])
```

Rubi [A] time = 0.0820603, antiderivative size = 198, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {170, 419}

$$\frac{2\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Elli
pticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x]
)], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[b*g - a*
h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x)))])
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx = \frac{\left(2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(fg-eh)\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

Mathematica [A] time = 1.2895, size = 227, normalized size = 1.41

$$-\frac{2\sqrt{a+bx}\sqrt{g+hx}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(g+hx)(af-be)}{(a+bx)(fg-eh)}}\right), \frac{(ad-bc)(eh-fg)}{(be-af)(dg-ch)}\right)}{\sqrt{c+dx}\sqrt{e+fx}(bg-ah)\sqrt{\frac{(g+hx)(af-be)}{(a+bx)(fg-eh)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqr
t[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[
ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c)
+ a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])
```

Maple [A] time = 0.059, size = 270, normalized size = 1.7

$$2\frac{af^2hx^2-bf^2gx^2+2aefhx-2befgx+ae^2h-be^2g}{\sqrt{hx+g}\sqrt{fx+e}\sqrt{dx+c}\sqrt{bx+a}(eh-fg)(af-be)}\sqrt{\frac{(af-be)(hx+g)}{(ah-bg)(fx+e)}}\sqrt{\frac{(eh-fg)(dx+c)}{(ch-dg)(fx+e)}}\sqrt{\frac{(eh-fg)(bx+a)}{(ah-bg)(fx+e)}}\text{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*((a*f-b*e)*(h*x+g
)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*
h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b
*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*(a*f^2*
h*x^2-b*f^2*g*x^2+2*a*e*f*h*x-2*b*e*f*g*x+a*e^2*h-b*e^2*g)/(e*h-f*g)/(a*f-b
*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bdfhx^4+aceg+\left(bdfg+\left(bde+(bc+ad)f\right)h\right)x^3+\left(\left(bde+(bc+ad)f\right)g+\left(acf+(bc+ad)e\right)h\right)x^2+\left(aceh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*d*f*h*x
^4 + a*c*e*g + (b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^3 + ((b*d*e + (b*c +
 a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^2 + (a*c*e*h + (a*c*f + (b*c + a*
d)*e)*g)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.110 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=429

$$-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e+fx}\sqrt{bg-ah}}{\sqrt{a+bx}\sqrt{fg-eh}}\right),-\frac{(bc-ad)(fg-eh)}{(bg-ah)(de-cf)}\right)}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\,E\left(\sin^{-1}\right.}{\sqrt{g+hx}(bc-ad)(be-af)\sqrt{bg}}$$

```
[Out] (-2*b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*
g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a
*h)))]])/((b*c - a*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]) - (2*d*Sqrt[((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*
h))/((d*e - c*f)*(b*g - a*h)))]])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*
h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rubi [A] time = 0.256346, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {171, 170, 419, 176, 424}

$$-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}}{\sqrt{fg-eh}}\right.\right.}{\sqrt{g+hx}(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\ }}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*
g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a
*h)))]])/((b*c - a*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]) - (2*d*Sqrt[((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*
h))/((d*e - c*f)*(b*g - a*h)))]])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*
h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rule 171

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> -Dist[d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
```

```
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} - \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad}$$

$$= -\frac{\left(2d\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right.}{(bc-ad)(fg-eh)\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= -\frac{2b\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)\middle| -\frac{(bc-ad)(fg-}{(de-cf)(bg-}\right.}{(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

Mathematica [B] time = 14.3374, size = 3247, normalized size = 7.57

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*b^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)
*(b*g - a*h)*Sqrt[a + b*x]) - (2*(-((b*(c + d*x)^(3/2)*(f + (d*e)/(c + d*x)
 - (c*f)/(c + d*x))*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x))*Sqrt[a + ((c +
d*x)*(b - (b*c)/(c + d*x)))/d])/(Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x))
```

```
/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])) + ((b*c - a*d)*f*(b*g -
 a*h)*(-(d*g) + c*h)*Sqrt[c + d*x]*Sqrt[(b - (b*c)/(c + d*x) + (a*d)/(c + d
*x))*(f + (d*e)/(c + d*x) - (c*f)/(c + d*x))*(h + (d*g)/(c + d*x) - (c*h)/(
c + d*x))]*Sqrt[a + ((c + d*x)*(b - (b*c)/(c + d*x)))/d]*((d*e*Sqrt[-(((b*c
 - a*d)*(-(d*g) + c*h)*(-(b/(b*c - a*d)) + (c + d*x)^(-1)))/(-(b*d*g) + a*d
*h))]*(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))*Sqrt[(-(h/(-(d*g) + c*h)) + (c
 + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*(((-(b*d*g) + a*d*h)*E
llipticE[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/
(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g)
+ a*h))])/((b*c - a*d)*(-(d*g) + c*h)) - (b*EllipticF[ArcSin[Sqrt[((d*e - c
*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c -
a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(b*c - a*d)))/(Sqrt[
(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c
*h))]*Sqrt[(b + (-(b*c) + a*d)/(c + d*x))*(f + (d*e - c*f)/(c + d*x))*(h +
(d*g - c*h)/(c + d*x))]) - (c*f*Sqrt[-(((b*c - a*d)*(-(d*g) + c*h)*(-(b/(b*
c - a*d)) + (c + d*x)^(-1)))/(-(b*d*g) + a*d*h))]*(-(f/(-(d*e) + c*f)) + (c
 + d*x)^(-1))*Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f)
 - h/(-(d*g) + c*h))]*(((-(b*d*g) + a*d*h)*EllipticE[ArcSin[Sqrt[((d*e - c*
f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a
*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/((b*c - a*d)*(-(d*g)
+ c*h)) - (b*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h
)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) +
 c*f)*(-(b*g) + a*h))])/(b*c - a*d)))/(Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x
)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*Sqrt[(b + (-(b*c) + a*d)/
(c + d*x))*(f + (d*e - c*f)/(c + d*x))*(h + (d*g - c*h)/(c + d*x))]) + (f*S
qrt[(-(b/(b*c - a*d)) + (c + d*x)^(-1))/(-(b/(b*c - a*d)) + h/(-(d*g) + c*h
))]*Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-
(d*g) + c*h))]*(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))*EllipticF[ArcSin[Sqrt
[((-(d*e) + c*f)*(-h - (d*g)/(c + d*x) + (c*h)/(c + d*x)))/(d*(-(f*g) + e*h
))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(Sqrt[
(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h)
)]*Sqrt[(b + (-(b*c) + a*d)/(c + d*x))*(f + (d*e - c*f)/(c + d*x))*(h + (d*
g - c*h)/(c + d*x))])))/((f*g - e*h)*(b - (b*c)/(c + d*x) + (a*d)/(c + d*x)
)*Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))/d]*Sqrt[g + ((c + d*x)*(h - (c
*h)/(c + d*x)))/d]) - ((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*h*Sqrt[c + d*
x]*Sqrt[(b - (b*c)/(c + d*x) + (a*d)/(c + d*x))*(f + (d*e)/(c + d*x) - (c*f
)/(c + d*x))*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x))]*Sqrt[a + ((c + d*x)*(
b - (b*c)/(c + d*x)))/d]*((d*g*Sqrt[-(((b*c - a*d)*(-(d*g) + c*h)*(-(b/(b*c
 - a*d)) + (c + d*x)^(-1)))/(-(b*d*g) + a*d*h))]*(-(f/(-(d*e) + c*f)) + (c
+ d*x)^(-1))*Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f)
- h/(-(d*g) + c*h))]*(((-(b*d*g) + a*d*h)*EllipticE[ArcSin[Sqrt[((d*e - c*f
)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*
d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/((b*c - a*d)*(-(d*g) +
 c*h)) - (b*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)
/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) +
c*f)*(-(b*g) + a*h))])/(b*c - a*d)))/(Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)
^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*Sqrt[(b + (-(b*c) + a*d)/(
c + d*x))*(f + (d*e - c*f)/(c + d*x))*(h + (d*g - c*h)/(c + d*x))]) - (c*h*
Sqrt[-(((b*c - a*d)*(-(d*g) + c*h)*(-(b/(b*c - a*d)) + (c + d*x)^(-1)))/(-(
b*d*g) + a*d*h))]*(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))*Sqrt[(-(h/(-(d*g)
+ c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*(((-(b*d*g
) + a*d*h)*EllipticE[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/
(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c
*f)*(-(b*g) + a*h))])/((b*c - a*d)*(-(d*g) + c*h)) - (b*EllipticF[ArcSin[Sq
rt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))
]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(b*c - a
*d)))/(Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h
/(-(d*g) + c*h))]*Sqrt[(b + (-(b*c) + a*d)/(c + d*x))*(f + (d*e - c*f)/(c +
 d*x))*(h + (d*g - c*h)/(c + d*x))]) + (h*Sqrt[(-(b/(b*c - a*d)) + (c + d*x
```

```
)^(-1))/(-(b/(b*c - a*d)) + h/(-(d*g) + c*h))]*Sqrt[(-(f/(-(d*e) + c*f)) +
(c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*(-(h/(-(d*g) + c*
h)) + (c + d*x)^(-1))*EllipticF[ArcSin[Sqrt[((-(d*e) + c*f)*(-h - (d*g)/(c
+ d*x) + (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h
))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)
^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*Sqrt[(b + (-(b*c) + a*d)/(c +
 d*x))*(f + (d*e - c*f)/(c + d*x))*(h + (d*g - c*h)/(c + d*x))]])))/((f*g -
e*h)*(b - (b*c)/(c + d*x) + (a*d)/(c + d*x))*Sqrt[e + ((c + d*x)*(f - (c*f)
/(c + d*x)))/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])))/(d*(b*c -
a*d)*(b*e - a*f)*(b*g - a*h))
```

Maple [B] time = 0.095, size = 4660, normalized size = 10.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(x^2*b^2*d*e^2*h^
2+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g
)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b^2*d*f^2*g^2*((e*h-f*g)*(d*x+c)/(c*h-d*g
)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*
x+g)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e
))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a^2*d*f^2*h^2
*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(
f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticE(((a*f-b
*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*
e))^(1/2))*a*b*c*e^2*h^2*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-
f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))
^(1/2)-EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*
h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b^2*c*e^2*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*
g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h
*x+g)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+
e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*b*c*e^2*h^2*((
e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x
+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticF(((a*f-b*e)
*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))
^(1/2))*b^2*c*e^2*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g
)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1
/2)-2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h
-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*d*e*f*g*h*((e*h-f*g)*(d*x+c)/(c*h-d
*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(
h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)-x*a*b*d*e*f*g*h+EllipticE(((a*f-b*e)*(h*x+g
)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))
*b^2*d*e^2*g^2*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+
a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)+x*b
^2*c*e^2*h^2+a*b*c*f^2*g^2+b^2*c*e^2*g*h-b^2*c*e*f*g^2+x*a*b*d*f^2*g^2+x*b^
2*d*e^2*g*h-x*b^2*d*e*f*g^2-x^2*a*b*d*e*f*h^2+x^2*a*b*d*f^2*g*h-x^2*b^2*d*e
*f*g*h-x*a*b*c*e*f*h^2+x*a*b*c*f^2*g*h-x*b^2*c*e*f*g*h-2*EllipticF(((a*f-b*
e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e
))^(1/2))*x*a*b*d*e*f*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h
-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e)
)^(1/2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a
*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a^2*d*e^2*h^2*((e*h-f*g)*(d*x+c)/(c*h-d
*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(
h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)-a*b*c*e*f*g*h-EllipticE(((a*f-b*e)*(h*x+g)/
```

```
(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x
^2*a*b*d*f^2*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*
x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)-E
llipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(
c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*d*f^2*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(
f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g
)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e)
)^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*c*e*f*h^2*((
e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x
+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)-2*EllipticE(((a*f-b*
e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e
))^(1/2))*x*b^2*c*e*f*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h
-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e)
)^(1/2)-2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*
(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*c*e*f*h^2*((e*h-f*g)*(d*x+c)/(c
*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*
e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b^2*c*e
*f*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-
b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticE((
(a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(
a*f-b*e))^(1/2))*a*b*d*e^2*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*
((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f
*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*
e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*b*d*e^2*g*h*((e*h-f*g)*(d*x+c)/(
c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b
*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)
/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*c*
f^2*h^2*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h
-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticE(
((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/
(a*f-b*e))^(1/2))*x^2*b^2*c*f^2*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(
1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*
g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c
*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*c*f^2*h^2*((e*h-f*g)*
(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2
)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticF(((a*f-b*e)*(h*x+g)/
(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x
^2*b^2*c*f^2*g*h*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*
x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)+2
*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)
/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b^2*d*e*f*g^2*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(
f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g
)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e)
)^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a^2*d*e*f*h^2*((
e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x
+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2))/(a*h-b*g)/(e*h-f*g)
/(a*f-b*e)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+}}{b^2dfhx^5 + a^2ceg + \left(b^2dfg + \left(b^2de + \left(b^2c + 2\,abd\right)f\right)h\right)x^4 + \left(\left(b^2de + \left(b^2c + 2\,abd\right)f\right)g + \left(\left(b^2c + 2\,abd\right)e + \right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d*f*h
*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a*b*d)*f)*h)*x^4 + ((
b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)
*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*f)*g + (a^2*c*f + (2
*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*a*b*c + a^2*d)*e)*g)
*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.111 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=786

$$-\frac{4bd\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e+fx}\sqrt{bg-ah}}{\sqrt{a+bx}\sqrt{fg-eh}}\right),-\frac{(bc-ad)(fg-eh)}{(bg-ah)(de-cf)}\right)}{\sqrt{c+dx}(bc-ad)^2\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2d^2fh-abd^2\right.}{\sqrt{a+bx}(bc-ad)^2(be-af}$$

```
[Out] (-2*d^3*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(d*e - c*
f)*(d*g - c*h)*Sqrt[c + d*x]) - (2*b^3*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x])/((b*c - a*d)^2*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + (2*b*(a^2*d^
2*f*h - a*b*d^2*(f*g + e*h) + b^2*(2*d^2*e*g + c^2*f*h - c*d*(f*g + e*h)))*
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(b*e - a*f)*(d*e
- c*f)*(b*g - a*h)*(d*g - c*h)*Sqrt[a + b*x]) - (2*Sqrt[f*g - e*h]*(a^2*d^2
*f*h - a*b*d^2*(f*g + e*h) + b^2*(2*d^2*e*g + c^2*f*h - c*d*(f*g + e*h)))*S
qrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]*Ellip
ticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])
], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d)^2*
(b*e - a*f)*(d*e - c*f)*Sqrt[b*g - a*h]*(d*g - c*h)*Sqrt[((b*e - a*f)*(c +
d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]) - (4*b*d*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
- a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g
 - e*h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d)^2*Sqrt[b*g - a*h]*Sqrt[f
*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*
x)))])
```

Rubi [F] time = 0.010114, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Defer[Int][1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Mathematica [B] time = 17.4584, size = 7061, normalized size = 8.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.224, size = 21094, normalized size = 26.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{}{b^2d^2fhx^6 + a^2c^2eg + \left(b^2d^2fg + \left(b^2d^2e + 2\left(b^2cd + abd^2\right)f\right)h\right)x^5 + \left(\left(b^2d^2e + 2\left(b^2cd + abd^2\right)f\right)g + \left(2\left(b^2cd\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d^2*f
*h*x^6 + a^2*c^2*e*g + (b^2*d^2*f*g + (b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)
*h)*x^5 + ((b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)*g + (2*(b^2*c*d + a*b*d^2)
*e + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*h)*x^4 + ((2*(b^2*c*d + a*b*d^2)*e
+ (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*g + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e
 + 2*(a*b*c^2 + a^2*c*d)*f)*h)*x^3 + (((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e +
2*(a*b*c^2 + a^2*c*d)*f)*g + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*h)*x^2 +
 (a^2*c^2*e*h + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*g)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

3.112 $\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=319

$$\frac{\left(a^2d^2+abcd+b^2c^2\right)(e+fx)^{n+1}}{b^3d^3f(n+1)}-\frac{a^4(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^3(n+1)(bc-ad)(be-af)}+\frac{e(ad+bc)(e+fx)^{n+1}}{b^2d^2f^2(n+1)}-\frac{(ad+bc)(e+}{b^2d^2f^2(n}$$

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*d*f^3*(1 + n)) + ((b*c + a*d)*e*(e + f*x)^(1 + n
))/(b^2*d^2*f^2*(1 + n)) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(e + f*x)^(1 + n)
)/(b^3*d^3*f*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*d*f^3*(2 + n)) - ((b*c +
 a*d)*(e + f*x)^(2 + n))/(b^2*d^2*f^2*(2 + n)) + (e + f*x)^(3 + n)/(b*d*f^3
*(3 + n)) - (a^4*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
 + f*x))/(b*e - a*f)])/(b^3*(b*c - a*d)*(b*e - a*f)*(1 + n)) + (c^4*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(
d^3*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rubi [A] time = 0.279362, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {180, 43, 68}

$$\frac{\left(a^2d^2+abcd+b^2c^2\right)(e+fx)^{n+1}}{b^3d^3f(n+1)}-\frac{a^4(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^3(n+1)(bc-ad)(be-af)}+\frac{e(ad+bc)(e+fx)^{n+1}}{b^2d^2f^2(n+1)}-\frac{(ad+bc)(e+}{b^2d^2f^2(n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```


```
[Out] (e^2*(e + f*x)^(1 + n))/(b*d*f^3*(1 + n)) + ((b*c + a*d)*e*(e + f*x)^(1 + n
))/(b^2*d^2*f^2*(1 + n)) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(e + f*x)^(1 + n)
)/(b^3*d^3*f*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*d*f^3*(2 + n)) - ((b*c +
 a*d)*(e + f*x)^(2 + n))/(b^2*d^2*f^2*(2 + n)) + (e + f*x)^(3 + n)/(b*d*f^3
*(3 + n)) - (a^4*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
 + f*x))/(b*e - a*f)])/(b^3*(b*c - a*d)*(b*e - a*f)*(1 + n)) + (c^4*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(
d^3*(b*c - a*d)*(d*e - c*f)*(1 + n))
```


Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{\left(b^2c^2+abcd+a^2d^2\right)(e+fx)^n}{b^3d^3} - \frac{(bc+ad)x(e+fx)^n}{b^2d^2} + \frac{x^2(e+fx)^n}{bd} + \frac{a^4(e+fx)^n}{b^3(bc-ad)(a+bx)}\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)(e+fx)^{1+n}}{b^3d^3f(1+n)} + \frac{\int x^2(e+fx)^n\,dx}{bd} + \frac{a^4\int\frac{(e+fx)^n}{a+bx}\,dx}{b^3(bc-ad)} - \frac{c^4\int\frac{(e+fx)^n}{c+dx}\,dx}{d^3(bc-ad)} - \frac{(bc+}{}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{a^4(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^3(bc-ad)(be-af)(1+n)} + \frac{c^4(e+fx)^{1+n}\,{}_2}{d^3(bc-}$$

$$= \frac{e^2(e+fx)^{1+n}}{bdf^3(1+n)} + \frac{(bc+ad)e(e+fx)^{1+n}}{b^2d^2f^2(1+n)} + \frac{\left(b^2c^2+abcd+a^2d^2\right)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{2e(e+fx)^{2+n}}{bdf^3(2+n)} - \frac{(b}{}$$

Mathematica [A] time = 1.32229, size = 285, normalized size = 0.89

$$\frac{(e+fx)^{n+1}\left(\frac{b^3c^4f^3\left(n^2+5n+6\right){}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)-(bc-ad)(cf-de)\left(a^2d^2f^2\left(n^2+5n+6\right)+abdf(n+3)(cf(n+2)+d(e-f(n+1)x))+b^2\left(c^2f^2\left(n^2+5n+6\right)+cdf(\right.\right.}{f^3(n+2)(n+3)(ad-bc)(cf-de)}\right.}{b^3d^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((a^4*d^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f
*x))/(b*e - a*f)])/((b*c - a*d)*(b*e - a*f))) + (-((b*c - a*d)*(-(d*e) + c*
f)*(a^2*d^2*f^2*(6 + 5*n + n^2) + a*b*d*f*(3 + n)*(c*f*(2 + n) + d*(e - f*(
1 + n)*x)) + b^2*(c^2*f^2*(6 + 5*n + n^2) + c*d*f*(3 + n)*(e - f*(1 + n)*x)
 + d^2*(2*e^2 - 2*e*f*(1 + n)*x + f^2*(2 + 3*n + n^2)*x^2)))) + b^3*c^4*f^3
*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*
f)])/((-(b*c) + a*d)*f^3*(-(d*e) + c*f)*(2 + n)*(3 + n))))/(b^3*d^3*(1 + n)
)
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^4}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^4}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^4}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(fx+e)^n x^4}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

3.113 $\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=216

$$\frac{a^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^2(n+1)(bc-ad)(be-af)}-\frac{(ad+bc)(e+fx)^{n+1}}{b^2d^2f(n+1)}-\frac{c^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^2(n+1)(bc-ad)(de-cf)}-\frac{e(e+fx}{bdf^2(n}$$

```
[Out] -((e*(e + f*x)^(1 + n))/(b*d*f^2*(1 + n))) - ((b*c + a*d)*(e + f*x)^(1 + n)
)/(b^2*d^2*f*(1 + n)) + (e + f*x)^(2 + n)/(b*d*f^2*(2 + n)) + (a^3*(e + f*x
)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b
^2*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (c^3*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*(b*c - a*d)*(d*e - c*
f)*(1 + n))
```

Rubi [A] time = 0.15416, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {180, 43, 68}

$$\frac{a^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^2(n+1)(bc-ad)(be-af)}-\frac{(ad+bc)(e+fx)^{n+1}}{b^2d^2f(n+1)}-\frac{c^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^2(n+1)(bc-ad)(de-cf)}-\frac{e(e+fx}{bdf^2(n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*d*f^2*(1 + n))) - ((b*c + a*d)*(e + f*x)^(1 + n)
)/(b^2*d^2*f*(1 + n)) + (e + f*x)^(2 + n)/(b*d*f^2*(2 + n)) + (a^3*(e + f*x
)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b
^2*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (c^3*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*(b*c - a*d)*(d*e - c*
f)*(1 + n))
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{(-bc-ad)(e+fx)^n}{b^2d^2} + \frac{x(e+fx)^n}{bd} - \frac{a^3(e+fx)^n}{b^2(bc-ad)(a+bx)} - \frac{c^3(e+fx)^n}{d^2(-bc+ad)(c+dx)}\right) dx$$

$$= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{\int x(e+fx)^n\,dx}{bd} - \frac{a^3\int\frac{(e+fx)^n}{a+bx}\,dx}{b^2(bc-ad)} + \frac{c^3\int\frac{(e+fx)^n}{c+dx}\,dx}{d^2(bc-ad)}$$

$$= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)} - \frac{c^3(e+fx)^{1+n}\,{}_2F_1\left(1,1\right.}{d^2(bc-ad)(de-}$$

$$= -\frac{e(e+fx)^{1+n}}{bdf^2(1+n)} - \frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{(e+fx)^{2+n}}{bdf^2(2+n)} + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e}{be}\right.}{b^2(bc-ad)(be-af)(1+n)}$$

Mathematica [A] time = 0.496155, size = 174, normalized size = 0.81

$$\frac{(e+fx)^{n+1}\left(\frac{a^3\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{be-af} + \frac{(bc-ad)(cf-de)(adf(n+2)+bcf(n+2)+bd(e-f(n+1)x))-b^2c^3f^2(n+2)\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^2f^2(n+2)(de-cf)}\right)}{b^2(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((a^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(b*e - a*f) + ((b*c - a*d)*(-(d*e) + c*f)*(b*c*f*(2 + n) + a*d
*f*(2 + n) + b*d*(e - f*(1 + n)*x)) - b^2*c^3*f^2*(2 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*f^2*(d*e - c*f)*(2 + n))
))/(b^2*(b*c - a*d)*(1 + n))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x^{3}}{bdx^{2}+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\left(e+fx\right)^{n}}{(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(x**3*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}x^{3}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

3.114 $\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{a^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b(n+1)(bc-ad)(be-af)}+\frac{c^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d(n+1)(bc-ad)(de-cf)}+\frac{(e+fx)^{n+1}}{bdf(n+1)}$$

```
[Out] (e + f*x)^(1 + n)/(b*d*f*(1 + n)) - (a^2*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b*(b*c - a*d)*(b*e - a*f)*(
1 + n)) + (c^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e +
 f*x))/(d*e - c*f)])/(d*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rubi [A] time = 0.107468, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {180, 68}

$$-\frac{a^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b(n+1)(bc-ad)(be-af)}+\frac{c^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d(n+1)(bc-ad)(de-cf)}+\frac{(e+fx)^{n+1}}{bdf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*d*f*(1 + n)) - (a^2*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b*(b*c - a*d)*(b*e - a*f)*(
1 + n)) + (c^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e +
 f*x))/(d*e - c*f)])/(d*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx &= \int \left(\frac{(e+fx)^n}{bd}+\frac{a^2(e+fx)^n}{b(bc-ad)(a+bx)}+\frac{c^2(e+fx)^n}{d(-bc+ad)(c+dx)}\right)dx\\
&= \frac{(e+fx)^{1+n}}{bdf(1+n)}+\frac{a^2\int \frac{(e+fx)^n}{a+bx}\,dx}{b(bc-ad)}-\frac{c^2\int \frac{(e+fx)^n}{c+dx}\,dx}{d(bc-ad)}\\
&= \frac{(e+fx)^{1+n}}{bdf(1+n)}-\frac{a^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b(bc-ad)(be-af)(1+n)}+\frac{c^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;
\end{aligned}$$

Mathematica [A] time = 0.0740055, size = 153, normalized size = 0.97

$$\frac{(e+fx)^{n+1}\left(a^2df(cf-de)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+(be-af)\left(bc^2f\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)-(bc-ad)(cf-de)\right)\right)}{bdf(n+1)(bc-ad)(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(a^2*d*f*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 +
n, (b*(e + f*x))/(b*e - a*f)] + (b*e - a*f)*(-((b*c - a*d)*(-(d*e) + c*f))
+ b*c^2*f*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])))/
(b*d*(b*c - a*d)*f*(b*e - a*f)*(d*e - c*f)*(1 + n))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n} x^{2}}{\left(bx+a\right)\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n} x^{2}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^{n} x^{2}}{bdx^{2}+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^2/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^2}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

3.115 $\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}-\frac{c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}$$

```
[Out] (a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e
- a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n)) - (c*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e - c
*f)*(1 + n))
```

Rubi [A] time = 0.0385097, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {156, 68}

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}-\frac{c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e
- a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n)) - (c*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e - c
*f)*(1 + n))
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx &= -\frac{a\int \frac{(e+fx)^n}{a+bx}\,dx}{bc-ad}+\frac{c\int \frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}\\ &= \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)}-\frac{c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0262487, size = 116, normalized size = 0.94

$$\frac{(e+fx)^{n+1}\left(a(cf-de)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+c(be-af)\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)\right)}{(n+1)(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(a*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*
(e + f*x))/(b*e - a*f)] + c*(b*e - a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

3.116 $\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{d(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}-\frac{b(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}$$

```
[Out] -((b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n))) + (d*(e + f*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e
- c*f)*(1 + n))
```

Rubi [A] time = 0.0331753, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {86, 68}

$$\frac{d(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}-\frac{b(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n))) + (d*(e + f*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e
- c*f)*(1 + n))
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx &= \frac{b\int \frac{(e+fx)^n}{a+bx}\,dx}{bc-ad}-\frac{d\int \frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}\\ &= -\frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)}+\frac{d(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0297939, size = 116, normalized size = 0.94

$$\frac{(e+fx)^{n+1}\left(b(de-cf)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+d(af-be)\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)\right)}{(n+1)(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(b*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
+ f*x))/(b*e - a*f)] + d*(-(b*e) + a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

3.117 $\int \frac{(e+fx)^n}{x(a+bx)(c+dx)} dx$

Optimal. Leaf size=175

$$\frac{b^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a(n+1)(bc-ad)(be-af)}-\frac{d^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{c(n+1)(bc-ad)(de-cf)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\right.}{ace(n+1)}$$

```
[Out] (b^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/(a*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (d^2*(e + f*x)^(1 + n)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c*(b*c - a*d)*
(d*e - c*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n
, 1 + (f*x)/e])/(a*c*e*(1 + n))
```

Rubi [A] time = 0.119487, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {180, 65, 68}

$$\frac{b^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a(n+1)(bc-ad)(be-af)}-\frac{d^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{c(n+1)(bc-ad)(de-cf)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\right.}{ace(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] (b^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/(a*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (d^2*(e + f*x)^(1 + n)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c*(b*c - a*d)*
(d*e - c*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n
, 1 + (f*x)/e])/(a*c*e*(1 + n))
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \left(\frac{(e+fx)^n}{acx} + \frac{b^2(e+fx)^n}{a(-bc+ad)(a+bx)} + \frac{d^2(e+fx)^n}{c(bc-ad)(c+dx)}\right) dx$$
$$= \frac{\int \frac{(e+fx)^n}{x}\,dx}{ac} - \frac{b^2\int \frac{(e+fx)^n}{a+bx}\,dx}{a(bc-ad)} + \frac{d^2\int \frac{(e+fx)^n}{c+dx}\,dx}{c(bc-ad)}$$
$$= \frac{b^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a(bc-ad)(be-af)(1+n)} - \frac{d^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{c(bc-ad)(de-cf)(1+n)} - \frac{(e}{\ldots}$$

Mathematica [A] time = 0.183661, size = 170, normalized size = 0.97

$$-\frac{(e+fx)^{n+1}\left(b^2ce(de-cf)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+(af-be)\left(ad^2e\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)-(bc-ad)(cf-\ldots\right.\right.}{ace(n+1)(ad-bc)(af-be)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] -(((e + f*x)^(1 + n)*(b^2*c*e*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n
, (b*(e + f*x))/(b*e - a*f)] + (-(b*e) + a*f)*(a*d^2*e*Hypergeometric2F1[1,
 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)] - (b*c - a*d)*(-(d*e) + c*f)*Hype
rgeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])))/(a*c*(-(b*c) + a*d)*e*(-(b*e
) + a*f)*(-(d*e) + c*f)*(1 + n)))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n}{x\,(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n}{(bx+a)(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^n}{bdx^3+acx+(bc+ad)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}}{(bx+a)(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

3.118 $\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=222

$$-\frac{b^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a^2(n+1)(bc-ad)(be-af)}+\frac{(ad+bc)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{a^2c^2e(n+1)}+\frac{d^3(e+fx)^{n+1}\,{}_2F_1\left(1\right.}{c^2(n+1)(bc}$$

```
[Out] -((b^3*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f)*(1 + n))) + (d^3*(e + f*x)^(1 + n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c^2*(b*c
- a*d)*(d*e - c*f)*(1 + n)) + ((b*c + a*d)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*c^2*e*(1 + n)) + (f*(e + f*x)^(1 +
n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*c*e^2*(1 + n))
```

Rubi [A] time = 0.151256, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {180, 65, 68}

$$-\frac{b^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a^2(n+1)(bc-ad)(be-af)}+\frac{(ad+bc)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{a^2c^2e(n+1)}+\frac{d^3(e+fx)^{n+1}\,{}_2F_1\left(1\right.}{c^2(n+1)(bc}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```


```
[Out] -((b^3*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f)*(1 + n))) + (d^3*(e + f*x)^(1 + n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c^2*(b*c
- a*d)*(d*e - c*f)*(1 + n)) + ((b*c + a*d)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*c^2*e*(1 + n)) + (f*(e + f*x)^(1 +
n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*c*e^2*(1 + n))
```


Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \left(\frac{(e+fx)^n}{acx^2} + \frac{(-bc-ad)(e+fx)^n}{a^2c^2x} - \frac{b^3(e+fx)^n}{a^2(-bc+ad)(a+bx)} - \frac{d^3(e+fx)^n}{c^2(bc-ad)(c+dx)}\right) dx$$
$$= \frac{\int \frac{(e+fx)^n}{x^2}\,dx}{ac} + \frac{b^3 \int \frac{(e+fx)^n}{a+bx}\,dx}{a^2(bc-ad)} - \frac{d^3 \int \frac{(e+fx)^n}{c+dx}\,dx}{c^2(bc-ad)} - \frac{(bc+ad)\int \frac{(e+fx)^n}{x}\,dx}{a^2c^2}$$
$$= -\frac{b^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)} + \frac{d^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{c^2(bc-ad)(de-cf)(1+n)} + \text{—} \ (b\ldots$$

Mathematica [A] time = 0.267072, size = 177, normalized size = 0.8

$$\frac{(e+fx)^{n+1}\left(\frac{\frac{e(ad+bc)\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)+acf\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{a^2e^2}-\frac{d^3\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(cf-de)}}{c^2}-\frac{b^3\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((b^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))
/(b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f))) + (-((d^3*Hypergeometric2F1[1
, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(-(d*e) + c*f))) +
 ((b*c + a*d)*e*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e] + a*c*f*Hyp
ergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*e^2))/c^2))/(1 + n)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{x^2(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n}{bdx^4+acx^2+(bc+ad)x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^4 + a*c*x^2 + (b*c + a*d)*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

3.119 $\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=167

$$\frac{(a+bx)^{m+2}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}-\frac{(a+bx}{}$$

```
[Out] ((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(1 + m))/(b^4*(1 + m)) + ((3
*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(
a + b*x)^(2 + m))/(b^4*(2 + m)) - ((3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*
(a + b*x)^(3 + m))/(b^4*(3 + m)) + (d*f*h*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rubi [A] time = 0.130565, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {142}

$$\frac{(a+bx)^{m+2}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}-\frac{(a+bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(1 + m))/(b^4*(1 + m)) + ((3
*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(
a + b*x)^(2 + m))/(b^4*(2 + m)) - ((3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*
(a + b*x)^(3 + m))/(b^4*(3 + m)) + (d*f*h*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx=\int \left(\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^m}{b^3}+\frac{\left(3a^2dfh+b^2(deg+cfg+ceh)-2\right.}{b}\right.$$

$$=\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)}+\frac{\left(3a^2dfh+b^2(deg+cfg+ceh)-2ab\right.}{b^4(2+}$$

Mathematica [A] time = 0.233434, size = 149, normalized size = 0.89

$$\frac{(a+bx)^{m+1}\left(\frac{(a+bx)\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{m+2}+\frac{(a+bx)^2(b(cfh+deh+dfg)-3adfh)}{m+3}+\frac{(bc-ad)(be-af)(bg-ah)}{m+1}+\frac{dfh(a+bx)^3}{m+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(((b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(1 + m) + ((3*a^2
*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(a +
b*x))/(2 + m) + ((-3*a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(a + b*x)^2)/(3 +
 m) + (d*f*h*(a + b*x)^3)/(4 + m)))/b^4
```

Maple [B] time = 0.007, size = 726, normalized size = 4.4

$$-\frac{(bx+a)^{1+m}\left(-b^3dfhm^3x^3-b^3cfhm^3x^2-b^3dehm^3x^2-b^3dfgm^3x^2-6\,b^3dfhm^2x^3+3\,ab^2dfhm^2x^2-b^3cehm^3x-b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(1+m)*(-b^3*d*f*h*m^3*x^3-b^3*c*f*h*m^3*x^2-b^3*d*e*h*m^3*x^2-b^3*
d*f*g*m^3*x^2-6*b^3*d*f*h*m^2*x^3+3*a*b^2*d*f*h*m^2*x^2-b^3*c*e*h*m^3*x-b^3
*c*f*g*m^3*x-7*b^3*c*f*h*m^2*x^2-b^3*d*e*g*m^3*x-7*b^3*d*e*h*m^2*x^2-7*b^3*
d*f*g*m^2*x^2-11*b^3*d*f*h*m*x^3+2*a*b^2*c*f*h*m^2*x+2*a*b^2*d*e*h*m^2*x+2*
a*b^2*d*f*g*m^2*x+9*a*b^2*d*f*h*m*x^2-b^3*c*e*g*m^3-8*b^3*c*e*h*m^2*x-8*b^3
*c*f*g*m^2*x-14*b^3*c*f*h*m*x^2-8*b^3*d*e*g*m^2*x-14*b^3*d*e*h*m*x^2-14*b^3
*d*f*g*m*x^2-6*b^3*d*f*h*x^3-6*a^2*b*d*f*h*m*x+a*b^2*c*e*h*m^2+a*b^2*c*f*g*
m^2+10*a*b^2*c*f*h*m*x+a*b^2*d*e*g*m^2+10*a*b^2*d*e*h*m*x+10*a*b^2*d*f*g*m*
x+6*a*b^2*d*f*h*x^2-9*b^3*c*e*g*m^2-19*b^3*c*e*h*m*x-19*b^3*c*f*g*m*x-8*b^3
*c*f*h*x^2-19*b^3*d*e*g*m*x-8*b^3*d*e*h*x^2-8*b^3*d*f*g*x^2-2*a^2*b*c*f*h*m
-2*a^2*b*d*e*h*m-2*a^2*b*d*f*g*m-6*a^2*b*d*f*h*x+7*a*b^2*c*e*h*m+7*a*b^2*c*
f*g*m+8*a*b^2*c*f*h*x+7*a*b^2*d*e*g*m+8*a*b^2*d*e*h*x+8*a*b^2*d*f*g*x-26*b^
3*c*e*g*m-12*b^3*c*e*h*x-12*b^3*c*f*g*x-12*b^3*d*e*g*x+6*a^3*d*f*h-8*a^2*b*
c*f*h-8*a^2*b*d*e*h-8*a^2*b*d*f*g+12*a*b^2*c*e*h+12*a*b^2*c*f*g+12*a*b^2*d*
e*g-24*b^3*c*e*g)/b^4/(m^4+10*m^3+35*m^2+50*m+24)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60691, size = 1847, normalized size = 11.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*e*g*m^3 + (b^4*d*f*h*m^3 + 6*b^4*d*f*h*m^2 + 11*b^4*d*f*h*m + 6*b^
4*d*f*h)*x^4 + (8*b^4*d*f*g + (b^4*d*f*g + (b^4*d*e + (b^4*c + a*b^3*d)*f)*
h)*m^3 + (7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + 3*a*b^3*d)*f)*h)*m^2 + 8*(b
```

```
^4*d*e + b^4*c*f)*h + 2*(7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + a*b^3*d)*f)*
h)*m)*x^3 - (a^2*b^2*c*e*h + (a^2*b^2*c*f - (9*a*b^3*c - a^2*b^2*d)*e)*g)*m
^2 + (12*b^4*c*e*h + ((b^4*d*e + (b^4*c + a*b^3*d)*f)*g + (a*b^3*c*f + (b^4
*c + a*b^3*d)*e)*h)*m^3 + ((8*b^4*d*e + (8*b^4*c + 5*a*b^3*d)*f)*g + ((8*b^
4*c + 5*a*b^3*d)*e + (5*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m^2 + 12*(b^4*d*e + b^
4*c*f)*g + ((19*b^4*d*e + (19*b^4*c + 4*a*b^3*d)*f)*g + ((19*b^4*c + 4*a*b^
3*d)*e + (4*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m)*x^2 + 4*(3*(2*a*b^3*c - a^2*b^2
*d)*e - (3*a^2*b^2*c - 2*a^3*b*d)*f)*g - 2*(2*(3*a^2*b^2*c - 2*a^3*b*d)*e -
 (4*a^3*b*c - 3*a^4*d)*f)*h + (((26*a*b^3*c - 7*a^2*b^2*d)*e - (7*a^2*b^2*c
 - 2*a^3*b*d)*f)*g + (2*a^3*b*c*f - (7*a^2*b^2*c - 2*a^3*b*d)*e)*h)*m + (24
*b^4*c*e*g + (a*b^3*c*e*h + (a*b^3*c*f + (b^4*c + a*b^3*d)*e)*g)*m^3 + (((9
*b^4*c + 7*a*b^3*d)*e + (7*a*b^3*c - 2*a^2*b^2*d)*f)*g - (2*a^2*b^2*c*f - (
7*a*b^3*c - 2*a^2*b^2*d)*e)*h)*m^2 + 2*(((13*b^4*c + 6*a*b^3*d)*e + 2*(3*a*
b^3*c - 2*a^2*b^2*d)*f)*g + (2*(3*a*b^3*c - 2*a^2*b^2*d)*e - (4*a^2*b^2*c -
 3*a^3*b*d)*f)*h)*m)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50
*b^4*m + 24*b^4)
```

Sympy [A] time = 7.09604, size = 8218, normalized size = 49.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] Piecewise((a**m*(c*e*g*x + c*e*h*x**2/2 + c*f*g*x**2/2 + c*f*h*x**3/3 + d*e
*g*x**2/2 + d*e*h*x**3/3 + d*f*g*x**3/3 + d*f*h*x**4/4), Eq(b, 0)), (6*a**3
*d*f*h*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7
*x**3) + 11*a**3*d*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3) - 2*a**2*b*c*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
 6*b**7*x**3) - 2*a**2*b*d*e*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) - 2*a**2*b*d*f*g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*f*h*x*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d*f*h*x/(6*a**3*b**4
+ 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*e*h/(6*a**3*b**
4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*f*g/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*c*f*h*x/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*d*e*g/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*d*e
*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b*
*2*d*f*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*f*h*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 18*a*b**2*d*f*h*x**2/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c*e*g/(6*a**3*b**4 + 18*a**2*b**5*x
 + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*e*h*x/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*f*g*x/(6*a**3*b**4 + 18*a**
2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*c*f*h*x**2/(6*a**3*b**4 +
 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*d*e*g*x/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*e*h*x**2/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*f*g*x
**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*
d*f*h*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*
b**7*x**3), Eq(m, -4)), (-6*a**4*d*f*h*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b
**5*x + 2*a*b**6*x**2) - 3*a**4*d*f*h/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b*
*6*x**2) + 2*a**3*b*c*f*h*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b
**6*x**2) + a**3*b*c*f*h/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 2*
a**3*b*d*e*h*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + a
```

```
**3*b*d*e*h/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 2*a**3*b*d*f*g*
log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + a**3*b*d*f*g/(
2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 12*a**3*b*d*f*h*x*log(a/b +
x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 4*a**2*b**2*c*f*h*x*log(
a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 4*a**2*b**2*d*e*h*
x*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 4*a**2*b**2*
d*f*g*x*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 6*a**2
*b**2*d*f*h*x**2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2)
 + 6*a**2*b**2*d*f*h*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - a
*b**3*c*e*g/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 2*a*b**3*c*f*h*
x**2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 2*a*b**3*
c*f*h*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 2*a*b**3*d*e*h*x
**2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 2*a*b**3*d
*e*h*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 2*a*b**3*d*f*g*x*
*2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 2*a*b**3*d*
f*g*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 2*a*b**3*d*f*h*x**
3/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + b**4*c*e*h*x**2/(2*a**3*b
**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + b**4*c*f*g*x**2/(2*a**3*b**4 + 4*a**
2*b**5*x + 2*a*b**6*x**2) + b**4*d*e*g*x**2/(2*a**3*b**4 + 4*a**2*b**5*x +
2*a*b**6*x**2), Eq(m, -3)), (6*a**3*d*f*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x
) + 6*a**3*d*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h*log(a/b + x)/(2*a*b
**4 + 2*b**5*x) - 4*a**2*b*c*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*e*h*log
(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*e*h/(2*a*b**4 + 2*b**5*x) - 4*
a**2*b*d*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*f*g/(2*a*b**4
+ 2*b**5*x) + 6*a**2*b*d*f*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**
2*c*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*e*h/(2*a*b**4 + 2*b
**5*x) + 2*a*b**2*c*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*f*g
/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x
) + 2*a*b**2*d*e*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*d*e*g/(2*a
*b**4 + 2*b**5*x) - 4*a*b**2*d*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4
*a*b**2*d*f*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*f*h*x**2/(2
*a*b**4 + 2*b**5*x) - 2*b**3*c*e*g/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*e*h*x*l
og(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*g*x*log(a/b + x)/(2*a*b**4 +
 2*b**5*x) + 2*b**3*c*f*h*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e*g*x*log(a
/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e*h*x**2/(2*a*b**4 + 2*b**5*x) + 2
*b**3*d*f*g*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d*f*h*x**3/(2*a*b**4 + 2*b**5
*x), Eq(m, -2)), (-a**3*d*f*h*log(a/b + x)/b**4 + a**2*c*f*h*log(a/b + x)/b
**3 + a**2*d*e*h*log(a/b + x)/b**3 + a**2*d*f*g*log(a/b + x)/b**3 + a**2*d*
f*h*x/b**3 - a*c*e*h*log(a/b + x)/b**2 - a*c*f*g*log(a/b + x)/b**2 - a*c*f*
h*x/b**2 - a*d*e*g*log(a/b + x)/b**2 - a*d*e*h*x/b**2 - a*d*f*g*x/b**2 - a*
d*f*h*x**2/(2*b**2) + c*e*g*log(a/b + x)/b + c*e*h*x/b + c*f*g*x/b + c*f*h*
x**2/(2*b) + d*e*g*x/b + d*e*h*x**2/(2*b) + d*f*g*x**2/(2*b) + d*f*h*x**3/(
3*b), Eq(m, -1)), (-6*a**4*d*f*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
5*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*c*f*h*m*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*b*c*f*h*(a
 + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
+ 2*a**3*b*d*e*h*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
50*b**4*m + 24*b**4) + 8*a**3*b*d*e*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**
3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*d*f*g*m*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*b*d*f
*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 6*a**3*b*d*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*e*h*m**2*(a + b*x)**m/(b**4*m**4
+ 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*c*e*h*m*
(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4
) - 12*a**2*b**2*c*e*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**
2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*f*g*m**2*(a + b*x)**m/(b**4*m**4 + 1
0*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*c*f*g*m*(a
+ b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) -
```

12*a**2*b**2*c*f*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b**2*c*f*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*c*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*d*e*g*m**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*d*e*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 12*a**2*b**2*d*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b**2*d*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*d*e*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b**2*d*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*d*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d*f*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*g*m**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c*e*g*m**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*a*b**3*c*e*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*a*b**3*c*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*h*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b**3*c*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 12*a*b**3*c*e*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*g*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b**3*c*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 12*a*b**3*c*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*h*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 5*a*b**3*c*f*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*c*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*g*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b**3*d*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 12*a*b**3*d*e*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*h*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 5*a*b**3*d*e*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*e*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*g*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 5*a*b**3*d*f*g*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*f*g*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*h*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*d*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a*b**3*d*f*h*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*e*g*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*b**4*c*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*b**4*c*e*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b**4*c*e*g*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*e*h*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*e*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*c*e*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +

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12*b**4*c*e*h*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
50*b**4*m + 24*b**4) + b**4*c*f*g*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b*
*4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*g*m**2*x**2*(a +
 b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
19*b**4*c*f*g*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 12*b**4*c*f*g*x**2*(a + b*x)**m/(b**4*m**4 + 10*b*
*4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*f*h*m**3*x**3*(a + b
*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*
b**4*c*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 14*b**4*c*f*h*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*h*x**3*(a + b*
x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**
4*d*e*g*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 5
0*b**4*m + 24*b**4) + 8*b**4*d*e*g*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b
**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*d*e*g*m*x**2*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 1
2*b**4*d*e*g*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 5
0*b**4*m + 24*b**4) + b**4*d*e*h*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**
4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*b**4*d*e*h*m**2*x**3*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 1
4*b**4*d*e*h*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
 50*b**4*m + 24*b**4) + 8*b**4*d*e*h*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4
*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*d*f*g*m**3*x**3*(a + b*x
)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*b*
*4*d*f*g*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
50*b**4*m + 24*b**4) + 14*b**4*d*f*g*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b*
*4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*f*g*x**3*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*
d*f*h*m**3*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 6*b**4*d*f*h*m**2*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**
4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 11*b**4*d*f*h*m*x**4*(a + b*
x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*b
**4*d*f*h*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b
**4*m + 24*b**4), True))
```

Giac [B] time = 2.65803, size = 2248, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d*f*h*m^3*x^4 + (b*x + a)^m*b^4*d*f*g*m^3*x^3 + (b*x + a)^
m*b^4*c*f*h*m^3*x^3 + (b*x + a)^m*a*b^3*d*f*h*m^3*x^3 + 6*(b*x + a)^m*b^4*d
*f*h*m^2*x^4 + (b*x + a)^m*b^4*d*h*m^3*x^3*e + (b*x + a)^m*b^4*c*f*g*m^3*x^
2 + (b*x + a)^m*a*b^3*d*f*g*m^3*x^2 + (b*x + a)^m*a*b^3*c*f*h*m^3*x^2 + 7*(
b*x + a)^m*b^4*d*f*g*m^2*x^3 + 7*(b*x + a)^m*b^4*c*f*h*m^2*x^3 + 3*(b*x + a
)^m*a*b^3*d*f*h*m^2*x^3 + 11*(b*x + a)^m*b^4*d*f*h*m*x^4 + (b*x + a)^m*b^4*
d*g*m^3*x^2*e + (b*x + a)^m*b^4*c*h*m^3*x^2*e + (b*x + a)^m*a*b^3*d*h*m^3*x
^2*e + 7*(b*x + a)^m*b^4*d*h*m^2*x^3*e + (b*x + a)^m*a*b^3*c*f*g*m^3*x + 8*
(b*x + a)^m*b^4*c*f*g*m^2*x^2 + 5*(b*x + a)^m*a*b^3*d*f*g*m^2*x^2 + 5*(b*x
+ a)^m*a*b^3*c*f*h*m^2*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m^2*x^2 + 14*(b*x
+ a)^m*b^4*d*f*g*m*x^3 + 14*(b*x + a)^m*b^4*c*f*h*m*x^3 + 2*(b*x + a)^m*a*b
^3*d*f*h*m*x^3 + 6*(b*x + a)^m*b^4*d*f*h*x^4 + (b*x + a)^m*b^4*c*g*m^3*x*e
+ (b*x + a)^m*a*b^3*d*g*m^3*x*e + (b*x + a)^m*a*b^3*c*h*m^3*x*e + 8*(b*x +
a)^m*b^4*d*g*m^2*x^2*e + 8*(b*x + a)^m*b^4*c*h*m^2*x^2*e + 5*(b*x + a)^m*a*
```

```
b^3*d*h*m^2*x^2*e + 14*(b*x + a)^m*b^4*d*h*m*x^3*e + 7*(b*x + a)^m*a*b^3*c*
f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*c*f*h
*m^2*x + 19*(b*x + a)^m*b^4*c*f*g*m*x^2 + 4*(b*x + a)^m*a*b^3*d*f*g*m*x^2 +
 4*(b*x + a)^m*a*b^3*c*f*h*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m*x^2 + 8*(b
*x + a)^m*b^4*d*f*g*x^3 + 8*(b*x + a)^m*b^4*c*f*h*x^3 + (b*x + a)^m*a*b^3*c
*g*m^3*e + 9*(b*x + a)^m*b^4*c*g*m^2*x*e + 7*(b*x + a)^m*a*b^3*d*g*m^2*x*e
+ 7*(b*x + a)^m*a*b^3*c*h*m^2*x*e - 2*(b*x + a)^m*a^2*b^2*d*h*m^2*x*e + 19*
(b*x + a)^m*b^4*d*g*m*x^2*e + 19*(b*x + a)^m*b^4*c*h*m*x^2*e + 4*(b*x + a)^
m*a*b^3*d*h*m*x^2*e + 8*(b*x + a)^m*b^4*d*h*x^3*e - (b*x + a)^m*a^2*b^2*c*f
*g*m^2 + 12*(b*x + a)^m*a*b^3*c*f*g*m*x - 8*(b*x + a)^m*a^2*b^2*d*f*g*m*x -
 8*(b*x + a)^m*a^2*b^2*c*f*h*m*x + 6*(b*x + a)^m*a^3*b*d*f*h*m*x + 12*(b*x
+ a)^m*b^4*c*f*g*x^2 + 9*(b*x + a)^m*a*b^3*c*g*m^2*e - (b*x + a)^m*a^2*b^2*
d*g*m^2*e - (b*x + a)^m*a^2*b^2*c*h*m^2*e + 26*(b*x + a)^m*b^4*c*g*m*x*e +
12*(b*x + a)^m*a*b^3*d*g*m*x*e + 12*(b*x + a)^m*a*b^3*c*h*m*x*e - 8*(b*x +
a)^m*a^2*b^2*d*h*m*x*e + 12*(b*x + a)^m*b^4*d*g*x^2*e + 12*(b*x + a)^m*b^4*
c*h*x^2*e - 7*(b*x + a)^m*a^2*b^2*c*f*g*m + 2*(b*x + a)^m*a^3*b*d*f*g*m + 2
*(b*x + a)^m*a^3*b*c*f*h*m + 26*(b*x + a)^m*a*b^3*c*g*m*e - 7*(b*x + a)^m*a
^2*b^2*d*g*m*e - 7*(b*x + a)^m*a^2*b^2*c*h*m*e + 2*(b*x + a)^m*a^3*b*d*h*m*
e + 24*(b*x + a)^m*b^4*c*g*x*e - 12*(b*x + a)^m*a^2*b^2*c*f*g + 8*(b*x + a)
^m*a^3*b*d*f*g + 8*(b*x + a)^m*a^3*b*c*f*h - 6*(b*x + a)^m*a^4*d*f*h + 24*(
b*x + a)^m*a*b^3*c*g*e - 12*(b*x + a)^m*a^2*b^2*d*g*e - 12*(b*x + a)^m*a^2*
b^2*c*h*e + 8*(b*x + a)^m*a^3*b*d*h*e)/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 +
 50*b^4*m + 24*b^4)
```

3.120 $\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx$

Optimal. Leaf size=134

$$\frac{(a+bx)^{m+1}(dg-ch)(fg-eh)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}-\frac{(a+bx)^{m+1}(adfh+b(m+2)(-cfh-deh+dfg)-bd}{b^2h^2(m+1)(m+2)}$$

```
[Out] -(((a + b*x)^(1 + m)*(a*d*f*h + b*(d*f*g - d*e*h - c*f*h)*(2 + m) - b*d*f*h
*(1 + m)*x))/(b^2*h^2*(1 + m)*(2 + m))) + ((d*g - c*h)*(f*g - e*h)*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])
/(h^2*(b*g - a*h)*(1 + m))
```

Rubi [A] time = 0.0860064, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {147, 68}

$$\frac{(a+bx)^{m+1}(dg-ch)(fg-eh)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}-\frac{(a+bx)^{m+1}(adfh+b(m+2)(-cfh-deh+dfg)-bd}{b^2h^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*d*f*h + b*(d*f*g - d*e*h - c*f*h)*(2 + m) - b*d*f*h
*(1 + m)*x))/(b^2*h^2*(1 + m)*(2 + m))) + ((d*g - c*h)*(f*g - e*h)*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])
/(h^2*(b*g - a*h)*(1 + m))
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = -\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)} + \frac{((dg-ch)(fg-e}{h}$$

$$= -\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)} + \frac{(dg-ch)(fg-e}{}$$

Mathematica [A] time = 0.190052, size = 120, normalized size = 0.9

$$\frac{(a+bx)^{m+1}\left(\frac{b(cfh+deh-dfg)-adfh}{b^2(m+1)}+\frac{dfh(a+bx)}{b^2(m+2)}+\frac{(dg-ch)(fg-eh)\,{}_2F_1\left(1,m+1;m+2;\frac{h(a+bx)}{ah-bg}\right)}{(m+1)(bg-ah)}\right)}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*((-(a*d*f*h) + b*(-(d*f*g) + d*e*h + c*f*h))/(b^2*(1 + m
)) + (d*f*h*(a + b*x))/(b^2*(2 + m)) + ((d*g - c*h)*(f*g - e*h)*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/((b*g - a*h)*(1 + m)
)))/h^2
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(fx+e)(dx+c)(bx+a)^m}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)(fx+e)(bx+a)^m}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dfx^2+ce+(de+cf)x)(bx+a)^m}{hx+g},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((d*f*x^2 + c*e + (d*e + c*f)*x)*(b*x + a)^m/(h*x + g), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m (c+dx) \left(e+fx\right)}{g+hx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] Integral((a + b*x)**m*(c + d*x)*(e + f*x)/(g + h*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)\left(fx+e\right)(bx+a)^m}{hx+g}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

3.121 $\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=140

$$\frac{(a+bx)^{m+1}(dg-ch)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}-\frac{(a+bx)^{m+1}(de-cf)\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

```
[Out] -(((d*e - c*f)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a
 + b*x))/(b*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m))) + ((d*g - c*h)*(
a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g -
a*h))])/((b*g - a*h)*(f*g - e*h)*(1 + m))
```

Rubi [A] time = 0.0591633, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {156, 68}

$$\frac{(a+bx)^{m+1}(dg-ch)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}-\frac{(a+bx)^{m+1}(de-cf)\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```


```
[Out] -(((d*e - c*f)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a
 + b*x))/(b*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m))) + ((d*g - c*h)*(
a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g -
a*h))])/((b*g - a*h)*(f*g - e*h)*(1 + m))
```


Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx &= -\frac{(de-cf)\int \frac{(a+bx)^m}{e+fx}\,dx}{fg-eh}+\frac{(dg-ch)\int \frac{(a+bx)^m}{g+hx}\,dx}{fg-eh}\\ &= -\frac{(de-cf)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)}+\frac{(dg-ch)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+\right.}{(bg-ah)(fg-eh)(1+m}\end{aligned}$$

Mathematica [A] time = 0.0939513, size = 115, normalized size = 0.82

$$\frac{(a+bx)^{m+1}\left(\frac{(dg-ch)\,{}_2F_1\left(1,m+1;m+2;\frac{h(a+bx)}{ah-bg}\right)}{bg-ah}-\frac{(de-cf)\,{}_2F_1\left(1,m+1;m+2;\frac{f(a+bx)}{af-be}\right)}{be-af}\right)}{(m+1)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(((d*e - c*f)*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(a
 + b*x))/(-(b*e) + a*f)])/(b*e - a*f)) + ((d*g - c*h)*Hypergeometric2F1[1,
1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/(b*g - a*h)))/((f*g - e*h)*(1
+ m))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(dx+c)\,(bx+a)^m}{(fx+e)\,(hx+g)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)(bx+a)^m}{fhx^2+eg+(fg+eh)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*(b*x + a)^m/(f*h*x^2 + e*g + (f*g + e*h)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m (c+dx)}{(e+fx)(g+hx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Integral((a + b*x)**m*(c + d*x)/((e + f*x)*(g + h*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

3.122 $\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=224

$$\frac{d^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)}-\frac{f^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)}+\frac{h^2(a+bx)^{m+1}\,{}_2F_1\left(1,m\right.}{(m+1)(bg-ah)(}$$

```
[Out] (d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*(1 + m)) - (f^2*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f))])
/((b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(1 + m)) + (h^2*(a + b*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g - a*h)
*(d*g - c*h)*(f*g - e*h)*(1 + m))
```

Rubi [A] time = 0.194952, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {180, 68}

$$\frac{d^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)}-\frac{f^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)}+\frac{h^2(a+bx)^{m+1}\,{}_2F_1\left(1,m\right.}{(m+1)(bg-ah)(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*(1 + m)) - (f^2*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f))])
/((b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(1 + m)) + (h^2*(a + b*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g - a*h)
*(d*g - c*h)*(f*g - e*h)*(1 + m))
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \left(\frac{d^2(a+bx)^m}{(de-cf)(dg-ch)(c+dx)} + \frac{f^2(a+bx)^m}{(de-cf)(-fg+eh)(e+fx)} + \frac{h^2(a+bx)^m}{(dg-ch)(fg-eh)(g+\right.$$

$$= \frac{d^2\int\frac{(a+bx)^m}{c+dx}\,dx}{(de-cf)(dg-ch)} - \frac{f^2\int\frac{(a+bx)^m}{e+fx}\,dx}{(de-cf)(fg-eh)} + \frac{h^2\int\frac{(a+bx)^m}{g+hx}\,dx}{(dg-ch)(fg-eh)}$$

$$= \frac{d^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)} - \frac{f^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a}{be}\right.}{(be-af)(de-cf)(fg-eh)(1+m)}$$

Mathematica [A] time = 0.395389, size = 193, normalized size = 0.86

$$\frac{(a+bx)^{m+1}\left(\frac{d^2\,{}_2F_1\left(1,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)}{(bc-ad)(cf-de)(ch-dg)} + \frac{f^2\,{}_2F_1\left(1,m+1;m+2;\frac{f(a+bx)}{af-be}\right)}{(be-af)(de-cf)(eh-fg)} + \frac{h^2\,{}_2F_1\left(1,m+1;m+2;\frac{h(a+bx)}{ah-bg}\right)}{(bg-ah)(dg-ch)(fg-eh)}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^(1 + m)*((d^2*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(
-(b*c) + a*d)])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) + (f^2*Hypergeo
metric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b*e) + a*f)])/((b*e - a*f)*(d*e
 - c*f)*(-(f*g) + e*h)) + (h^2*Hypergeometric2F1[1, 1 + m, 2 + m, (h*(a + b
*x))/(-(b*g) + a*h)])/((b*g - a*h)*(d*g - c*h)*(f*g - e*h))))/(1 + m)
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(hx+g)(fx+e)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^m}{dfhx^3+ceg+\left(dfg+\left(de+cf\right)h\right)x^2+\left(ceh+\left(de+cf\right)g\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*f*h*x^3 + c*e*g + (d*f*g + (d*e + c*f)*h)*x^2 + (c*
e*h + (d*e + c*f)*g)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)\left(fx+e\right)\left(hx+g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

3.123 $\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)} \, dx$

Optimal. Leaf size=140

$$\frac{bx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\right.}{c(m+1)(bc-ad)}$$

```
[Out] (b*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b*x)/
a)])/(a*(b*c - a*d)*(1 + m)*(1 + (f*x)/e)^n) - (d*x^(1 + m)*(e + f*x)^n*App
ellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)])/(c*(b*c - a*d)*(1 + m)*
(1 + (f*x)/e)^n)
```

Rubi [A] time = 0.122773, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {180, 135, 133}

$$\frac{bx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\right.}{c(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (b*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b*x)/
a)])/(a*(b*c - a*d)*(1 + m)*(1 + (f*x)/e)^n) - (d*x^(1 + m)*(e + f*x)^n*App
ellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)])/(c*(b*c - a*d)*(1 + m)*
(1 + (f*x)/e)^n)
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{bx^m(e+fx)^n}{(bc-ad)(a+bx)} - \frac{dx^m(e+fx)^n}{(bc-ad)(c+dx)}\right)dx$$
$$= \frac{b\int \frac{x^m(e+fx)^n}{a+bx}\,dx}{bc-ad} - \frac{d\int \frac{x^m(e+fx)^n}{c+dx}\,dx}{bc-ad}$$
$$= \frac{\left(b(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{a+bx}\,dx}{bc-ad} - \frac{\left(d(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{c+dx}\,dx}{bc-ad}$$
$$= \frac{bx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(bc-ad)(1+m)} - \frac{dx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F}{c(bc-}$$

Mathematica [A] time = 0.216623, size = 104, normalized size = 0.74

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\left(adF_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{dx}{c}\right)-bcF_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{bx}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*(-(b*c*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((
b*x)/a)]) + a*d*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)]))/(a*
c*(-(b*c) + a*d)*(1 + m)*(1 + (f*x)/e)^n)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x^{m}}{bdx^{2}+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^m/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}x^{m}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

3.124 $\int (a+bx)^m (c+dx)^n (e+fx)(g+hx)\,dx$

Optimal. Leaf size=266

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m+1)-d\right.}{b^3d^2(m+1)(m+n}$$

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*h*(2 + m) + a*d*f*h*(2 + n) -
 b*d*(f*g + e*h)*(3 + m + n) - b*d*f*h*(2 + m + n)*x))/(b^2*d^2*(2 + m + n)
*(3 + m + n))) + ((a^2*d^2*f*h*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*h*(1
+ m) - d*(f*g + e*h)*(3 + m + n)) + b^2*(c^2*f*h*(1 + m)*(2 + m) - c*d*(f*g
 + e*h)*(1 + m)*(3 + m + n) + d^2*e*g*(2 + m + n)*(3 + m + n)))*(a + b*x)^(
1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(b*c - a
*d))^n)
```

Rubi [A] time = 0.165118, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {147, 70, 69}

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m+1)-d\right.}{b^3d^2(m+1)(m+n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```


```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*h*(2 + m) + a*d*f*h*(2 + n) -
 b*d*(f*g + e*h)*(3 + m + n) - b*d*f*h*(2 + m + n)*x))/(b^2*d^2*(2 + m + n)
*(3 + m + n))) + ((a^2*d^2*f*h*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*h*(1
+ m) - d*(f*g + e*h)*(3 + m + n)) + b^2*(c^2*f*h*(1 + m)*(2 + m) - c*d*(f*g
 + e*h)*(1 + m)*(3 + m + n) + d^2*e*g*(2 + m + n)*(3 + m + n)))*(a + b*x)^(
1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(b*c - a
*d))^n)
```


Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n}{b^2d^2(2+m+n)(3+m+n)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n}{b^2d^2(2+m+n)(3+m+n)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n}{b^2d^2(2+m+n)(3+m+n)}$$

Mathematica [A] time = 0.235778, size = 195, normalized size = 0.73

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(b\left(b(de-cf)(dg-ch)\,{}_2F_1\left(m+1,-n;m+2;\frac{d(a+bx)}{ad-bc}\right)-(bc-ad)(2cfh-d(eh+fg))\,{}_2F_1\right.\right.}{b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*((b*c - a*d)^2*f*h*Hypergeometric2F1[1 + m,
-2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h -
d*(f*g + e*h))*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (d*(a + b*x))/(-(b*c
) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[1 + m, -n, 2 + m,
(d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*((b*(c + d*x))/(b*c - a*d
))^n)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int (bx+a)^m(dx+c)^n(fx+e)(hx+g)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fhx^2 + eg + \left(fg + eh\right)x\right)(bx + a)^m (dx + c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\left(hx + g\right)(bx + a)^m (dx + c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

3.125 $\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=245

$$\frac{(bc-ad)(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m-1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-5m+6\right)-2abd(2-m)(2d(e}{12b^4d^2(m+1)}$$

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(2 - m)*(4*b*d*(f*g + e*h) - a*d*f*h*(3 - m) -
 b*c*f*h*(2 + m) + 3*b*d*f*h*x))/(12*b^2*d^2) + ((b*c - a*d)*(a^2*d^2*f*h*(
6 - 5*m + m^2) - 2*a*b*d*(2 - m)*(2*d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(1
2*d^2*e*g - 4*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)
^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[-1 + m, 1 + m, 2 +
 m, -((d*(a + b*x))/(b*c - a*d))])/(12*b^4*d^2*(1 + m)*(c + d*x)^m)
```

Rubi [A] time = 0.154641, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {147, 70, 69}

$$\frac{(bc-ad)(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m-1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-5m+6\right)-2abd(2-m)(2d(e}{12b^4d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```


```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(2 - m)*(4*b*d*(f*g + e*h) - a*d*f*h*(3 - m) -
 b*c*f*h*(2 + m) + 3*b*d*f*h*x))/(12*b^2*d^2) + ((b*c - a*d)*(a^2*d^2*f*h*(
6 - 5*m + m^2) - 2*a*b*d*(2 - m)*(2*d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(1
2*d^2*e*g - 4*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)
^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[-1 + m, 1 + m, 2 +
 m, -((d*(a + b*x))/(b*c - a*d))])/(12*b^4*d^2*(1 + m)*(c + d*x)^m)
```


Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3b}{12b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3b}{12b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3b}{12b^2d^2}$$

Mathematica [A] time = 0.260858, size = 195, normalized size = 0.8

$$\frac{(a+bx)^{m+1}(c+dx)^{1-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{m-1}\left(b\left(b(de-cf)(dg-ch)\,{}_2F_1\left(m-1,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)-(bc-ad)(2cfh-d(eh\right.\right.}{b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*((b*(c + d*x))/(b*c - a*d))^(-1 + m)*(
(b*c - a*d)^2*f*h*Hypergeometric2F1[-3 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(
b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric2F1
[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(d*g
- c*h)*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)
])))/(b^3*d^2*(1 + m))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^{1-m} (fx+e)(hx+g)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fhx^2 + eg + \left(fg + eh\right)x\right)(bx + a)^m(dx + c)^{-m+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\left(hx + g\right)(bx + a)^m(dx + c)^{-m+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

3.126 $\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=235

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-3m+2\right)-abd(1-m)(3d(eh+fg)-2\right.}{6b^3d^2(m+1)}$$

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*(3*b*d*(f*g + e*h) - a*d*f*h*(2 - m) -
 b*c*f*h*(2 + m) + 2*b*d*f*h*x))/(6*b^2*d^2) + ((a^2*d^2*f*h*(2 - 3*m + m^2
) - a*b*d*(1 - m)*(3*d*(f*g + e*h) - 2*c*f*h*(1 + m)) + b^2*(6*d^2*e*g - 3*
c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*((b*(
c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))
/(b*c - a*d))])/(6*b^3*d^2*(1 + m)*(c + d*x)^m)
```

Rubi [A] time = 0.13751, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {147, 70, 69}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-3m+2\right)-abd(1-m)(3d(eh+fg)-2\right.}{6b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```


```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*(3*b*d*(f*g + e*h) - a*d*f*h*(2 - m) -
 b*c*f*h*(2 + m) + 2*b*d*f*h*x))/(6*b^2*d^2) + ((a^2*d^2*f*h*(2 - 3*m + m^2
) - a*b*d*(1 - m)*(3*d*(f*g + e*h) - 2*c*f*h*(1 + m)) + b^2*(6*d^2*e*g - 3*
c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*((b*(
c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))
/(b*c - a*d))])/(6*b^3*d^2*(1 + m)*(c + d*x)^m)
```


Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfh}{6b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfh}{6b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfh}{6b^2d^2}$$

Mathematica [A] time = 0.20556, size = 189, normalized size = 0.8

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(b\left(b(de-cf)(dg-ch)\,{}_2F_1\left(m,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)-(bc-ad)(2cfh-d(eh+fg))\,{}_2F_1\right.\right.}{b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*((b*c - a*d)^2*f*h*Hyperge
ometric2F1[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c
- a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (d
*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[
m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*(c + d*x
)^m)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{(hx+g)(fx+e)(bx+a)^m}{(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

```
[Out] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fhx^2 + eg + \left(fg + eh\right)x\right)(bx + a)^m}{(dx + c)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m/(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(f*x+e)*(h*x+g)/((d*x+c)**m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx + e\right)\left(hx + g\right)(bx + a)^m}{(dx + c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

3.127 $\int (a+bx)^m (c+dx)^{-1-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=261

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-\frac{(a+bx)^{m+1}(c+dx)^{-m}}{}$$

```
[Out] ((a + b*x)^(1 + m)*(2*b*d^2*e*g + b*c^2*f*h*(2 + m) - c*d*(2*b*(f*g + e*h)
+ a*f*h*m) + d*(b*c - a*d)*f*h*m*x))/(2*b*d^2*(b*c - a*d)*m*(c + d*x)^m) -
((b^2*c^2*f*h*(1 + m)*(2 + m) - 2*b*c*d*(1 + m)*(b*f*g + b*e*h + a*f*h*m) +
 d^2*(2*b^2*e*g + 2*a*b*(f*g + e*h)*m - a^2*f*h*(1 - m)*m))*(a + b*x)^(1 +
m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a
 + b*x))/(b*c - a*d))])/(2*b^2*d^2*(b*c - a*d)*m*(1 + m)*(c + d*x)^m)
```

Rubi [A] time = 0.186199, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {146, 70, 69}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-\frac{(a+bx)^{m+1}(c+dx)^{-m}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(2*b*d^2*e*g + b*c^2*f*h*(2 + m) - c*d*(2*b*(f*g + e*h)
+ a*f*h*m) + d*(b*c - a*d)*f*h*m*x))/(2*b*d^2*(b*c - a*d)*m*(c + d*x)^m) -
((b^2*c^2*f*h*(1 + m)*(2 + m) - 2*b*c*d*(1 + m)*(b*f*g + b*e*h + a*f*h*m) +
 d^2*(2*b^2*e*g + 2*a*b*(f*g + e*h)*m - a^2*f*h*(1 - m)*m))*(a + b*x)^(1 +
m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a
 + b*x))/(b*c - a*d))])/(2*b^2*d^2*(b*c - a*d)*m*(1 + m)*(c + d*x)^m)
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afh\right.}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afh\right.}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afh\right.}{2bd^2(bc-ad)m}$$

Mathematica [A] time = 0.186019, size = 221, normalized size = 0.85

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)\left(a^2d^2fh(m-1)m+2abdm(d(eh+fg)-cfh(m+1))+b^2\left(c^2fh\left(m^2+3m+2\right)-2cd(m+1)(eh+fg\right.\right.}{m+1}\right.}{2b^2d^2m(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(b*(a*d*f*h*m*(c + d*x) - b*(2*d^2*e*g + c^2*f*h*(2 + m)
 + c*d*(-2*f*g - 2*e*h + f*h*m*x))) + ((a^2*d^2*f*h*(-1 + m)*m + 2*a*b*d*m*
(d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(2*d^2*e*g - 2*c*d*(f*g + e*h)*(1 + m
) + c^2*f*h*(2 + 3*m + m^2)))*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2
F1[m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(1 + m)))/(2*b^2*d^2*(-(
b*c) + a*d)*m*(c + d*x)^m)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^{-1-m} (fx+e)(hx+g)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fhx^2 + eg + \left(fg + eh\right)x\right)(bx + a)^m (dx + c)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\left(hx + g\right)(bx + a)^m (dx + c)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

3.128 $\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=203

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-cd(afh(m+1)+b(eh+fg))+dfh(m+1)x(bc-ad)+bc^2fh(m+2)+bd^2eg\right)}{bd^2(m+1)(bc-ad)}-\frac{(a+bx)^{}}{}$$

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*(b*d^2*e*g + b*c^2*f*h*(2 + m) - c*d*
(b*(f*g + e*h) + a*f*h*(1 + m)) + d*(b*c - a*d)*f*h*(1 + m)*x))/(b*d^2*(b*c
 - a*d)*(1 + m)) - ((a*d*f*h*m + b*(d*(f*g + e*h) - c*f*h*(2 + m)))*(a + b*
x)^m*Hypergeometric2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(b*d^3*m*
(-((d*(a + b*x))/(b*c - a*d)))^m*(c + d*x)^m)
```

Rubi [A] time = 0.106725, antiderivative size = 205, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {143, 70, 69}

$$-\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-dfh(m+1)x(bc-ad)+acdfh(m+1)-b\left(c^2fh(m+2)-cd(eh+fg)+d^2eg\right)\right)}{bd^2(m+1)(bc-ad)}-\frac{(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*(a*c*d*f*h*(1 + m) - b*(d^2*e*g - c
*d*(f*g + e*h) + c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(1 + m)*x))/(b*d^2*(b
*c - a*d)*(1 + m))) - ((b*d*(f*g + e*h) + a*d*f*h*m - b*c*f*h*(2 + m))*(a +
 b*x)^m*Hypergeometric2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(b*d^3
*m*(-((d*(a + b*x))/(b*c - a*d)))^m*(c + d*x)^m)
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && !(SumSimplerQ[n, 1] && !SumSimplerQ[m, 1])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b\left(d^2eg-cd(fg+eh)+c^2fh(2\right.\right.}{bd^2(bc-ad)(1+m)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b\left(d^2eg-cd(fg+eh)+c^2fh(2\right.\right.}{bd^2(bc-ad)(1+m)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b\left(d^2eg-cd(fg+eh)+c^2fh(2\right.\right.}{bd^2(bc-ad)(1+m)}$$

Mathematica [A] time = 0.249263, size = 198, normalized size = 0.98

$$\frac{(a+bx)^m(c+dx)^{-m}\left(\frac{(m+1)(bc-ad)\left(\frac{d(a+bx)}{ad-bc}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{b(c+dx)}{bc-ad}\right)(-adfhm+bcfh(m+2)-bd(eh+fg))}{m}-\frac{d(a+bx)\left(adfh(m+1)(c+dx)-b\left(c^2fh(m+2\right.\right.}{c+dx}\right.}{bd^3(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^m*(-((d*(a + b*x)*(a*d*f*h*(1 + m)*(c + d*x) - b*(d^2*e*g + c^2*
f*h*(2 + m) + c*d*(-(f*g) - e*h + f*h*(1 + m)*x))))/(c + d*x)) + ((b*c - a*
d)*(1 + m)*(-(b*d*(f*g + e*h)) - a*d*f*h*m + b*c*f*h*(2 + m))*Hypergeometri
c2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(m*((d*(a + b*x))/(-(b*c) +
 a*d))^m)))/(b*d^3*(b*c - a*d)*(1 + m)*(c + d*x)^m)
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int (bx+a)^m(dx+c)^{-2-m}(fx+e)(hx+g)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fhx^2+eg+\left(fg+eh\right)x\right)(bx+a)^m(dx+c)^{-m-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-2-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

3.129 $\int (a+bx)^m (c+dx)^{-3-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=246

$$\frac{fh(a+bx)^{m+3}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m+3,m+3;m+4;-\frac{d(a+bx)}{bc-ad}\right)}{(m+3)(bc-ad)^3}-\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(-bx\left(a^2dfh(2m+3)\right.\right.}{}$$

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-2 - m)*(a^2*b*c*f*h*m - a^3*d*f*h*(1 + m)
- b^3*c*e*g*(2 + m) + a*b^2*(c*(f*g + e*h) + d*e*g*(1 + m)) - b*(a^2*d*f*h*
(3 + 2*m) + b^2*(d*e*g + c*(f*g + e*h)*(1 + m)) - a*b*(2*c*f*h*(1 + m) + d*
(f*g + e*h)*(2 + m)))*x))/(b^2*(b*c - a*d)^2*(1 + m)*(2 + m))) + (f*h*(a +
b*x)^(3 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[3 + m, 3 + m,
4 + m, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(3 + m)*(c + d*x)^m)
```

Rubi [A] time = 0.159763, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {145, 70, 69}

$$\frac{fh(a+bx)^{m+3}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m+3,m+3;m+4;-\frac{d(a+bx)}{bc-ad}\right)}{(m+3)(bc-ad)^3}-\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(-bx\left(a^2dfh(2m+3)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```


```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-2 - m)*(a^2*b*c*f*h*m - a^3*d*f*h*(1 + m)
- b^3*c*e*g*(2 + m) + a*b^2*(c*(f*g + e*h) + d*e*g*(1 + m)) - b*(a^2*d*f*h*
(3 + 2*m) + b^2*(d*e*g + c*(f*g + e*h)*(1 + m)) - a*b*(2*c*f*h*(1 + m) + d*
(f*g + e*h)*(2 + m)))*x))/(b^2*(b*c - a*d)^2*(1 + m)*(2 + m))) + (f*h*(a +
b*x)^(3 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[3 + m, 3 + m,
4 + m, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(3 + m)*(c + d*x)^m)
```


Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2+m) + \right.}{}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2+m) + \right.}{}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2+m) + \right.}{}$$

Mathematica [A] time = 0.331029, size = 237, normalized size = 0.96

$$-\frac{(a+bx)^m(c+dx)^{-m-2}\left(d^3(a+bx)\left(a^2bfh(cm-d(2m+3)x)+a^3(-d)fh(m+1)+ab^2(ceh+cf(g+2h(m+1)x)+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^m*(c + d*x)^(-2 - m)*(d^3*(a + b*x)*(-(a^3*d*f*h*(1 + m)) + a^
2*b*f*h*(c*m - d*(3 + 2*m)*x) + a*b^2*(c*e*h + d*e*g*(1 + m) + d*f*g*(2 + m
)*x + d*e*h*(2 + m)*x + c*f*(g + 2*h*(1 + m)*x)) - b^3*(d*e*g*x + c*(e*g*(2
 + m) + f*g*(1 + m)*x + e*h*(1 + m)*x))) + ((b*c - a*d)^4*f*h*(1 + m)*Hyper
geometric2F1[-2 - m, -2 - m, -1 - m, (b*(c + d*x))/(b*c - a*d)])/((d*(a + b
*x))/(-(b*c) + a*d))^m))/(b^2*d^3*(b*c - a*d)^2*(1 + m)*(2 + m)))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^{-3-m} (fx+e)(hx+g)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fhx^2 + eg + \left(fg + eh\right)x\right)(bx + a)^m (dx + c)^{-m-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-3-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\left(hx + g\right)(bx + a)^m (dx + c)^{-m-3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

3.130 $\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=362

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+\right.\right.}{bd^2(m+2)(m+3)(bc-ad)^2}$$

```
[Out] ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 +
 m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))
*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(b*d^2*(b*c - a*d)^2*(2 + m)*(3 + m)
) + ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*
(1 + m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^
2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c - a*d)^3*(1 + m)*(2 +
m)*(3 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*c*d*f*h*(3 + m) + b*
(d^2*e*g - c*d*(f*g + e*h) - c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(3 + m)*x
))/(b*d^2*(b*c - a*d)*(3 + m))
```

Rubi [A] time = 0.361866, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {146, 45, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+\right.\right.}{bd^2(m+2)(m+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 +
 m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))
*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(b*d^2*(b*c - a*d)^2*(2 + m)*(3 + m)
) + ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*
(1 + m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^
2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c - a*d)^3*(1 + m)*(2 +
m)*(3 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*c*d*f*h*(3 + m) + b*
(d^2*e*g - c*d*(f*g + e*h) - c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(3 + m)*x
))/(b*d^2*(b*c - a*d)*(3 + m))
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^m (c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m)+b\left(d^2eg-cd(fg+eh)-c^2fh(2\right.\right.}{bd^2(bc-ad)(3+m)}$$

$$= \frac{\left(a^2d^2fh\left(6+5m+m^2\right)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2\left(2d\right.\right.}{bd^2(bc-ad)^2(}$$

$$= \frac{\left(a^2d^2fh\left(6+5m+m^2\right)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2\left(2d\right.\right.}{bd^2(bc-ad)^2(}$$

Mathematica [A] time = 0.54954, size = 220, normalized size = 0.61

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(\frac{(c+dx)(-ad(m+1)+bc(m+2)+bdx)\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+cd(m+1)(eh+\right.\right.}{(m+1)(m+2)(bc-ad)^2}\right.}{bd^2(m+3)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*d*f*h*(3 + m)*(c + d*x) + ((a^2*d^
2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 + m)) + b
^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(-(a*d*(1 + m)) + b*c*(2 + m) + b*d*x))/((b*c - a*d)^2*(1 + m)*(2 + m))
+ b*(d^2*e*g - c^2*f*h*(2 + m) - c*d*(e*h + f*(g + h*(3 + m)*x)))))/(b*d^2*
(b*c - a*d)*(3 + m))
```

Maple [B] time = 0.008, size = 894, normalized size = 2.5

$$-\frac{(bx+a)^{1+m}(dx+c)^{-3-m}\left(a^2d^2fhm^2x^2-2\,abcdfhm^2x^2+b^2c^2fhm^2x^2+a^2d^2ehm^2x+a^2d^2fgm^2x+5\,a^2d^2fhmx^2-2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(-3-m)*(a^2*d^2*f*h*m^2*x^2-2*a*b*c*d*f*h*m^2*x^2+b^
2*c^2*f*h*m^2*x^2+a^2*d^2*e*h*m^2*x+a^2*d^2*f*g*m^2*x+5*a^2*d^2*f*h*m*x^2-2
*a*b*c*d*e*h*m^2*x-2*a*b*c*d*f*g*m^2*x-8*a*b*c*d*f*h*m*x^2-a*b*d^2*e*h*m*x^
2-a*b*d^2*f*g*m*x^2+b^2*c^2*e*h*m^2*x+b^2*c^2*f*g*m^2*x+3*b^2*c^2*f*h*m*x^2
+b^2*c*d*e*h*m*x^2+b^2*c*d*f*g*m*x^2+2*a^2*c*d*f*h*m*x+a^2*d^2*e*g*m^2+4*a^
```

```
2*d^2*e*h*m*x+4*a^2*d^2*f*g*m*x+6*a^2*d^2*f*h*x^2-2*a*b*c^2*f*h*m*x-2*a*b*c
*d*e*g*m^2-8*a*b*c*d*e*h*m*x-8*a*b*c*d*f*g*m*x-6*a*b*c*d*f*h*x^2-2*a*b*d^2*
e*g*m*x-3*a*b*d^2*e*h*x^2-3*a*b*d^2*f*g*x^2+b^2*c^2*e*g*m^2+4*b^2*c^2*e*h*m
*x+4*b^2*c^2*f*g*m*x+2*b^2*c^2*f*h*x^2+2*b^2*c*d*e*g*m*x+b^2*c*d*e*h*x^2+b^
2*c*d*f*g*x^2+2*b^2*d^2*e*g*x^2+a^2*c*d*e*h*m+a^2*c*d*f*g*m+6*a^2*c*d*f*h*x
+3*a^2*d^2*e*g*m+3*a^2*d^2*e*h*x+3*a^2*d^2*f*g*x-a*b*c^2*e*h*m-a*b*c^2*f*g*
m-2*a*b*c^2*f*h*x-8*a*b*c*d*e*g*m-10*a*b*c*d*e*h*x-10*a*b*c*d*f*g*x-2*a*b*d
^2*e*g*x+5*b^2*c^2*e*g*m+3*b^2*c^2*e*h*x+3*b^2*c^2*f*g*x+6*b^2*c*d*e*g*x+2*
a^2*c^2*f*h+a^2*c*d*e*h+a^2*c*d*f*g+2*a^2*d^2*e*g-3*a*b*c^2*e*h-3*a*b*c^2*f
*g-6*a*b*c*d*e*g+6*b^2*c^2*e*g)/(a^3*d^3*m^3-3*a^2*b*c*d^2*m^3+3*a*b^2*c^2*
d*m^3-b^3*c^3*m^3+6*a^3*d^3*m^2-18*a^2*b*c*d^2*m^2+18*a*b^2*c^2*d*m^2-6*b^3
*c^3*m^2+11*a^3*d^3*m-33*a^2*b*c*d^2*m+33*a*b^2*c^2*d*m-11*b^3*c^3*m+6*a^3*
d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

Fricas [B] time = 1.4598, size = 3309, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*g*m^2 + ((b^3*c^2*d - 2*a*b^2*c*
d^2 + a^2*b*d^3)*f*h*m^2 + (2*b^3*d^3*e + (b^3*c*d^2 - 3*a*b^2*d^3)*f)*g +
((b^3*c*d^2 - 3*a*b^2*d^3)*e + 2*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*
f)*h + ((b^3*c*d^2 - a*b^2*d^3)*f*g + ((b^3*c*d^2 - a*b^2*d^3)*e + (3*b^3*c
^2*d - 8*a*b^2*c*d^2 + 5*a^2*b*d^3)*f)*h)*m)*x^4 + (((b^3*c^2*d - 2*a*b^2*c
*d^2 + a^2*b*d^3)*f*g + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c
^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*h)*m^2 + 4*(2*b^3*c*d^2*e + (b
^3*c^2*d - 3*a*b^2*c*d^2)*f)*g + 2*(2*(b^3*c^2*d - 3*a*b^2*c*d^2)*e + (b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 3*a^3*d^3)*f)*h + ((2*(b^3*c*d^2 - a*
b^2*d^3)*e + (5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*f)*g + ((5*b^3*c^2
*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*e + (3*b^3*c^3 - 7*a*b^2*c^2*d - a^2*b*c*
d^2 + 5*a^3*d^3)*f)*h)*m)*x^3 + ((((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
e + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + ((b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*f)*h)*m^2 + 3*(4*b^3*c^2*d*e + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 a^3*d^3)*f)*g + 3*(4*a^3*c*d^2*f + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2 + a^3*d^3)*e)*h + (((7*b^3*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*e + 4*(b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + (4*(b^3*c^3 - a*b^2*c^2*d
 - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 8*a^2*b*c^2*d + 7*a^3*c*d^2)*f)*
h)*m)*x^2 + (2*(3*a*b^2*c^3 - 3*a^2*b*c^2*d + a^3*c*d^2)*e - (3*a^2*b*c^3 -
 a^3*c^2*d)*f)*g + (2*a^3*c^3*f - (3*a^2*b*c^3 - a^3*c^2*d)*e)*h - ((a^2*b*
c^3 - a^3*c^2*d)*e*h - ((5*a*b^2*c^3 - 8*a^2*b*c^2*d + 3*a^3*c*d^2)*e - (a^
```

```
2*b*c^3 - a^3*c^2*d)*f)*g)*m + (((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*
h + ((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2
*b*c^2*d + a^3*c*d^2)*f)*g)*m^2 + 2*((3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + a^3*d^3)*e - 2*(3*a^2*b*c^2*d - a^3*c*d^2)*f)*g + 4*(2*a^3*c^2*d*f -
 (3*a^2*b*c^2*d - a^3*c*d^2)*e)*h + (((5*b^3*c^3 - a*b^2*c^2*d - 7*a^2*b*c*
d^2 + 3*a^3*d^3)*e + (3*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*f)*g + ((3
*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*e - 2*(a^2*b*c^3 - a^3*c^2*d)*f)*
h)*m)*x)*(b*x + a)^m*(d*x + c)^(-m - 4)/(6*b^3*c^3 - 18*a*b^2*c^2*d + 18*a^
2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*m^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m^2 + 11*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-4-m)*(f*x+e)*(h*x+g),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="giac")

[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

3.131 $\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=507

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)\right.\right.}{2bd^2(m+3)(m+4)(bc-ad)^2}$$

```
[Out] ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1
+ m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2
)))*(a + b*x)^(1 + m)*(c + d*x)^(-3 - m))/(2*b*d^2*(b*c - a*d)^2*(3 + m)*(4
 + m)) + ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) +
c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 +
3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(d^2*(b*c - a*d)^3*(2 +
m)*(3 + m)*(4 + m)) + (b*(a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d
*(f*g + e*h) + c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m)
+ c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c
 - a*d)^4*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(
-4 - m)*(a*c*d*f*h*(4 + m) + b*(2*d^2*e*g - 2*c*d*(f*g + e*h) - c^2*f*h*(2
+ m)) - d*(b*c - a*d)*f*h*(4 + m)*x))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Rubi [A] time = 0.585929, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {146, 45, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)\right.\right.}{2bd^2(m+3)(m+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1
+ m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2
)))*(a + b*x)^(1 + m)*(c + d*x)^(-3 - m))/(2*b*d^2*(b*c - a*d)^2*(3 + m)*(4
 + m)) + ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) +
c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 +
3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(d^2*(b*c - a*d)^3*(2 +
m)*(3 + m)*(4 + m)) + (b*(a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d
*(f*g + e*h) + c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m)
+ c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c
 - a*d)^4*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(
-4 - m)*(a*c*d*f*h*(4 + m) + b*(2*d^2*e*g - 2*c*d*(f*g + e*h) - c^2*f*h*(2
+ m)) - d*(b*c - a*d)*f*h*(4 + m)*x))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(acdfh(4+m)+b\left(2d^2eg-2cd(fg+eh)-c^2fh(\right.\right.}{2bd^2(bc-ad)(4+m)}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2\left(6\right.\right.}{2bd^2(bc-ad)}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2\left(6\right.\right.}{2bd^2(bc-ad)}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2\left(6\right.\right.}{2bd^2(bc-ad)}$$

Mathematica [A] time = 0.76909, size = 279, normalized size = 0.55

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-4}\left(\frac{(c+dx)\left(a^2d^2\left(m^2+3m+2\right)-2abd(m+1)(c(m+3)+dx)+b^2\left(c^2\left(m^2+5m+6\right)+2cd(m+3)x+2d^2x^2\right)\right)\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)\right.}{(m+1)(m+2)(m+3)(bc-ad)^3}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-4 - m)*(a*d*f*h*(4 + m)*(c + d*x) + b*(2*d^2
*e*g - c^2*f*h*(2 + m) - c*d*(2*f*g + 2*e*h + f*h*(4 + m)*x)) + ((a^2*d^2*f
*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1 + m)) + b^2
*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(a^2*d^2*(2 + 3*m + m^2) - 2*a*b*d*(1 + m)*(c*(3 + m) + d*x) + b^2*(c^2*
(6 + 5*m + m^2) + 2*c*d*(3 + m)*x + 2*d^2*x^2)))/((b*c - a*d)^3*(1 + m)*(2
+ m)*(3 + m))))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Maple [B] time = 0.01, size = 2343, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(-4-m)*(a^3*d^3*f*h*m^3*x^2-3*a^2*b*c*d^2*f*h*m^3*x^
2-a^2*b*d^3*f*h*m^2*x^3+3*a*b^2*c^2*d*f*h*m^3*x^2+2*a*b^2*c*d^2*f*h*m^2*x^3
-b^3*c^3*f*h*m^3*x^2-b^3*c^2*d*f*h*m^2*x^3+a^3*d^3*e*h*m^3*x+a^3*d^3*f*g*m^
3*x+8*a^3*d^3*f*h*m^2*x^2-3*a^2*b*c*d^2*e*h*m^3*x-3*a^2*b*c*d^2*f*g*m^3*x-2
3*a^2*b*c*d^2*f*h*m^2*x^2-2*a^2*b*d^3*e*h*m^2*x^2-2*a^2*b*d^3*f*g*m^2*x^2-7
*a^2*b*d^3*f*h*m*x^3+3*a*b^2*c^2*d*e*h*m^3*x+3*a*b^2*c^2*d*f*g*m^3*x+22*a*b
^2*c^2*d*f*h*m^2*x^2+4*a*b^2*c*d^2*e*h*m^2*x^2+4*a*b^2*c*d^2*f*g*m^2*x^2+10
*a*b^2*c*d^2*f*h*m*x^3+2*a*b^2*d^3*e*h*m*x^3+2*a*b^2*d^3*f*g*m*x^3-b^3*c^3*
e*h*m^3*x-b^3*c^3*f*g*m^3*x-7*b^3*c^3*f*h*m^2*x^2-2*b^3*c^2*d*e*h*m^2*x^2-2
*b^3*c^2*d*f*g*m^2*x^2-3*b^3*c^2*d*f*h*m*x^3-2*b^3*c*d^2*e*h*m*x^3-2*b^3*c*
d^2*f*g*m*x^3+2*a^3*c*d^2*f*h*m^2*x+a^3*d^3*e*g*m^3+7*a^3*d^3*e*h*m^2*x+7*a
^3*d^3*f*g*m^2*x+19*a^3*d^3*f*h*m*x^2-4*a^2*b*c^2*d*f*h*m^2*x-3*a^2*b*c*d^2
*e*g*m^3-22*a^2*b*c*d^2*e*h*m^2*x-22*a^2*b*c*d^2*f*g*m^2*x-58*a^2*b*c*d^2*f
*h*m*x^2-3*a^2*b*d^3*e*g*m^2*x-10*a^2*b*d^3*e*h*m*x^2-10*a^2*b*d^3*f*g*m*x^
2-12*a^2*b*d^3*f*h*x^3+2*a*b^2*c^3*f*h*m^2*x+3*a*b^2*c^2*d*e*g*m^3+23*a*b^2
*c^2*d*e*h*m^2*x+23*a*b^2*c^2*d*f*g*m^2*x+53*a*b^2*c^2*d*f*h*m*x^2+6*a*b^2*
c*d^2*e*g*m^2*x+20*a*b^2*c*d^2*e*h*m*x^2+20*a*b^2*c*d^2*f*g*m*x^2+8*a*b^2*c
*d^2*f*h*x^3+6*a*b^2*d^3*e*g*m*x^2+8*a*b^2*d^3*e*h*x^3+8*a*b^2*d^3*f*g*x^3-
b^3*c^3*e*g*m^3-8*b^3*c^3*e*h*m^2*x-8*b^3*c^3*f*g*m^2*x-14*b^3*c^3*f*h*m*x^
2-3*b^3*c^2*d*e*g*m^2*x-10*b^3*c^2*d*e*h*m*x^2-10*b^3*c^2*d*f*g*m*x^2-2*b^3
*c^2*d*f*h*x^3-6*b^3*c*d^2*e*g*m*x^2-2*b^3*c*d^2*e*h*x^3-2*b^3*c*d^2*f*g*x^
3-6*b^3*d^3*e*g*x^3+a^3*c*d^2*e*h*m^2+a^3*c*d^2*f*g*m^2+10*a^3*c*d^2*f*h*m*
x+6*a^3*d^3*e*g*m^2+14*a^3*d^3*e*h*m*x+14*a^3*d^3*f*g*m*x+12*a^3*d^3*f*h*x^
2-2*a^2*b*c^2*d*e*h*m^2-2*a^2*b*c^2*d*f*g*m^2-20*a^2*b*c^2*d*f*h*m*x-21*a^2
*b*c*d^2*e*g*m^2-53*a^2*b*c*d^2*e*h*m*x-53*a^2*b*c*d^2*f*g*m*x-56*a^2*b*c*d
^2*f*h*x^2-9*a^2*b*d^3*e*g*m*x-8*a^2*b*d^3*e*h*x^2-8*a^2*b*d^3*f*g*x^2+a*b^
2*c^3*e*h*m^2+a*b^2*c^3*f*g*m^2+10*a*b^2*c^3*f*h*m*x+24*a*b^2*c^2*d*e*g*m^2
+58*a*b^2*c^2*d*e*h*m*x+58*a*b^2*c^2*d*f*g*m*x+34*a*b^2*c^2*d*f*h*x^2+30*a*
b^2*c*d^2*e*g*m*x+34*a*b^2*c*d^2*e*h*x^2+34*a*b^2*c*d^2*f*g*x^2+6*a*b^2*d^3
*e*g*x^2-9*b^3*c^3*e*g*m^2-19*b^3*c^3*e*h*m*x-19*b^3*c^3*f*g*m*x-8*b^3*c^3*
f*h*x^2-21*b^3*c^2*d*e*g*m*x-8*b^3*c^2*d*e*h*x^2-8*b^3*c^2*d*f*g*x^2-24*b^3
*c*d^2*e*g*x^2+2*a^3*c^2*d*f*h*m+3*a^3*c*d^2*e*h*m+3*a^3*c*d^2*f*g*m+8*a^3*
c*d^2*f*h*x+11*a^3*d^3*e*g*m+8*a^3*d^3*e*h*x+8*a^3*d^3*f*g*x-2*a^2*b*c^3*f*
h*m-10*a^2*b*c^2*d*e*h*m-10*a^2*b*c^2*d*f*g*m-34*a^2*b*c^2*d*f*h*x-42*a^2*b
*c*d^2*e*g*m-34*a^2*b*c*d^2*e*h*x-34*a^2*b*c*d^2*f*g*x-6*a^2*b*d^3*e*g*x+7*
a*b^2*c^3*e*h*m+7*a*b^2*c^3*f*g*m+8*a*b^2*c^3*f*h*x+57*a*b^2*c^2*d*e*g*m+56
*a*b^2*c^2*d*e*h*x+56*a*b^2*c^2*d*f*g*x+24*a*b^2*c*d^2*e*g*x-26*b^3*c^3*e*g
*m-12*b^3*c^3*e*h*x-12*b^3*c^3*f*g*x-36*b^3*c^2*d*e*g*x+2*a^3*c^2*d*f*h+2*a
^3*c*d^2*e*h+2*a^3*c*d^2*f*g+6*a^3*d^3*e*g-8*a^2*b*c^3*f*h-8*a^2*b*c^2*d*e*
h-8*a^2*b*c^2*d*f*g-24*a^2*b*c*d^2*e*g+12*a*b^2*c^3*e*h+12*a*b^2*c^3*f*g+36
*a*b^2*c^2*d*e*g-24*b^3*c^3*e*g)/(a^4*d^4*m^4-4*a^3*b*c*d^3*m^4+6*a^2*b^2*c
^2*d^2*m^4-4*a*b^3*c^3*d*m^4+b^4*c^4*m^4+10*a^4*d^4*m^3-40*a^3*b*c*d^3*m^3+
60*a^2*b^2*c^2*d^2*m^3-40*a*b^3*c^3*d*m^3+10*b^4*c^4*m^3+35*a^4*d^4*m^2-140
*a^3*b*c*d^3*m^2+210*a^2*b^2*c^2*d^2*m^2-140*a*b^3*c^3*d*m^2+35*b^4*c^4*m^2
+50*a^4*d^4*m-200*a^3*b*c*d^3*m+300*a^2*b^2*c^2*d^2*m-200*a*b^3*c^3*d*m+50*
b^4*c^4*m+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d^2-96*a*b^3*c^3*d+24*b
^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^{m}(dx+c)^{-m-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

Fricas [B] time = 1.97079, size = 6969, normalized size = 13.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*e*g*m^3 + ((b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*h*m^2 + 2*(3*b^4*d^4*e + (b^4*c*
d^3 - 4*a*b^3*d^4)*f)*g + 2*((b^4*c*d^3 - 4*a*b^3*d^4)*e + (b^4*c^2*d^2 - 4
*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*f)*h + (2*(b^4*c*d^3 - a*b^3*d^4)*f*g + (2*(b
^4*c*d^3 - a*b^3*d^4)*e + (3*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + 7*a^2*b^2*d^4)*
f)*h)*m)*x^5 + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)
*f*h*m^3 + (2*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*g + (2*(b^4*c^2
*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + (8*b^4*c^3*d - 23*a*b^3*c^2*d^2 + 2
2*a^2*b^2*c*d^3 - 7*a^3*b*d^4)*f)*h)*m^2 + 10*(3*b^4*c*d^3*e + (b^4*c^2*d^2
 - 4*a*b^3*c*d^3)*f)*g + 10*((b^4*c^2*d^2 - 4*a*b^3*c*d^3)*e + (b^4*c^3*d -
 4*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3)*f)*h + (2*(3*(b^4*c*d^3 - a*b^3*d^4)*e
+ 2*(3*b^4*c^2*d^2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*f)*g + (4*(3*b^4*c^2*d^
2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*e + (17*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 5
5*a^2*b^2*c*d^3 - 12*a^3*b*d^4)*f)*h)*m)*x^4 + (((b^4*c^3*d - 3*a*b^3*c^2*d
^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*f*g + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a
^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^
4*d^4)*f)*h)*m^3 + ((3*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + 5*(2
*b^4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*f)*g + (5*(2*b^
4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (7*b^4*c^4 - 1
6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 8*a^4*d^4)*f)*h)*m^2 +
 20*(3*b^4*c^2*d^2*e + (b^4*c^3*d - 4*a*b^3*c^2*d^2)*f)*g + 4*(5*(b^4*c^3*d
 - 4*a*b^3*c^2*d^2)*e + (2*b^4*c^4 - 8*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 + 1
2*a^3*b*c*d^3 - 3*a^4*d^4)*f)*h + ((3*(9*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + a^2
*b^2*d^4)*e + (29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*f)*g + ((29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*e + (14*b^4*c^4 - 46*a*b^3*c^3*d + 15*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^
3 - 19*a^4*d^4)*f)*h)*m)*x^3 - ((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)
*e*h - (3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^2 - 2*a^4*c*d^3)*e
 - (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g)*m^2 + ((((b^4*c^3*d -
3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d
 + 2*a^3*b*c*d^3 - a^4*d^4)*f)*g + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^
3 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3
)*f)*h)*m^3 + ((3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - a^3*b*
d^4)*e + (8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 -
 7*a^4*d^4)*f)*g + ((8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^
3*b*c*d^3 - 7*a^4*d^4)*e + 5*(a*b^3*c^4 - 4*a^2*b^2*c^3*d + 5*a^3*b*c^2*d^2
 - 2*a^4*c*d^3)*f)*h)*m^2 + 4*(15*b^4*c^3*d*e + (3*b^4*c^4 - 12*a*b^3*c^3*d
 - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*f)*g + 4*((3*b^4*c^4 - 1
2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*e + 5*(4*a^
3*b*c^2*d^2 - a^4*c*d^3)*f)*h + (((47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2
*b^2*c*d^3 - 2*a^3*b*d^4)*e + (19*b^4*c^4 - 36*a*b^3*c^3*d - 15*a^2*b^2*c^2
*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*f)*g + ((19*b^4*c^4 - 36*a*b^3*c^3*d -
15*a^2*b^2*c^2*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*e + (4*a*b^3*c^4 - 41*a^2
*b^2*c^3*d + 66*a^3*b*c^2*d^2 - 29*a^4*c*d^3)*f)*h)*m)*x^2 + 2*(3*(4*a*b^3*
c^4 - 6*a^2*b^2*c^3*d + 4*a^3*b*c^2*d^2 - a^4*c*d^3)*e - (6*a^2*b^2*c^4 - 4
*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g - 2*((6*a^2*b^2*c^4 - 4*a^3*b*c^3*d + a^4*
c^2*d^2)*e - (4*a^3*b*c^4 - a^4*c^3*d)*f)*h + (((26*a*b^3*c^4 - 57*a^2*b^2*
```

```
c^3*d + 42*a^3*b*c^2*d^2 - 11*a^4*c*d^3)*e - (7*a^2*b^2*c^4 - 10*a^3*b*c^3*
d + 3*a^4*c^2*d^2)*f)*g - ((7*a^2*b^2*c^4 - 10*a^3*b*c^3*d + 3*a^4*c^2*d^2)
*e - 2*(a^3*b*c^4 - a^4*c^3*d)*f)*h)*m + (((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3
*a^3*b*c^2*d^2 - a^4*c*d^3)*e*h + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3
 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)
*f)*g)*m^3 + ((3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c
*d^3 - 2*a^4*d^4)*e + (7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 -
8*a^4*c*d^3)*f)*g + ((7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 - 8
*a^4*c*d^3)*e - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*h)*m^2 + 2
*(3*(4*b^4*c^4 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^
4)*e - 5*(6*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*f)*g - 10*((6*a^2*
b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*e - (4*a^3*b*c^3*d - a^4*c^2*d^2)*
f)*h + (((26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3
 - 11*a^4*d^4)*e + (12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 17
*a^4*c*d^3)*f)*g + ((12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 1
7*a^4*c*d^3)*e - 4*(2*a^2*b^2*c^4 - 5*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f)*h)*m)
*x)*(b*x + a)^m*(d*x + c)^(-m - 5)/(24*b^4*c^4 - 96*a*b^3*c^3*d + 144*a^2*b
^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^4 + 10*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^3 + 35*(b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^2 + 50*(b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-5-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

3.132 $\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=815

$$\frac{h(a+bx)^3(e+fx)^{m+1}(c+dx)^{-m-3}}{df}+\frac{(bc-ad)^2(adf+b(cf(m+2)-de(m+3)))(cfh(m+4)-d(fg+eh(m+3)))(e+f}{d^4f^2(de-cf)(m+3)}$$

```
[Out] ((b*c - a*d)^2*(a*d*f + b*(c*f*(2 + m) - d*e*(3 + m)))*(c*f*h*(4 + m) - d*(
f*g + e*h*(3 + m)))*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c
*f)*(3 + m)) - (b*(b*c - a*d)*(c*f*h*(4 + m) - d*(f*g + e*h*(3 + m)))*(a +
b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^3*f^2) + (h*(a + b*x)^3*(c +
d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*f) - ((b*c - a*d)^2*(3*a*d*f*h - b*(c*f
*h*(4 + m) - d*(f*g + e*h*m)))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4*f
*(d*e - c*f)*(2 + m)) + ((b*c - a*d)*(c*f*h*(4 + m) - d*(f*g + e*h*(3 + m))
)*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^2*(c^2*f^2*(2
+ 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m + m^2)))*(c + d
*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c*f)^2*(2 + m)*(3 + m)) - (
(b*c - a*d)*(a*d*f - b*(2*d*e*(2 + m) - c*f*(3 + 2*m)))*(3*a*d*f*h - b*(c*f
*h*(4 + m) - d*(f*g + e*h*m)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^4*f
*(d*e - c*f)^2*(1 + m)*(2 + m)) - ((b*c - a*d)*(c*f*h*(4 + m) - d*(f*g + e*
h*(3 + m)))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^2*(c
^2*f^2*(2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m + m^2
)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^4*f*(d*e - c*f)^3*(1 + m)*(2 +
 m)*(3 + m)) - (b^2*(3*a*d*f*h - b*(c*f*h*(4 + m) - d*(f*g + e*h*m)))*(e +
f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f*(c + d*x))/(d*e - c*f))])/(d^5
*f*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m)
```

Rubi [A] time = 1.43461, antiderivative size = 803, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {153, 159, 89, 79, 70, 69, 90, 45, 37}

$$\frac{h(a+bx)^3(e+fx)^{m+1}(c+dx)^{-m-3}}{df}+\frac{(bc-ad)^2(adf+bc(m+2)f-bde(m+3))(cfh(m+4)-d(fg+eh(m+3)))(e+f}{d^4f^2(de-cf)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)^2*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c*f*h*(4 + m) - d*(
f*g + e*h*(3 + m)))*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c
*f)*(3 + m)) - (b*(b*c - a*d)*(c*f*h*(4 + m) - d*(f*g + e*h*(3 + m)))*(a +
b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^3*f^2) + (h*(a + b*x)^3*(c +
d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*f) - ((b*c - a*d)^2*(b*d*f*g + 3*a*d*f*
h + b*d*e*h*m - b*c*f*h*(4 + m))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4
*f*(d*e - c*f)*(2 + m)) - ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) - c*f*h*(4 +
m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^2*(c^2*f^2*(
2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m + m^2)))*(c +
 d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c*f)^2*(2 + m)*(3 + m)) -
 ((b*c - a*d)*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))*(a*d*f -
2*b*d*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d
^4*f*(d*e - c*f)^2*(1 + m)*(2 + m)) + ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) -
 c*f*h*(4 + m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^
2*(c^2*f^2*(2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m +
 m^2)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^4*f*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))
```

```
*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f*(c + d*x))/(d*e - c*f))]
)/(d^5*f*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m)
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} + \frac{\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(-$$

$$= \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} - \frac{((bc-ad)(dfg+deh(3+m)-cfh($$

$$= \frac{b(bc-ad)(dfg+deh(3+m)-cfh(4+m))(a+bx)(c+dx)^{-3-m}(e+fx)^1}{d^3f^2}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m)-cfh(4-}{d^4f^2(de-cf)(3+m)}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m)-cfh(4-}{d^4f^2(de-cf)(3+m)}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m)-cfh(4-}{d^4f^2(de-cf)(3+m)}$$

Mathematica [C] time = 48.597, size = 3579, normalized size = 4.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] (3*a*b^2*c^3*g*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^(3 + m)*(
-2*c^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*x - 6*c*d^2*e^
3*x^2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m*x^2 - c*d^2*e
^3*m^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*d^2*e^2*f*x^3
```

```
+ 6*c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8*c^2*d*e*f^2*m
*x^3 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*m^2*x^3 - c^3*
f^3*m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c^2*d*e^3*x*((c
*e + d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)/(c*(e + f*x))
)^m + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d*e - c*f)^3*(1
 + m)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f*x)/e)^m*(1 +
(f*x)/e)^3) + (3*a^2*b*c^3*h*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e +
f*x)^(3 + m)*(-2*c^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*
x - 6*c*d^2*e^3*x^2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m
*x^2 - c*d^2*e^3*m^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*
d^2*e^2*f*x^3 + 6*c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8
*c^2*d*e*f^2*m*x^3 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*
m^2*x^3 - c^3*f^3*m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c
^2*d*e^3*x*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)
/(c*(e + f*x)))^m + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d
*e - c*f)^3*(1 + m)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f
*x)/e)^m*(1 + (f*x)/e)^3) + (b^3*g*x^4*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4
+ m)*(e + f*x)^m*AppellF1[4, 4 + m, -m, 5, -((d*x)/c), -((f*x)/e)])/(4*((e
+ f*x)/e)^m) + (3*a*b^2*h*x^4*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(e +
 f*x)^m*AppellF1[4, 4 + m, -m, 5, -((d*x)/c), -((f*x)/e)])/(4*((e + f*x)/e)
^m) + (b^3*h*x^5*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^m*Appel
lF1[5, 4 + m, -m, 6, -((d*x)/c), -((f*x)/e)])/(5*((e + f*x)/e)^m) + (3*a^2*
b*g*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(1 + (d*x)/c)^(-4 - m)*(e + f*
x)^m*((c*(e + f*x))/(e*(c + d*x)))^(-1 - m)*(1 + (f*x)/e)^(1 + m)*(c*(4 + m
)*(3*e + f*x)*(-2*d^3*e^3*x^3 + c^3*(-2*e^2*f*m*x*((c*(e + f*x))/(e*(c + d*
x)))^m + e*f^2*m*(1 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + f^3*(2 + 3*m
 + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e^3*(-1 + ((c*(e + f*x))/(e
*(c + d*x)))^m)) - 2*c^2*d*e*x*(e*f*m*(3 + m)*x*((c*(e + f*x))/(e*(c + d*x)
))^m + f^2*(3 + 4*m + m^2)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m - e^2*(-3 +
3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m)) + c
*d^2*e^2*x^2*(f*(6 + 5*m + m^2)*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*(-6 +
 6*((c*(e + f*x))/(e*(c + d*x)))^m + 5*m*((c*(e + f*x))/(e*(c + d*x)))^m +
m^2*((c*(e + f*x))/(e*(c + d*x)))^m)))*Gamma[4 + m] - (2*d^4*e^4*(1 + m)*x^
4 - 2*c*d^3*e^3*x^3*(-3*e*m + f*(4 + m)*x) + c^4*(e^2*f^2*(-5 + m)*m*x^2*((
c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f^3*m*(1 + m)*x^3*((c*(e + f*x))/(e*(c
+ d*x)))^m + f^4*(2 + 3*m + m^2)*x^4*((c*(e + f*x))/(e*(c + d*x)))^m + 6*e^
4*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m) - 2*e^3*f*x*(4 + m - 4*((c*(e + f*
x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^3*d*e*x*
(2*e*f^2*m*(4 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + f^3*(4 + 5*m + m^2
)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + e^2*f*(4 + m)*x*(3 - 3*((c*(e + f*x
))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m) - e^3*(-8 + m + 8*
((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*x)))^m)) + c
^2*d^2*e^2*x^2*(f^2*(12 + 7*m + m^2)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m +
2*e*f*(4 + m)*x*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/
(e*(c + d*x)))^m) + e^2*(m^2*((c*(e + f*x))/(e*(c + d*x)))^m + 12*(-1 + ((c
*(e + f*x))/(e*(c + d*x)))^m) + m*(6 + 7*((c*(e + f*x))/(e*(c + d*x)))^m)))
)*Gamma[5 + m]))/(2*c*e*(-(d*e) + c*f)^3*(1 + m)*(2 + m)*(3 + m)*(4 + m)*x*
((e + f*x)/e)^m*Gamma[4 + m]) + (a^3*h*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4
+ m)*(1 + (d*x)/c)^(-4 - m)*(e + f*x)^m*((c*(e + f*x))/(e*(c + d*x)))^(-1 -
 m)*(1 + (f*x)/e)^(1 + m)*(c*(4 + m)*(3*e + f*x)*(-2*d^3*e^3*x^3 + c^3*(-2*
e^2*f*m*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*f^2*m*(1 + m)*x^2*((c*(e + f*
x))/(e*(c + d*x)))^m + f^3*(2 + 3*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x))
)^m + 2*e^3*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^2*d*e*x*(e*f*m*(3
 + m)*x*((c*(e + f*x))/(e*(c + d*x)))^m + f^2*(3 + 4*m + m^2)*x^2*((c*(e +
f*x))/(e*(c + d*x)))^m - e^2*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((
c*(e + f*x))/(e*(c + d*x)))^m)) + c*d^2*e^2*x^2*(f*(6 + 5*m + m^2)*x*((c*(e
 + f*x))/(e*(c + d*x)))^m + e*(-6 + 6*((c*(e + f*x))/(e*(c + d*x)))^m + 5*m
*((c*(e + f*x))/(e*(c + d*x)))^m + m^2*((c*(e + f*x))/(e*(c + d*x)))^m)))*G
amma[4 + m] - (2*d^4*e^4*(1 + m)*x^4 - 2*c*d^3*e^3*x^3*(-3*e*m + f*(4 + m)*
```

```
x) + c^4*(e^2*f^2*(-5 + m)*m*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f^3*
m*(1 + m)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + f^4*(2 + 3*m + m^2)*x^4*((c
*(e + f*x))/(e*(c + d*x)))^m + 6*e^4*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m)
 - 2*e^3*f*x*(4 + m - 4*((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x)
)/(e*(c + d*x)))^m)) - 2*c^3*d*e*x*(2*e*f^2*m*(4 + m)*x^2*((c*(e + f*x))/(e
*(c + d*x)))^m + f^3*(4 + 5*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m +
e^2*f*(4 + m)*x*(3 - 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(
e*(c + d*x)))^m) - e^3*(-8 + m + 8*((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((
c*(e + f*x))/(e*(c + d*x)))^m)) + c^2*d^2*e^2*x^2*(f^2*(12 + 7*m + m^2)*x^2
*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f*(4 + m)*x*(-3 + 3*((c*(e + f*x))/(
e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m) + e^2*(m^2*((c*(e + f*
x))/(e*(c + d*x)))^m + 12*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m) + m*(6 + 7
*((c*(e + f*x))/(e*(c + d*x)))^m))))*Gamma[5 + m]))/(2*c*e*(-(d*e) + c*f)^3
*(1 + m)*(2 + m)*(3 + m)*(4 + m)*x*((e + f*x)/e)^m*Gamma[4 + m]) + (a^3*f^3
*g*(e + f*x)^(1 + m)*(1 + (d*(e + f*x))/((c - (d*e)/f)*f))^m*Hypergeometric
2F1[1 + m, 4 + m, 2 + m, -((d*(e + f*x))/((c - (d*e)/f)*f))])/((-(d*e) + c*
f)^4*(1 + m)*(c - (d*e)/f + (d*(e + f*x))/f)^m)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (bx+a)^3 (dx+c)^{-4-m} \left(fx+e\right)^m \left(hx+g\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] int((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^3 \left(hx+g\right)(dx+c)^{-m-4} \left(fx+e\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3hx^4 + a^3g + \left(b^3g + 3\,ab^2h\right)x^3 + 3\left(ab^2g + a^2bh\right)x^2 + \left(3\,a^2bg + a^3h\right)x\right)(dx+c)^{-m-4}\left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b^3*h*x^4 + a^3*g + (b^3*g + 3*a*b^2*h)*x^3 + 3*(a*b^2*g + a^2*b*
h)*x^2 + (3*a^2*b*g + a^3*h)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^3 (hx+g)(dx+c)^{-m-4} (fx+e)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")

[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

3.133 $\int (a+bx)^2 (c+dx)^{-4-m} (e+fx)^m (g+hx)\, dx$

Optimal. Leaf size=572

$$-\frac{(dg-ch)(c+dx)^{-m-2}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df\left(a^2df+ab(cf(m+1)-de(m+3)\right)\right)}{d^3f(m+2)(m+3)(de-cf)^2}$$

```
[Out] ((b*c - a*d)*(d*g - c*h)*(a*d*f + b*(c*f*(2 + m) - d*e*(3 + m)))*(c + d*x)^
(-3 - m)*(e + f*x)^(1 + m))/(d^3*f*(d*e - c*f)*(3 + m)) - (b*(d*g - c*h)*(a
 + b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^2*f) - ((b*c - a*d)^2*h*(c
 + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)*(2 + m)) - ((d*g - c*h
)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(1 + m) - d*e*(3 + m)) - 2*d*f*(b^2*c*e + a
^2*d*f + a*b*(c*f*(1 + m) - d*e*(3 + m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1
+ m))/(d^3*f*(d*e - c*f)^2*(2 + m)*(3 + m)) - ((b*c - a*d)*h*(a*d*f - b*(2*
d*e*(2 + m) - c*f*(3 + 2*m)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d
*e - c*f)^2*(1 + m)*(2 + m)) + ((d*g - c*h)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(
1 + m) - d*e*(3 + m)) - 2*d*f*(b^2*c*e + a^2*d*f + a*b*(c*f*(1 + m) - d*e*(
3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*h*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f
*(c + d*x))/(d*e - c*f))])/(d^4*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m
)
```

Rubi [A] time = 0.662017, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {159, 89, 79, 70, 69, 90, 45, 37}

$$-\frac{(dg-ch)(c+dx)^{-m-2}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df\left(a^2df+b(acf(m+1)-ade(m+3\right)\right)}{d^3f(m+2)(m+3)(de-cf)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(d*g - c*h)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c + d*x)^
(-3 - m)*(e + f*x)^(1 + m))/(d^3*f*(d*e - c*f)*(3 + m)) - (b*(d*g - c*h)*(a
 + b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^2*f) - ((b*c - a*d)^2*h*(c
 + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)*(2 + m)) - ((d*g - c*h
)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b
*(b*c*e + a*c*f*(1 + m) - a*d*e*(3 + m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1
+ m))/(d^3*f*(d*e - c*f)^2*(2 + m)*(3 + m)) - ((b*c - a*d)*h*(a*d*f - 2*b*d
*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d
*e - c*f)^2*(1 + m)*(2 + m)) + ((d*g - c*h)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(
1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b*(b*c*e + a*c*f*(1 + m) - a*d*e*(
3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*h*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f
*(c + d*x))/(d*e - c*f))])/(d^4*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m
)
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

1]

Rubi steps

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{h\int (a+bx)^2(c+dx)^{-3-m}(e+fx)^m\,dx}{d} + \frac{(dg-ch)\int (a+bx)^2(c+dx)^{-4-m}}{d}$$

$$= -\frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f} - \frac{(bc-ad)^2h(c+dx)^{-2-m}(e}{d^3(de-cf)(2+m}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-m}(e+fx)^{1}}{d^3f(de-cf)(3+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-m}(e+fx)^{1}}{d^3f(de-cf)(3+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-m}(e+fx)^{1}}{d^3f(de-cf)(3+m)}$$

Mathematica [A] time = 1.8164, size = 422, normalized size = 0.74

$$\frac{(c+dx)^{-m-3}(e+fx)^m\left(-h(m+3)(c+dx)(de-cf)\left(df(m+1)(e+fx)(bc-ad)^2(de-cf)-(c+dx)\left(d(e+fx)\left(a^2d^2f^2+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^m*(-(d*(d*g - c*h)*(e + f*x)*(-((b*c - a*d)*(
d*e - c*f)^2*(1 + m)*(2 + m)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))) + b*d
*(d*e - c*f)^3*(1 + m)*(2 + m)*(3 + m)*(a + b*x) + (b^2*(d*e - c*f)*(2 + m)
*(c*f*(1 + m) - d*e*(3 + m)) + 2*d*f*(-(a^2*d*f) - b*(b*c*e + a*c*f*(1 + m)
 - a*d*e*(3 + m))))*(c + d*x)*(-(c*f*(2 + m)) + d*(e + e*m - f*x)))) - (d*e
 - c*f)*h*(3 + m)*(c + d*x)*(d*(b*c - a*d)^2*f*(d*e - c*f)*(1 + m)*(e + f*x
) - (c + d*x)*(d*(a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(2 + m)) + b^2
*(-(c^2*f^2*(1 + m)) + d^2*e^2*(2 + m)))*(e + f*x) - (b^2*(d*e - c*f)^3*(2
+ m)*Hypergeometric2F1[-1 - m, -1 - m, -m, (f*(c + d*x))/(-(d*e) + c*f)])/(
(d*(e + f*x))/(d*e - c*f))^m))))/(d^4*f*(d*e - c*f)^3*(1 + m)*(2 + m)*(3 +
m))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (bx+a)^2(dx+c)^{-4-m}(fx+e)^m(hx+g)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2hx^3 + a^2g + \left(b^2g + 2\,abh\right)x^2 + \left(2\,abg + a^2h\right)x\right)(dx+c)^{-m-4}\left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b^2*h*x^3 + a^2*g + (b^2*g + 2*a*b*h)*x^2 + (2*a*b*g + a^2*h)*x)*
(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

3.134 $\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=363

$$\frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)+d(2fg-eh(m+3)))+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2eh(m+3\right.\right.}{d^2f(m+2)(m+3)(de-cf)^2}$$

```
[Out] ((b*(c^2*f^2*h*(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*
(1 + m)*(f*g - 2*e*h*(3 + m))) + a*d*f*(c*f*h*(1 + m) + d*(2*f*g - e*h*(3 +
 m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^2*f*(d*e - c*f)^2*(2 + m)*(
3 + m)) - ((b*(c^2*f^2*h*(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)
) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 + m))) + a*d*f*(c*f*h*(1 + m) + d*(2*f*g
- e*h*(3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^2*(d*e - c*f)^3*(
1 + m)*(2 + m)*(3 + m)) - ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g
 - c*h) - b*c*(c*f*h*(2 + m) + d*(f*g - e*h*(3 + m))) + b*d*(d*e - c*f)*h*(
3 + m)*x))/(d^2*f*(d*e - c*f)*(3 + m))
```

Rubi [A] time = 0.396708, antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {146, 45, 37}

$$\frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2eh(m+3\right.\right.}{d^2f(m+2)(m+3)(de-cf)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m +
 m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 +
 m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^2*f*(d*e - c*f)^2*(2 + m)*(
3 + m)) - ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*
(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g -
2*e*h*(3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^2*(d*e - c*f)^3*(
1 + m)*(2 + m)*(3 + m)) - ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g
 - c*h) - b*c*(d*f*g + c*f*h*(2 + m) - d*e*h*(3 + m)) + b*d*(d*e - c*f)*h*(
3 + m)*x))/(d^2*f*(d*e - c*f)*(3 + m))
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(dfg+cfh(2+m)-deh(3}{d^2f(de-cf)(3+m)}$$

$$= \frac{\left(adf(2dfg+cfh(1+m)-deh(3+m))+b\left(c^2f^2h\left(2+3m+m^2\right)-d^2\right.\right.}{d^2f(de}$$

$$= \frac{\left(adf(2dfg+cfh(1+m)-deh(3+m))+b\left(c^2f^2h\left(2+3m+m^2\right)-d^2\right.\right.}{d^2f(de}$$

Mathematica [A] time = 0.573499, size = 227, normalized size = 0.63

$$\frac{(c+dx)^{-m-3}(e+fx)^{m+1}\left(\frac{(c+dx)(cf(m+2)-d(em+e-fx))\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2eh(m+3))+d^2e(m}{(m+1)(m+2)(de-cf)^2}\right.}{d^2f(m+3)(cf-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g - c*h) + ((a*d*f*(2*d*f*g
 + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m + m^2) + d^2*e*(3
 + m)*(-(f*g) + e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 + m))))*(c + d
*x)*(c*f*(2 + m) - d*(e + e*m - f*x)))/((d*e - c*f)^2*(1 + m)*(2 + m)) - b*
(c^2*f*h*(2 + m) - d^2*e*h*(3 + m)*x + c*d*(-(e*h*(3 + m)) + f*(g + h*(3 +
m)*x)))))/(d^2*f*(-(d*e) + c*f)*(3 + m))
```

Maple [B] time = 0.008, size = 906, normalized size = 2.5

$$-\frac{(dx+c)^{-3-m}\left(fx+e\right)^{1+m}\left(-bc^2f^2hm^2x^2+2\,bcdefhm^2x^2-bd^2e^2hm^2x^2-ac^2f^2hm^2x+2\,acdefhm^2x-acdf^2hmx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] -(d*x+c)^(-3-m)*(f*x+e)^(1+m)*(-b*c^2*f^2*h*m^2*x^2+2*b*c*d*e*f*h*m^2*x^2-b
*d^2*e^2*h*m^2*x^2-a*c^2*f^2*h*m^2*x+2*a*c*d*e*f*h*m^2*x-a*c*d*f^2*h*m*x^2-
a*d^2*e^2*h*m^2*x+a*d^2*e*f*h*m*x^2-b*c^2*f^2*g*m^2*x-3*b*c^2*f^2*h*m*x^2+2
*b*c*d*e*f*g*m^2*x+8*b*c*d*e*f*h*m*x^2-b*c*d*f^2*g*m*x^2-b*d^2*e^2*g*m^2*x-
5*b*d^2*e^2*h*m*x^2+b*d^2*e*f*g*m*x^2-a*c^2*f^2*g*m^2-4*a*c^2*f^2*h*m*x+2*a
```

```
*c*d*e*f*g*m^2+8*a*c*d*e*f*h*m*x-2*a*c*d*f^2*g*m*x-a*c*d*f^2*h*x^2-a*d^2*e^
2*g*m^2-4*a*d^2*e^2*h*m*x+2*a*d^2*e*f*g*m*x+3*a*d^2*e*f*h*x^2-2*a*d^2*f^2*g
*x^2+2*b*c^2*e*f*h*m*x-4*b*c^2*f^2*g*m*x-2*b*c^2*f^2*h*x^2-2*b*c*d*e^2*h*m*
x+8*b*c*d*e*f*g*m*x+6*b*c*d*e*f*h*x^2-b*c*d*f^2*g*x^2-4*b*d^2*e^2*g*m*x-6*b
*d^2*e^2*h*x^2+3*b*d^2*e*f*g*x^2+a*c^2*e*f*h*m-5*a*c^2*f^2*g*m-3*a*c^2*f^2*
h*x-a*c*d*e^2*h*m+8*a*c*d*e*f*g*m+10*a*c*d*e*f*h*x-6*a*c*d*f^2*g*x-3*a*d^2*
e^2*g*m-3*a*d^2*e^2*h*x+2*a*d^2*e*f*g*x+b*c^2*e*f*g*m+2*b*c^2*e*f*h*x-3*b*c
^2*f^2*g*x-b*c*d*e^2*g*m-6*b*c*d*e^2*h*x+10*b*c*d*e*f*g*x-3*b*d^2*e^2*g*x+3
*a*c^2*e*f*h-6*a*c^2*f^2*g-a*c*d*e^2*h+6*a*c*d*e*f*g-2*a*d^2*e^2*g-2*b*c^2*
e^2*h+3*b*c^2*e*f*g-b*c*d*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2
*f*m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^
3*e^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3
*f^3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)\left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [B] time = 1.60383, size = 3272, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -(((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*h*m^2 - (3*b*d^3*e*f^2 - (
b*c*d^2 + 2*a*d^3)*f^3)*g + (6*b*d^3*e^2*f - 3*(2*b*c*d^2 + a*d^3)*e*f^2 +
(2*b*c^2*d + a*c*d^2)*f^3)*h - ((b*d^3*e*f^2 - b*c*d^2*f^3)*g - (5*b*d^3*e^
2*f - (8*b*c*d^2 + a*d^3)*e*f^2 + (3*b*c^2*d + a*c*d^2)*f^3)*h)*m)*x^4 + (a
*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3*e*f^2)*g*m^2 + (((b*d^3*e^2*f - 2*b*c*
d^2*e*f^2 + b*c^2*d*f^3)*g + (b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*
d + 2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*h)*m^2 - 4*(3*b*c*d^2*e*f^2 -
 (b*c^2*d + 2*a*c*d^2)*f^3)*g + 2*(3*b*d^3*e^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2
*d + 2*a*c*d^2)*e*f^2 + (b*c^3 + 2*a*c^2*d)*f^3)*h + ((3*b*d^3*e^2*f - 2*(4
*b*c*d^2 + a*d^3)*e*f^2 + (5*b*c^2*d + 2*a*c*d^2)*f^3)*g + (5*b*d^3*e^3 - (
b*c*d^2 - 3*a*d^3)*e^2*f - (7*b*c^2*d + 8*a*c*d^2)*e*f^2 + (3*b*c^3 + 5*a*c
^2*d)*f^3)*h)*m)*x^3 + (((b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*d +
2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*g + (a*c^3*f^3 + (b*c*d^2 + a*d^3
)*e^3 - (2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*h)*m^2 + 3*(
b*d^3*e^3 - 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + (b*c^3 + 4*a*c^2*d)*f^3)*g
- 3*(3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3 - (4*b*c*d^2 + a*d^3)*e^
3)*h + ((4*b*d^3*e^3 - (4*b*c*d^2 - a*d^3)*e^2*f - 4*(b*c^2*d + 2*a*c*d^2)*
e*f^2 + (4*b*c^3 + 7*a*c^2*d)*f^3)*g + (4*a*c^3*f^3 + (7*b*c*d^2 + 4*a*d^3)
*e^3 - 4*(2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - 4*a*c^2*d)*e*f^2)*h)*m)*x^2
 + (6*a*c^3*e*f^2 + (b*c^2*d + 2*a*c*d^2)*e^3 - 3*(b*c^3 + 2*a*c^2*d)*e^2*f
)*g - (3*a*c^3*e^2*f - (2*b*c^3 + a*c^2*d)*e^3)*h + ((5*a*c^3*e*f^2 + (b*c^
2*d + 3*a*c*d^2)*e^3 - (b*c^3 + 8*a*c^2*d)*e^2*f)*g + (a*c^2*d*e^3 - a*c^3*
```

```
e^2*f)*h)*m + (((a*c^3*f^3 + (b*c*d^2 + a*d^3)*e^3 - (2*b*c^2*d + a*c*d^2)*
e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*g + (a*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3
*e*f^2)*h)*m^2 + 2*(3*a*c^2*d*e*f^2 + 3*a*c^3*f^3 + (2*b*c*d^2 + a*d^3)*e^3
 - 3*(2*b*c^2*d + a*c*d^2)*e^2*f)*g - 4*(3*a*c^2*d*e^2*f - (2*b*c^2*d + a*c
*d^2)*e^3)*h + ((5*a*c^3*f^3 + (5*b*c*d^2 + 3*a*d^3)*e^3 - (8*b*c^2*d + 7*a
*c*d^2)*e^2*f + (3*b*c^3 - a*c^2*d)*e*f^2)*g + (3*a*c^3*e*f^2 + (2*b*c^2*d
+ 5*a*c*d^2)*e^3 - 2*(b*c^3 + 4*a*c^2*d)*e^2*f)*h)*m)*x)*(d*x + c)^(-m - 4)
*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^
3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2
*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2
*d*e*f^2 - c^3*f^3)*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

3.135 $\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=188

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)+d(2fg-eh(m+3)))}{d(m+2)(m+3)(de-cf)^2}-\frac{f(c+dx)^{-m-1}(e+f\ldots}{d(m+\ldots}$$

```
[Out] -(((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m)
)) + ((c*f*h*(1 + m) + d*(2*f*g - e*h*(3 + m)))*(c + d*x)^(-2 - m)*(e + f*x
)^(1 + m))/(d*(d*e - c*f)^2*(2 + m)*(3 + m)) - (f*(c*f*h*(1 + m) + d*(2*f*g
 - e*h*(3 + m)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)^3*(1
+ m)*(2 + m)*(3 + m))
```

Rubi [A] time = 0.0982305, antiderivative size = 186, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {79, 45, 37}

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+2)(m+3)(de-cf)^2}-\frac{f(c+dx)^{-m-1}(e+f\ldots}{d(m+1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] -(((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m)
)) + ((2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m))*(c + d*x)^(-2 - m)*(e + f*x
)^(1 + m))/(d*(d*e - c*f)^2*(2 + m)*(3 + m)) - (f*(2*d*f*g + c*f*h*(1 + m)
- d*e*h*(3 + m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)^3*(1
+ m)*(2 + m)*(3 + m))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} - \frac{(2dfg+cfh(1+m)-deh(3+m))\int (c+}{d(de-cf)(3+m)}$$
$$= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} + \frac{(2dfg+cfh(1+m)-deh(3+m))(c+d}{d(de-cf)^2(2+m)(3+}$$
$$= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} + \frac{(2dfg+cfh(1+m)-deh(3+m))(c+d}{d(de-cf)^2(2+m)(3+}$$

Mathematica [A] time = 0.119008, size = 182, normalized size = 0.97

$$-\frac{(ch-dg)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(-m-3)(de-cf)} - \frac{\left(\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(-m-2)(de-cf)} + \frac{f(c+dx)^{-m-1}(e+fx)^{m+1}}{(-m-2)(-m-1)(de-cf)^2}\right)(-h(cf(m+1)+de(-m-3))-2dfg)}{d(-m-3)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] -(((-(d*g) + c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(-3
 - m))) - ((-2*d*f*g - h*(d*e*(-3 - m) + c*f*(1 + m)))*(((c + d*x)^(-2 - m)*
(e + f*x)^(1 + m))/((d*e - c*f)*(-2 - m)) + (f*(c + d*x)^(-1 - m)*(e + f*x)
^(1 + m))/((d*e - c*f)^2*(-2 - m)*(-1 - m))))/(d*(d*e - c*f)*(-3 - m))
```

Maple [B] time = 0.008, size = 509, normalized size = 2.7

$$-\frac{(dx+c)^{-3-m}\left(fx+e\right)^{1+m}\left(-c^2f^2hm^2x+2\,cdefhm^2x-cdf^2hmx^2-d^2e^2hm^2x+d^2efhmx^2-c^2f^2gm^2-4\,c^2f^2hmx\right.}{c^3f^3m^3-3\,c^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] -(d*x+c)^(-3-m)*(f*x+e)^(1+m)*(-c^2*f^2*h*m^2*x+2*c*d*e*f*h*m^2*x-c*d*f^2*h
*m*x^2-d^2*e^2*h*m^2*x+d^2*e*f*h*m*x^2-c^2*f^2*g*m^2-4*c^2*f^2*h*m*x+2*c*d*
e*f*g*m^2+8*c*d*e*f*h*m*x-2*c*d*f^2*g*m*x-c*d*f^2*h*x^2-d^2*e^2*g*m^2-4*d^2
*e^2*h*m*x+2*d^2*e*f*g*m*x+3*d^2*e*f*h*x^2-2*d^2*f^2*g*x^2+c^2*e*f*h*m-5*c^
2*f^2*g*m-3*c^2*f^2*h*x-c*d*e^2*h*m+8*c*d*e*f*g*m+10*c*d*e*f*h*x-6*c*d*f^2*
g*x-3*d^2*e^2*g*m-3*d^2*e^2*h*x+2*d^2*e*f*g*x+3*c^2*e*f*h-6*c^2*f^2*g-c*d*e
^2*h+6*c*d*e*f*g-2*d^2*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f*
m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3*e
^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3*f^
3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (hx+g)(dx+c)^{-m-4}\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [B] time = 1.50452, size = 1805, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -((2*d^3*f^3*g - (d^3*e*f^2 - c*d^2*f^3)*h*m - (3*d^3*e*f^2 - c*d^2*f^3)*h)
*x^4 + (c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*g*m^2 + (8*c*d^2*f^3*g + (d^
3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*h*m^2 - 4*(3*c*d^2*e*f^2 - c^2*d*f^3)*
h - (2*(d^3*e*f^2 - c*d^2*f^3)*g - (3*d^3*e^2*f - 8*c*d^2*e*f^2 + 5*c^2*d*f
^3)*h)*m)*x^3 + (12*c^2*d*f^3*g + ((d^3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*
g + (d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m^2 + 3*(d^3*e^3 - 3
*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*h + ((d^3*e^2*f - 8*c*d^2*e*f^2 + 7
*c^2*d*f^3)*g + 4*(d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m)*x^2
 + 2*(c*d^2*e^3 - 3*c^2*d*e^2*f + 3*c^3*e*f^2)*g + (c^2*d*e^3 - 3*c^3*e^2*f
)*h + ((3*c*d^2*e^3 - 8*c^2*d*e^2*f + 5*c^3*e*f^2)*g + (c^2*d*e^3 - c^3*e^2
*f)*h)*m + (((d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*g + (c*d^2*e^3
 - 2*c^2*d*e^2*f + c^3*e*f^2)*h)*m^2 + 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d
*e*f^2 + 3*c^3*f^3)*g + 4*(c*d^2*e^3 - 3*c^2*d*e^2*f)*h + ((3*d^3*e^3 - 7*c
*d^2*e^2*f - c^2*d*e*f^2 + 5*c^3*f^3)*g + (5*c*d^2*e^3 - 8*c^2*d*e^2*f + 3*
c^3*e*f^2)*h)*m)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^
2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2
 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

3.136 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx} dx$

Optimal. Leaf size=177

$$-\frac{(Ab-aB)(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F_1\left(n+1;1,-p;n+2;\frac{b(c+dx)}{bc-ad},-\frac{f(c+dx)}{de-cf}\right)}{b(n+1)(bc-ad)}-\frac{B(c+dx)^{n+1}(e+fx)^{p+1}{}_2F_1\left(1,\right.}{b(p+1)(de-}$$

```
[Out] -(((A*b - a*B)*(c + d*x)^(1 + n)*(e + f*x)^p*AppellF1[1 + n, 1, -p, 2 + n,
(b*(c + d*x))/(b*c - a*d), -((f*(c + d*x))/(d*e - c*f))])/(b*(b*c - a*d)*(1
 + n)*((d*(e + f*x))/(d*e - c*f))^p)) - (B*(c + d*x)^(1 + n)*(e + f*x)^(1 +
 p)*Hypergeometric2F1[1, 2 + n + p, 2 + p, (d*(e + f*x))/(d*e - c*f)])/(b*(
d*e - c*f)*(1 + p))
```

Rubi [A] time = 0.118519, antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {157, 70, 69, 137, 136}

$$\frac{B(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}{}_2F_1\left(n+1,-p;n+2;-\frac{f(c+dx)}{de-cf}\right)}{bd(n+1)}-\frac{(Ab-aB)(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F_1\left(n+\right.}{b(n+1)(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] -(((A*b - a*B)*(c + d*x)^(1 + n)*(e + f*x)^p*AppellF1[1 + n, -p, 1, 2 + n,
-((f*(c + d*x))/(d*e - c*f)), (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*(1
 + n)*((d*(e + f*x))/(d*e - c*f))^p)) + (B*(c + d*x)^(1 + n)*(e + f*x)^p*Hy
pergeometric2F1[1 + n, -p, 2 + n, -((f*(c + d*x))/(d*e - c*f))])/(b*d*(1 +
n)*((d*(e + f*x))/(d*e - c*f))^p)
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx = \frac{B\int (c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{a+bx}\,dx}{b}$$

$$= \frac{\left(B(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\right)\int (c+dx)^n\left(\frac{de}{de-cf}+\frac{dfx}{de-cf}\right)^p\,dx}{b} + \frac{\left((Ab-aB)(e+fx)^p\left(\frac{d(e+\ldots}{de-\ldots}\right.\right.}{\ldots}$$

$$= -\frac{(Ab-aB)(c+dx)^{1+n}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F_1\left(1+n;-p,1;2+n;-\frac{f(c+dx)}{de-cf},\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)(1+n)} - \ldots$$

Mathematica [A] time = 0.398779, size = 199, normalized size = 1.12

$$\frac{(c+dx)^n(e+fx)^p\left(\frac{(Ab-aB)\left(\frac{b(c+dx)}{d(a+bx)}\right)^{-n}\left(\frac{b(e+fx)}{f(a+bx)}\right)^{-p}F_1\left(-n-p;-n,-p;-n-p+1;\frac{ad-bc}{d(a+bx)},\frac{af-be}{f(a+bx)}\right)}{n+p}+\frac{bB(e+fx)\left(\frac{f(c+dx)}{cf-de}\right)^{-n}{}_2F_1\left(-n,p+1;p+2;\frac{d(e+fx)}{de-cf}\right)}{f(p+1)}\right)}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] ((c + d*x)^n*(e + f*x)^p*(((A*b - a*B)*AppellF1[-n - p, -n, -p, 1 - n - p,
(-(b*c) + a*d)/(d*(a + b*x)), (-(b*e) + a*f)/(f*(a + b*x))])/((n + p)*((b*(
c + d*x))/(d*(a + b*x)))^n*((b*(e + f*x))/(f*(a + b*x)))^p) + (b*B*(e + f*x
)*Hypergeometric2F1[-n, 1 + p, 2 + p, (d*(e + f*x))/(d*e - c*f)])/(f*(1 + p
)*((f*(c + d*x))/(-(d*e) + c*f))^n)))/b^2
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^p(dx+c)^n(Bx+A)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
```

```
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(dx+c)^n (fx+e)^p}{bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx+A)(dx+c)^n (fx+e)^p}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(dx+c)^n (fx+e)^p}{bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

3.137 $\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx$

Optimal. Leaf size=233

$$-\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\,{}_2F_1\left(1,-m;1-m;\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m}-\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\,{}_2F_1\left(m,m+1;m+2;\right.}{bf^2m(m+1)(bc}$$

```
[Out] -((d*(B*e - A*f)*(a + b*x)^(1 + m))/((b*c - a*d)*f^2*m*(c + d*x)^m)) - ((B*
e - A*f)*(a + b*x)^m*Hypergeometric2F1[1, -m, 1 - m, ((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))])/(f^2*m*(c + d*x)^m) - ((a*B*d*f*m - b*(B*d*e -
A*d*f + B*c*f*m))*(a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeom
etric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*f^2
*m*(1 + m)*(c + d*x)^m)
```

Rubi [A] time = 0.12704, antiderivative size = 220, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {157, 70, 69, 105, 131}

$$\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\,{}_2F_1\left(1,m;m+1;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m}-\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\,{}_2F_1\left(m,m;m+1\right.}{f^2m}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^m*Hypergeometric2F1[1, m, 1 + m, ((d*e - c*f)*(a + b
*x))/((b*e - a*f)*(c + d*x))])/(f^2*m*(c + d*x)^m) - ((B*e - A*f)*(a + b*x)
^m*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, m, 1 + m, -((d*(a + b
*x))/(b*c - a*d))])/(f^2*m*(c + d*x)^m) + (B*(a + b*x)^(1 + m)*((b*(c + d*x
))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))/(b*c -
 a*d))])/(b*f*(1 + m)*(c + d*x)^m)
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \frac{B\int (a+bx)^m(c+dx)^{-m}\,dx}{f} + \frac{(-Be+Af)\int \frac{(a+bx)^m(c+dx)^{-m}}{e+fx}\,dx}{f}$$

$$= -\frac{(b(Be-Af))\int (a+bx)^{-1+m}(c+dx)^{-m}\,dx}{f^2} + \frac{((be-af)(Be-Af))\int \frac{(a+bx)^{-1+m_{\cdot}}}{e+f}$$

$$= \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,m;1+m;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m} + \frac{B(a+bx)^{1+m}(c}{}$$

$$= \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,m;1+m;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m} - \frac{(Be-Af)(a+b}{}$$

Mathematica [A] time = 0.136575, size = 174, normalized size = 0.75

$$\frac{(a+bx)^m(c+dx)^{-m}\left(\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(Bfm(a+bx)\,{}_2F_1\left(m,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)-b(m+1)(Be-Af)\,{}_2F_1\left(m,m;m+1;\frac{d(}{a}\right.\right.\right.}{bf^2m(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] ((a + b*x)^m*(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[1, m, 1 + m, ((d*e -
c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))] + ((b*(c + d*x))/(b*c - a*d))^m*(-
(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[m, m, 1 + m, (d*(a + b*x))/(-(b*c)
 + a*d)]) + B*f*m*(a + b*x)*Hypergeometric2F1[m, 1 + m, 2 + m, (d*(a + b*x)
)/(-(b*c) + a*d)])))/(b*f^2*m*(1 + m)*(c + d*x)^m)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(bx+a)^m}{(dx+c)^m\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)/((d*x+c)**m)/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

3.138 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=250

$$\frac{2\sqrt{a+bx}(Ab-aB)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2}+\frac{2B(a+bx)^{3/2}(c+d}{}$$

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*AppellF1[1/2, -n, -p,
3/2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (2*B*(a + b*x)^(
3/2)*(c + d*x)^n*(e + f*x)^p*AppellF1[3/2, -n, -p, 5/2, -((d*(a + b*x))/(b*
c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(3*b^2*((b*(c + d*x))/(b*c - a*d)
)^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rubi [A] time = 0.217165, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {159, 140, 139, 138}

$$\frac{2\sqrt{a+bx}(Ab-aB)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2}+\frac{2B(a+bx)^{3/2}(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*AppellF1[1/2, -n, -p,
3/2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (2*B*(a + b*x)^(
3/2)*(c + d*x)^n*(e + f*x)^p*AppellF1[3/2, -n, -p, 5/2, -((d*(a + b*x))/(b*
c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(3*b^2*((b*(c + d*x))/(b*c - a*d)
)^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \frac{B\int \sqrt{a+bx}(c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b} + \frac{\left((Ab-aB)(c+d \cdots\right.}{\cdots}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bf \cdots}{be- \cdots}\right.}{b}$$

$$= \frac{2(Ab-aB)\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+b \cdots}{bc-a \cdots}\right.}{b^2}$$

Mathematica [A] time = 0.20808, size = 184, normalized size = 0.74

$$\frac{2\sqrt{a+bx}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\left(3(Ab-aB)F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};\frac{d(a+bx)}{ad-bc},\frac{f(a+bx)}{af-be}\right)+B(a+bx)F_1\left(\frac{3}{2};-n,-p; \cdots\right.\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*(3*(A*b - a*B)*AppellF1[1/2, -n, -
p, 3/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + B*(a
+ b*x)*AppellF1[3/2, -n, -p, 5/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x
))/(-(b*e) + a*f)]))/(3*b^2*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b
*e - a*f))^p)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (Bx+A)(dx+c)^n\left(fx+e\right)^p\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
```

```
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(dx+c)^n (fx+e)^p}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx+A)(dx+c)^n (fx+e)^p}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(dx+c)^n (fx+e)^p}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

3.139 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$

Optimal. Leaf size=530

$$\frac{3h^2(bg-ah)(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+3;-n,-p;m+4;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+3)}+\frac{(bg-ah}{}$$

```
[Out] ((b*g - a*h)^3*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^4*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(3*h*(b*g - a*h)^2*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m
, -n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))
])/(b^4*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p
) + (3*h^2*(b*g - a*h)*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3
 + m, -n, -p, 4 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a
*f))])/(b^4*(3 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f
))^p) + (h^3*(a + b*x)^(4 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[4 + m, -n,
-p, 5 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
4*(4 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rubi [A] time = 1.17864, antiderivative size = 530, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {181, 159, 140, 139, 138}

$$\frac{3h^2(bg-ah)(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+3;-n,-p;m+4;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+3)}+\frac{(bg-ah}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
```


```
[Out] ((b*g - a*h)^3*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^4*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(3*h*(b*g - a*h)^2*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m
, -n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))
])/(b^4*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p
) + (3*h^2*(b*g - a*h)*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3
 + m, -n, -p, 4 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a
*f))])/(b^4*(3 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f
))^p) + (h^3*(a + b*x)^(4 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[4 + m, -n,
-p, 5 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
4*(4 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```


Rule 181

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)^2\,dx}{b} + \frac{(bg-ah)\int (a+bx)^m(c

Mathematica [F] time = 4.52404, size = 0, normalized size = 0.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
```

```
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3, x]
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^n \left(fx+e\right)^p \left(hx+g\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(hx+g\right)^3 (bx+a)^m (dx+c)^n \left(fx+e\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)^3*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(h^3x^3+3\,gh^2x^2+3\,g^2hx+g^3\right)(bx+a)^m(dx+c)^n\left(fx+e\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*(b*x + a)^m*(d*x + c)^n*
(f*x + e)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.140 $\int (a+bx)^m (c+dx)^n (e+fx)^p (g+hx)^2 \, dx$

Optimal. Leaf size=393

$$\frac{(bg-ah)^2(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(m+1)}+\frac{2h(bg-ah}{}$$

```
[Out] ((b*g - a*h)^2*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^3*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(2*h*(b*g - a*h)*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m,
-n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])
/(b^3*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
+ (h^2*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3 + m, -n, -p, 4
+ m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^3*(3 +
 m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rubi [A] time = 0.492588, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {181, 159, 140, 139, 138}

$$\frac{(bg-ah)^2(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(m+1)}+\frac{2h(bg-ah}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
```

```
[Out] ((b*g - a*h)^2*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^3*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(2*h*(b*g - a*h)*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m,
-n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])
/(b^3*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
+ (h^2*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3 + m, -n, -p, 4
+ m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^3*(3 +
 m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 181

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b} + \frac{(bg-ah)\int (a+bx)^m(c+$$

$$= \frac{h^2\int (a+bx)^{2+m}(c+dx)^n(e+fx)^p\,dx}{b^2} + 2\frac{(h(bg-ah))\int (a+bx)^{1+m}(c+}{b^2}$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^2} + 2\cdot$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-a}\right.}{b^2}$$

$$= \frac{(bg-ah)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m\right.}{b^3(1+m)}$$

Mathematica [F] time = 1.486, size = 0, normalized size = 0.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
```

```
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2, x]
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^n \left(fx+e\right)^p \left(hx+g\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(hx+g\right)^2 (bx+a)^m (dx+c)^n \left(fx+e\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(h^2x^2+2\,ghx+g^2\right)(bx+a)^m(dx+c)^n\left(fx+e\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.141 $\int(a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$

Optimal. Leaf size=256

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+\frac{h(a+bx)}{}$$

```
[Out] ((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n,
-p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (h
*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m, -n, -p, 3 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((
b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rubi [A] time = 0.210448, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {159, 140, 139, 138}

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+\frac{h(a+bx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
```


```
[Out] ((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n,
-p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (h
*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m, -n, -p, 3 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((
b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```


Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p\,dx}{b} + \frac{(bg-ah)\int (a+bx)^m(c+dx)^n(e+fx}{b}$$

$$= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b} + \frac{\Big((bg-}{}$$

$$= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\Big(}{b}$$

$$= \frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\Big(1+m;-n,}{b^2(1+m)}$$

Mathematica [F] time = 0.905266, size = 0, normalized size = 0.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
```

```
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x), x]
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^n \left(fx+e\right)^p \left(hx+g\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((hx+g)(bx+a)^m(dx+c)^n(fx+e)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

3.142 $\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx$

Optimal. Leaf size=123

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(m+1)}$$

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rubi [A] time = 0.0830603, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {140, 139, 138}

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx = \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx$$
$$= \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right.$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d}{b}\right.}{b(1+m)}$$

Mathematica [A] time = 0.133515, size = 121, normalized size = 0.98

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;\frac{d(a+bx)}{ad-bc},\frac{f(a+bx)}{af-be}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, (
d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Maple [F] time = 0.002, size = 0, normalized size = 0.

$$\int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^m(dx+c)^n(fx+e)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^n (fx+e)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

3.143 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} dx$

Optimal. Leaf size=31

$$\text{CannotIntegrate}\left(\frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx},x\right)$$

```
[Out] Defer[Int][((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Rubi [A] time = 0.0075481, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
```

```
[Out] Defer[Int][((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Rubi steps

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} dx = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} dx$$

Mathematica [A] time = 0.25432, size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
```

```
[Out] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Maple [A] time = 0.157, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m (dx+c)^n \left(fx+e\right)^p}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

3.144 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-m-n}\, dx$

Optimal. Leaf size=268

$$\frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+$$

```
[Out] ((A*b - a*B)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x)
)/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b
*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(1 + m)*((b*(c + d*x))/(b*c
 - a*d))^n) + (B*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[2 + m, -n, m + n, 3 + m, -((d*(a + b*x)
)/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((b*(c + d*x))/
(b*c - a*d))^n)
```

Rubi [A] time = 0.211948, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {159, 140, 139, 138}

$$\frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
```

```
[Out] ((A*b - a*B)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x)
)/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b
*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(1 + m)*((b*(c + d*x))/(b*c
 - a*d))^n) + (B*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[2 + m, -n, m + n, 3 + m, -((d*(a + b*x)
)/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((b*(c + d*x))/
(b*c - a*d))^n)
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \frac{B\int (a+bx)^{1+m}(c+dx)^n(e+fx)^{-m-n}\,dx}{b} + \frac{(Ab-aB)\int (a+bx)^m(c+dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\right.}{b}$$

$$= \frac{(Ab-aB)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(\right.}{b^2(1+m)}$$

Mathematica [F] time = 0.636679, size = 0, normalized size = 0.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
```

```
[Out] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n), x]
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int (bx+a)^m(Bx+A)(dx+c)^n\left(fx+e\right)^{-n-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-n-m),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-n-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-m-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

3.145 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-1-m-n}\,dx$

Optimal. Leaf size=283

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n} F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{bf(m+1)} - \frac{(a+bx)^{m+\cdots}}{\cdots}$$

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b*c - a*d))
, -((f*(a + b*x))/(b*e - a*f))])/(b*f*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n
) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a +
b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*(
(b*(c + d*x))/(b*c - a*d))^n)
```

Rubi [A] time = 0.208066, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {159, 140, 139, 138}

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n} F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{bf(m+1)} - \frac{(a+bx)^{m+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b*c - a*d))
, -((f*(a + b*x))/(b*e - a*f))])/(b*f*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n
) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a +
b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*(
(b*(c + d*x))/(b*c - a*d))^n)
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx = \frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n}\,dx}{f} + \frac{(-Be+Af)\int (a+bx)^m(c}{f}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}\,d}{f}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^m\left(\frac{bc}{bc-a}\right.}{f}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+\right.}{bf(1+m)}$$

Mathematica [A] time = 0.320218, size = 208, normalized size = 0.73

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n+1}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n-1}\left(b(Af-Be)F_1\left(m+1;-n,m+n+1;m+2;\frac{d(a+bx)}{ad-bc},\frac{f(a}{af}\right.\right.}{f(m+1)(be-af)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(1 - m - n)*((b*(e + f*x))/(b*e -
a*f))^(-1 + m + n)*(B*(b*e - a*f)*AppellF1[1 + m, -n, m + n, 2 + m, (d*(a +
 b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + b*(-(B*e) + A*f)*App
ellF1[1 + m, -n, 1 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*
x))/(-(b*e) + a*f)]))/(f*(b*e - a*f)^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^
n)
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int (bx+a)^m(Bx+A)(dx+c)^n\left(fx+e\right)^{-1-m-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-1-m-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

3.146 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-2-m-n}\,dx$

Optimal. Leaf size=277

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n+1;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{f(m+1)(be-af)}-\frac{(a+\cdots}{\cdots}$$

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a + b*x))/(b*c - a
*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1
- m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*
c - a*d)*(e + f*x)))])/(f*(b*e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*
c - a*d)*(e + f*x)))^n)
```

Rubi [A] time = 0.147295, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {159, 140, 139, 138, 132}

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n+1;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{f(m+1)(be-af)}-\frac{(a+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```


```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a + b*x))/(b*c - a
*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1
- m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*
c - a*d)*(e + f*x)))])/(f*(b*e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*
c - a*d)*(e + f*x)))^n)
```


Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx = \frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-1-m-n}\,dx}{f} + \frac{(-Be+Af)\int (a+bx)^m(c \cdots}{f}$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1-m-n}\,{}_2F_1\left(1 \cdots\right.}{f(be-af)(1+m)}$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1-m-n}\,{}_2F_1\left(1 \cdots\right.}{f(be-af)(1+m)}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m \cdots\right.}{f(be-af)(1+m)}$$

Mathematica [A] time = 0.442717, size = 215, normalized size = 0.78

$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{n}\left((Af-Be)\,{}_2F_1\left(m+1,-n;m+2;\frac{(cf-de)(a+bx)}{(bc-ad)(e+fx)}\right)+B(e+fx)\left(\frac{b \cdots}{\cdots}\right.\right.}{f(m+1)(af-be)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*((b*(e + f*x))/(b*e
 - a*f))^n*(B*(e + f*x)*((b*(e + f*x))/(b*e - a*f))^m*AppellF1[1 + m, -n, 1
 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)
] + (-(B*e) + A*f)*Hypergeometric2F1[1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a +
 b*x))/((b*c - a*d)*(e + f*x))]))/(f*(-(b*e) + a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n))
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int (bx+a)^m (Bx+A)(dx+c)^n \left(fx+e\right)^{-2-m-n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-2-m-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

3.147 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-3-m-n}\,dx$

Optimal. Leaf size=263

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(m+1$$

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/((
b*e - a*f)*(d*e - c*f)*(2 + m + n)) - ((b*(B*c*e*(1 + m) + A*(c*f*(1 + n) -
 d*e*(2 + m + n))) + a*(A*d*f*(1 + m) + B*(d*e*(1 + n) - c*f*(2 + m + n))))
*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 +
 m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e -
 a*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*
d)*(e + f*x)))^n)
```

Rubi [A] time = 0.226597, antiderivative size = 261, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {155, 12, 132}

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(m+1$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/((
b*e - a*f)*(d*e - c*f)*(2 + m + n)) - ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) -
A*d*e*(2 + m + n)) + a*(A*d*f*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))
*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 +
 m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e -
 a*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*
d)*(e + f*x)))^n)
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
```

```
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \frac{\int (b(Bce(1+m)+ \ldots}{\ldots}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \frac{(b(Bce(1+m)+A \ldots}{\ldots}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \frac{(b(Bce(1+m)+A \ldots}{\ldots}$$

Mathematica [A] time = 0.31494, size = 223, normalized size = 0.85

$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-2}\left(\frac{(e+fx)\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(m+1)-Bcf(m+n+2)+Bde(n+1))+b(Acf(n+1)-Ade(m+n+2)+Bce(m+1)))\,{}_2F_1 \ldots}{(m+1)(be-af)} \ldots\right)}{(m+n+2)(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-2 - m - n)*((-(B*e) + A*f)*(c
+ d*x) + ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) - A*d*e*(2 + m + n)) + a*(A*d*f
*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))*(e + f*x)*Hypergeometric2F1[
1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*
e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n)))/((b
*e - a*f)*(d*e - c*f)*(2 + m + n)))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int (bx+a)^m (Bx+A)(dx+c)^n \left(fx+e\right)^{-3-m-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-3-m-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

3.148 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx$

Optimal. Leaf size=558

$$(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(bde((m+n+3)(A(-adf-bcf+bde)+aBcf)-(Be-$$

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-3 - m - n))/((
b*e - a*f)*(d*e - c*f)*(3 + m + n)) + ((a*f*(A*d*f*(2 + m) + B*(d*e*(1 + n)
 - c*f*(3 + m + n))) + b*(B*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*
(4 + m + n))))*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/
((b*e - a*f)^2*(d*e - c*f)^2*(2 + m + n)*(3 + m + n)) + (((2 + m + n)*(a*b*
c*d*f*(B*e - A*f) + b*d*e*((a*B*c*f + A*(b*d*e - b*c*f - a*d*f))*(3 + m + n
) - (B*e - A*f)*(b*c*(1 + m) + a*d*(1 + n))) - (b*c + a*d)*f*((a*B*c*f + A*
(b*d*e - b*c*f - a*d*f))*(3 + m + n) - (B*e - A*f)*(b*c*(1 + m) + a*d*(1 +
n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*f*(A*d*f*(2 + m) + B*(d*e*(1 + n) -
c*f*(3 + m + n))) + b*(B*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*(4
+ m + n)))))*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*Hypergeom
etric2F1[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)
))])/((b*e - a*f)^3*(d*e - c*f)^2*(1 + m)*(2 + m + n)*(3 + m + n)*(((b*e -
a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n)
```

Rubi [A] time = 0.9808, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {155, 12, 132}

$$(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(-bde(a(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-3 - m - n))/((
b*e - a*f)*(d*e - c*f)*(3 + m + n)) + ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n)
- B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*
(4 + m + n))))*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/
((b*e - a*f)^2*(d*e - c*f)^2*(2 + m + n)*(3 + m + n)) + (((2 + m + n)*(a*b*
c*d*f*(B*e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m
+ n)) + a*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d
)*f*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 +
m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*
f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(
1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(a + b*x)^(1 + m)*(c + d*x
)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*
f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)^3*(d*e - c*f)^2*(1 +
m)*(2 + m + n)*(3 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x))
)^n)
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} - \frac{\int (a+bx)^m(c+$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Adf(2+m}{}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Adf(2+m}{}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Adf(2+m}{}$$

Mathematica [A] time = 1.87166, size = 508, normalized size = 0.91

$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-3}\left(-\frac{(e+fx)^2\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(-bde(a(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Acf(n+2)-Ade(m}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-3 - m - n)*(-((B*e - A*f)*(c +
 d*x)) - ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e
*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n))))*(c + d*x)*(e +
 f*x))/((b*e - a*f)*(d*e - c*f)*(2 + m + n)) - (((2 + m + n)*(a*b*c*d*f*(B*
e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a
*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d)*f*(b*(B
*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 + m) + B*d*
e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*f*(A*d*f*
(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) +
 A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(e + f*x)^2*Hypergeometric2F1[1 + m
, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*e - a
*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)
```

```
*(e + f*x)))^n)))/((b*e - a*f)*(d*e - c*f)*(3 + m + n)))
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int (bx+a)^{m} (Bx+A) (dx+c)^{n} \left(fx+e\right)^{-4-m-n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx+A)(bx+a)^{m}(dx+c)^{n}\left(fx+e\right)^{-m-n-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((Bx+A)(bx+a)^{m}(dx+c)^{n}\left(fx+e\right)^{-m-n-4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-4-m-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

3.149 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

```
[Out] -(c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2) - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.142394, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1609, 1809, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2) - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x\left(-2c-3ad^2-3bd^2x\right)}{\sqrt{1-d^2x^2}}\,dx}{3d^2}$$

$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$

$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.0619055, size = 57, normalized size = 0.72

$$\frac{3bd\sin^{-1}(dx)-\sqrt{1-d^2x^2}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))) + 3*b*d*ArcSi
n[d*x])/(6*d^4)
```

Maple [C] time = 0.038, size = 139, normalized size = 1.8

$$-\frac{\mathrm{csgn}(d)}{6\,d^4}\sqrt{-dx+1}\sqrt{dx+1}\left(2\,\mathrm{csgn}(d)\,x^2cd^2\sqrt{-d^2x^2+1}+3\,\sqrt{-d^2x^2+1}\mathrm{csgn}(d)\,xbd^2+6\,\mathrm{csgn}(d)\,\sqrt{-d^2x^2+1}ad^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(-d^2*x^2+1)^(1/2)+3
*(-d^2*x^2+1)^(1/2)*csgn(d)*x*b*d^2+6*csgn(d)*(-d^2*x^2+1)^(1/2)*a*d^2+4*cs
gn(d)*(-d^2*x^2+1)^(1/2)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 3.06815, size = 134, normalized size = 1.7

$$-\frac{\sqrt{-d^2x^2+1}cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}bx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}a}{d^2}+\frac{b\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\,\sqrt{d^2}d^2}-\frac{2\,\sqrt{-d^2x^2+1}c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) - 2/3*sq
```

```
rt(-d^2*x^2 + 1)*c/d^4
```

Fricas [A] time = 1.04654, size = 189, normalized size = 2.39

$$-\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Sympy [C] time = 46.1254, size = 313, normalized size = 3.96

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0, & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - a*meijerg(((-1, -3/4, -1/2, -1/4,
0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I*pi)/(d**2*x
**2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + b*m
eijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1,
 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*c*meijerg(((-5
/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d*
*2*x**2))/(4*pi**(3/2)*d**4) - c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()
), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d**4)
```

Giac [A] time = 2.05163, size = 123, normalized size = 1.56

$$\frac{6\,bd^{10}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)-\left(6\,ad^{11}-3\,bd^{10}+6\,cd^9+\left(2\,(dx+1)cd^9+3\,bd^{10}-4\,cd^9\right)(dx+1)\right)\sqrt{dx+1}\sqrt{-dx+1}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(6*b*d^10*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^11 - 3*b*d^10 +
 6*c*d^9 + (2*(d*x + 1)*c*d^9 + 3*b*d^10 - 4*c*d^9)*(d*x + 1))*sqrt(d*x + 1
)*sqrt(-d*x + 1))/d
```

3.150 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.0613671, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-c-2ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(c+2ad^2\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.0308813, size = 45, normalized size = 0.71

$$\frac{\left(2ad^2+c\right)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

Maple [C] time = 0.015, size = 117, normalized size = 1.9

$$-\frac{\mathrm{csgn}(d)}{2\,d^3}\sqrt{-dx+1}\sqrt{dx+1}\left(\mathrm{csgn}(d)\,d\sqrt{-d^2x^2+1}xc - 2\arctan\left(\frac{\mathrm{csgn}(d)\,dx}{\sqrt{-d^2x^2+1}}\right)ad^2 + 2\,\mathrm{csgn}(d)\,d\sqrt{-d^2x^2+1}b - \mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [A] time = 4.26123, size = 105, normalized size = 1.67

$$\frac{a\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\,\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt
(-d^2*x^2 + 1)*b/d^2 + 1/2*c*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.04267, size = 167, normalized size = 2.65

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\left(2\,ad^2+c\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [C] time = 21.0093, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

Giac [A] time = 2.57145, size = 97, normalized size = 1.54

$$-\frac{\left((dx+1)cd^4+2\,bd^5-cd^4\right)\sqrt{dx+1}\sqrt{-dx+1}-2\left(2\,ad^6+cd^4\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*(((d*x + 1)*c*d^4 + 2*b*d^5 - c*d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
2*(2*a*d^6 + c*d^4)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
```

3.151 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rubi [A] time = 0.184023, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1809, 844, 216, 266, 63, 208}

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}-\frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx+b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right) \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-\frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0497883, size = 48, normalized size = 1.

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Maple [C] time = 0.024, size = 96, normalized size = 2.

$$\frac{\text{csgn}(d)}{d^2}\left(-\text{csgn}(d)\,\text{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)ad^2-\text{csgn}(d)\sqrt{-d^2x^2+1}c+\arctan\left(\text{csgn}(d)\,dx\frac{1}{\sqrt{-(dx+1)(dx-1)}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*d^2-csgn(d)*(-d^2*x^2+1)^(1/2)*c+
arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2))*b*d)*(-d*x+1)^(1/2)*(d*x+1)^(1
/2)/d^2*csgn(d)/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 3.42336, size = 89, normalized size = 1.85

$$-a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{\sqrt{-d^2x^2+1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d^2*x/sqrt(d^2))/
sqrt(d^2) - sqrt(-d^2*x^2 + 1)*c/d^2
```

Fricas [A] time = 1.14236, size = 196, normalized size = 4.08

$$\frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\sqrt{-dx+1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Sympy [C] time = 27.8669, size = 245, normalized size = 5.1

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bG_{6,6}^{2,6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
```

```
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.152 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rubi [A] time = 0.183313, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1807, 844, 216, 266, 63, 208}

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2} \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.054571, size = 48, normalized size = 1.

$$-\frac{a\sqrt{1-d^2x^2}}{x} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right) + \frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Maple [C] time = 0.02, size = 97, normalized size = 2.

$$\frac{\text{csgn}(d)}{dx}\left(-\text{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\text{csgn}(d)\,dxb - \text{csgn}(d)\,d\sqrt{-d^2x^2+1}a + \arctan\left(\text{csgn}(d)\,dx\frac{1}{\sqrt{-d^2x^2+1}}\right)xc\right)\sqrt{-d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-arctanh(1/(-d^2*x^2+1)^(1/2))*csgn(d)*d*x*b-csgn(d)*d*(-d^2*x^2+1)^(1/2)*
a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*x*c)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*
csgn(d)/(-d^2*x^2+1)^(1/2)/d/x
```

Maxima [A] time = 3.04543, size = 89, normalized size = 1.85

$$-b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{\sqrt{-d^2x^2+1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d^2*x/sqrt(d^2))/
sqrt(d^2) - sqrt(-d^2*x^2 + 1)*a/x
```

Fricas [A] time = 1.14896, size = 201, normalized size = 4.19

$$\frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\sqrt{-dx+1}ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

Sympy [C] time = 27.5663, size = 221, normalized size = 4.6

$$\frac{iadG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{adG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{2,6}\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
```

```
2))/(4*pi**(3/2)*d)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError
```

3.153 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

```
[Out] -(a*Sqrt[1 - d^2*x^2])/(2*x^2) - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rubi [A] time = 0.188045, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1609, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*Sqrt[1 - d^2*x^2])/(2*x^2) - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2b-\left(2c+ad^2\right)x}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{4}\left(-2c-ad^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right) \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(a+\frac{2c}{d^2}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right) \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0474664, size = 56, normalized size = 0.79

$$
-\frac{\sqrt{1-d^2x^2}(a+2bx)}{2x^2}-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a + 2*b*x)*Sqrt[1 - d^2*x^2])/(2*x^2) - ((2*c + a*d^2)*ArcTanh[Sqrt[1 -
d^2*x^2]])/2
```

Maple [C] time = 0.019, size = 108, normalized size = 1.5

$$
-\frac{(\mathrm{csgn}(d))^2}{2x^2}\sqrt{-dx+1}\sqrt{dx+1}\left(\mathrm{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)x^2ad^2+2\,\mathrm{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)x^2c+2\sqrt{-d^2x^2+1}xb+\sqrt{-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
x^2*a*d^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*x^2*c+2*(-d^2*x^2+1)^(1/2)*x*b+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

Maxima [A] time = 2.73431, size = 132, normalized size = 1.86

$$-\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-d^2x^2+1}b}{x}-\frac{\sqrt{-d^2x^2+1}a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

Fricas [A] time = 1.04925, size = 154, normalized size = 2.17

$$\frac{\left(ad^2+2\,c\right)x^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\sqrt{-dx+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

Sympy [C] time = 34.5905, size = 218, normalized size = 3.07

$$\frac{iad^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ad^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+ \dots bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) - a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1
), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) + I*b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)) + I*c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1
, 5/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - c*meijerg(((0, 1/4, 1/2,
 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*
x**2))/(4*pi**(3/2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError
```

3.154 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{dx-1}\sqrt{dx+1}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\cosh^{-1}(dx)}{2d^3}+\frac{cx^2\sqrt{dx-1}\sqrt{dx+1}}{3d^2}$$

```
[Out] (c*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(3*d^2) + (Sqrt[-1 + d*x]*Sqrt[1 + d*x
]*(2*(2*c + 3*a*d^2) + 3*b*d^2*x))/(6*d^4) + (b*ArcCosh[d*x])/(2*d^3)
```

Rubi [A] time = 0.146262, antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1610, 1809, 780, 217, 206}

$$-\frac{\left(1-d^2x^2\right)\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*(1 - d^2*x^2))/(3*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c + 3*
a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (
b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*
x]*Sqrt[1 + d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx &= \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\ &= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int \frac{x\left(2c+3ad^2+3bd^2x\right)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}\\ &= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}\\ &= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int\right.}{2d^2\sqrt{-1+dx}}\\ &= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}}+\frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{}{\sqrt{-1}}\right.}{2d^3\sqrt{-1+dx}\sqrt{1+d}}\end{aligned}$$

Mathematica [A] time = 0.331289, size = 149, normalized size = 1.71

$$\frac{\sqrt{-(dx-1)^2}\sqrt{dx+1}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)+6\sqrt{dx-1}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)(d(2ad-b)+2c)-12\sqrt{1-dx}\tanh^{-1}\left(\right.}{6d^4\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))
+ 6*(2*c + d*(-b + 2*a*d))*Sqrt[-1 + d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] - 1
2*(c + d*(-b + a*d))*Sqrt[1 - d*x]*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*
d^4*Sqrt[1 - d*x])
```

Maple [C] time = 0.023, size = 137, normalized size = 1.6

$$\frac{\text{csgn}(d)}{6\,d^4}\sqrt{dx-1}\sqrt{dx+1}\left(2\,\text{csgn}(d)\,x^2cd^2\sqrt{d^2x^2-1}+3\,\text{csgn}(d)\,\sqrt{d^2x^2-1}xbd^2+6\,\text{csgn}(d)\,\sqrt{d^2x^2-1}ad^2+4\,\text{csgn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(d^2*x^2-1)^(1/2)+3*cs
gn(d)*(d^2*x^2-1)^(1/2)*x*b*d^2+6*csgn(d)*(d^2*x^2-1)^(1/2)*a*d^2+4*csgn(d)
```

```
*(d^2*x^2-1)^(1/2)*c+3*ln((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*b*d)*csg
n(d)/d^4/(d^2*x^2-1)^(1/2)
```

Maxima [A] time = 1.35168, size = 147, normalized size = 1.69

$$\frac{\sqrt{d^2x^2-1}cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2}+\frac{2\,\sqrt{d^2x^2-1}c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(d^2))/(sqrt(d
^2)*d^2) + 2/3*sqrt(d^2*x^2 - 1)*c/d^4
```

Fricas [A] time = 1.05241, size = 176, normalized size = 2.02

$$-\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{dx-1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

Sympy [C] time = 44.7876, size = 308, normalized size = 3.54

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|{\frac{1}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|{\frac{e^{2i\pi}}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d**2) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1
, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*mei
jerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0
)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) + c*meijerg(((-5/4, -
3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d**2*x*
*2))/(4*pi**(3/2)*d**4) + I*c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()),
((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
```

```
(3/2)*d**4)
```

Giac [A] time = 2.12368, size = 130, normalized size = 1.49

$$-\frac{6\,bd^{10}\log\left(\left|-\sqrt{dx+1}+\sqrt{dx-1}\right|\right)-\left(6\,ad^{11}-3\,bd^{10}+6\,cd^{9}+\left(2\,(dx+1)cd^{9}+3\,bd^{10}-4\,cd^{9}\right)(dx+1)\right)\sqrt{dx+1}\sqrt{dx}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3840*(6*b*d^10*log(abs(-sqrt(d*x + 1) + sqrt(d*x - 1))) - (6*a*d^11 - 3*
b*d^10 + 6*c*d^9 + (2*(d*x + 1)*c*d^9 + 3*b*d^10 - 4*c*d^9)*(d*x + 1))*sqrt
(d*x + 1)*sqrt(d*x - 1))/d
```

3.155 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=52

$$\frac{(2ad^2+c)\cosh^{-1}(dx)}{2d^3}+\frac{\sqrt{dx-1}\sqrt{dx+1}(2b+cx)}{2d^2}$$

```
[Out] ((2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*d^2) + ((c + 2*a*d^2)*ArcCosh
[d*x])/(2*d^3)
```

Rubi [B] time = 0.0711419, antiderivative size = 135, normalized size of antiderivative = 2.6, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {901, 1815, 641, 217, 206}

$$\frac{\sqrt{d^2x^2-1}\,(2ad^2+c)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x}\right.}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right.}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [B] time = 0.210479, size = 126, normalized size = 2.42

$$\frac{4\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)(d(ad-b)+c)+d\sqrt{-(dx-1)^2}\sqrt{dx+1}(2b+cx)+2\sqrt{dx-1}(2bd-c)\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)}{2d^3\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x] + 2*(-c + 2*b*d)*Sqrt[-1 +
 d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + 4*(c + d*(-b + a*d))*Sqrt[1 - d*x]*Ar
cTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(2*d^3*Sqrt[1 - d*x])
```

Maple [C] time = 0.016, size = 120, normalized size = 2.3

$$\frac{\text{csgn}(d)}{2\,d^3}\sqrt{dx-1}\sqrt{dx+1}\left(\text{csgn}(d)\,d\sqrt{d^2x^2-1}xc+2\,\text{csgn}(d)\,d\sqrt{d^2x^2-1}b+2\,\ln\left(\left(\text{csgn}(d)\,\sqrt{d^2x^2-1}+dx\right)\text{csgn}(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(csgn(d)*d*(d^2*x^2-1)^(1/2)*x*c+2*csgn(d)*
d*(d^2*x^2-1)^(1/2)*b+2*ln((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*a*d^2+l
n((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*c)*csgn(d)/d^3/(d^2*x^2-1)^(1/2)
```

Maxima [B] time = 1.35955, size = 142, normalized size = 2.73

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{\sqrt{d^2}} + \frac{\sqrt{d^2x^2-1}cx}{2\,d^2} + \frac{\sqrt{d^2x^2-1}b}{d^2} + \frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(d^2))/sqrt(d^2) + 1/2*sqrt(d^2*x^2
 - 1)*c*x/d^2 + sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^
2 - 1)*sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.10158, size = 150, normalized size = 2.88

$$\frac{(cdx + 2\,bd)\sqrt{dx+1}\sqrt{dx-1} - \left(2\,ad^2 + c\right)\log\left(-dx + \sqrt{dx+1}\sqrt{dx-1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

Sympy [C] time = 20.7389, size = 277, normalized size = 5.33

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\ldots\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d) + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1
/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*meijerg(((-1, -3/4, -1
/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi
)/(d**2*x**2))/(4*pi**(3/2)*d**2) + c*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0
, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3
) - I*c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2
, -1, -1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

Giac [A] time = 2.5708, size = 104, normalized size = 2.

$$\frac{\left((dx+1)cd^4 + 2\,bd^5 - cd^4\right)\sqrt{dx+1}\sqrt{dx-1} - 2\left(2\,ad^6 + cd^4\right)\log\left(\left|-\sqrt{dx+1} + \sqrt{dx-1}\right|\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(((d*x + 1)*c*d^4 + 2*b*d^5 - c*d^4)*sqrt(d*x + 1)*sqrt(d*x - 1) - 2*
(2*a*d^6 + c*d^4)*log(abs(-sqrt(d*x + 1) + sqrt(d*x - 1))))/d
```

3.156 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$a\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{b\cosh^{-1}(dx)}{d}+\frac{c\sqrt{dx-1}\sqrt{dx+1}}{d^2}$$

```
[Out] (c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + (b*ArcCosh[d*x])/d + a*ArcTan[Sqrt[-
1 + d*x]*Sqrt[1 + d*x]]
```

Rubi [B] time = 0.184061, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1610, 1809, 844, 217, 206, 266, 63, 205}

$$\frac{a\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}-\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Sub}}{\sqrt{-1}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\right.}{d^2\sqrt{-1+dx}\sqrt{1+}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x}}\right.}{d\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [B] time = 0.399322, size = 128, normalized size = 2.33

$$\frac{\frac{ad^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)+cd^2x^2-2c\sqrt{1-d^2x^2}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)-c}{\sqrt{dx-1}\sqrt{dx+1}}-2(c-bd)\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-c + c*d^2*x^2 - 2*c*Sqrt[1 - d^2*x^2]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + a*
d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 +
 d*x]) - 2*(c - b*d)*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d^2
```

Maple [C] time = 0.019, size = 95, normalized size = 1.7

$$\frac{\operatorname{csgn}(d)}{d^2}\left(-\operatorname{csgn}(d)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)ad^2+\operatorname{csgn}(d)\sqrt{d^2x^2-1}c+\ln\left(\left(\operatorname{csgn}(d)\sqrt{(dx+1)(dx-1)}+dx\right)\operatorname{csgn}(d)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+csgn(d)*(d^2*x^2-1)^(1/2)*c+ln(
(csgn(d)*((d*x+1)*(d*x-1))^(1/2)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

Maxima [A] time = 3.91204, size = 86, normalized size = 1.56

$$-a\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{\sqrt{d^2}}+\frac{\sqrt{d^2x^2-1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(sqrt(d^2)*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(
d^2))/sqrt(d^2) + sqrt(d^2*x^2 - 1)*c/d^2
```

Fricas [A] time = 1.08568, size = 184, normalized size = 3.35

$$\frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

Sympy [C] time = 26.2475, size = 240, normalized size = 4.36

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [A] time = 1.98647, size = 96, normalized size = 1.75

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

3.157 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{a\sqrt{dx-1}\sqrt{dx+1}}{x}+b\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{c\cosh^{-1}(dx)}{d}$$

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + (c*ArcCosh[d*x])/d + b*ArcTan[Sqrt[-1
+ d*x]*Sqrt[1 + d*x]]
```

Rubi [B] time = 0.182098, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1610, 1807, 844, 217, 206, 266, 63, 205}

$$-\frac{a\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx,x,x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\text{Sub}}{\sqrt{-1}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\right.}{d^2\sqrt{-1+dx}\sqrt{1+}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right.}{d\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.167041, size = 89, normalized size = 1.62

$$\frac{a\left(d^2x^2-1\right)+bx\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{x\sqrt{dx-1}\sqrt{dx+1}} + \frac{2c\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*(-1 + d^2*x^2) + b*x*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(x*S
qrt[-1 + d*x]*Sqrt[1 + d*x]) + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] time = 0.017, size = 96, normalized size = 1.8

$$\frac{\operatorname{csgn}(d)}{dx}\left(-\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\operatorname{csgn}(d)\,dxb+\operatorname{csgn}(d)\,d\sqrt{d^2x^2-1}a+\ln\left(\left(\operatorname{csgn}(d)\sqrt{d^2x^2-1}+dx\right)\operatorname{csgn}(d)\right)xc\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-arctan(1/(d^2*x^2-1)^(1/2))*csgn(d)*d*x*b+csgn(d)*d*(d^2*x^2-1)^(1/2)*a+l
n((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*x*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)
*csgn(d)/(d^2*x^2-1)^(1/2)/d/x
```

Maxima [A] time = 2.01558, size = 86, normalized size = 1.56

$$-b\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)+\frac{c\log\left(2\,d^2x+2\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{\sqrt{d^2}}+\frac{\sqrt{d^2x^2-1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(sqrt(d^2)*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(
d^2))/sqrt(d^2) + sqrt(d^2*x^2 - 1)*a/x
```

Fricas [A] time = 1.14288, size = 203, normalized size = 3.69

$$\frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}ad-cx\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

Sympy [C] time = 27.9255, size = 216, normalized size = 3.93

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

Giac [A] time = 2.66535, size = 112, normalized size = 2.04

$$-\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)-\frac{8\,ad^{2}}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

3.158 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{2}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{2x^2}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{x}$$

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (b*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
/x + ((2*c + a*d^2)*ArcTan[Sqrt[-1 + d*x]*Sqrt[1 + d*x]])/2
```

Rubi [A] time = 0.188847, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1610, 1807, 807, 266, 63, 205}

$$\frac{\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*ArcTa
n[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+\left(2c+ad^2\right)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d}}\right.}{4\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\right.}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right.}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.12092, size = 82, normalized size = 0.99

$$\frac{\left(d^2x^2-1\right)(a+2bx)+x^2\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((a + 2*b*x)*(-1 + d^2*x^2) + (2*c + a*d^2)*x^2*Sqrt[-1 + d^2*x^2]*ArcTan[S
qrt[-1 + d^2*x^2]])/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Maple [C] time = 0.018, size = 103, normalized size = 1.2

$$-\frac{(\text{csgn}(d))^2}{2x^2}\sqrt{dx-1}\sqrt{dx+1}\left(\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)x^2ad^2+2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)x^2c-2\sqrt{d^2x^2-1}xb-\sqrt{d^2x^2-1}a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctan(1/(d^2*x^2-1)^(1/2))*x^2
*a*d^2+2*arctan(1/(d^2*x^2-1)^(1/2))*x^2*c-2*(d^2*x^2-1)^(1/2)*x*b-(d^2*x^2
-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^2
```

Maxima [A] time = 2.21014, size = 88, normalized size = 1.06

$$-\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)-c\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)+\frac{\sqrt{d^2x^2-1}b}{x}+\frac{\sqrt{d^2x^2-1}a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(sqrt(d^2)*abs(x))) - c*arcsin(1/(sqrt(d^2)*abs(x))) +
sqrt(d^2*x^2 - 1)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

Fricas [A] time = 1.05714, size = 173, normalized size = 2.08

$$\frac{2\,bdx^2+2\left(ad^2+2\,c\right)x^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\sqrt{dx-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

Sympy [C] time = 34.1775, size = 212, normalized size = 2.55

$$-\frac{ad^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iad^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4
, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2
))/(4*pi**(3/2)) - c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5
/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*c*meijerg(((0, 1/4, 1/2,
```

```
3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x*
*2))/(4*pi**(3/2))
```

Giac [B] time = 1.46926, size = 196, normalized size = 2.36

$$-\frac{\left(ad^3+2\,cd\right)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-16\,bd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

3.159 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{dx-1}\sqrt{dx+1}\left(2ad^2+3c\right)}{3x}+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{3x^3}+\frac{1}{2}bd^2\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{2x^2}$$

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(3*x^3) + (b*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
/(2*x^2) + ((3*c + 2*a*d^2)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(3*x) + (b*d^2*Ar
cTan[Sqrt[-1 + d*x]*Sqrt[1 + d*x]])/2
```

Rubi [A] time = 0.220846, antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1610, 1807, 835, 807, 266, 63, 205}

$$-\frac{\left(1-d^2x^2\right)\left(2ad^2+3c\right)}{3x\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(3*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(3*
x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1
+ d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+\left(3c+2ad^2\right)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2\left(3c+2ad^2\right)+3bd^2x}{x^2\sqrt{-1+d^2x^2}}\,dx}{6\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\sqrt{-1+d^2\right.}{2\sqrt{-1+}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\sqrt{-1+d^2\right.}{4}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{bd^2\sqrt{-1+d^2x}}{2\sqrt{-1}}$$

Mathematica [A] time = 0.116229, size = 94, normalized size = 0.81

$$\frac{\left(d^2x^2-1\right)\left(a\left(4d^2x^2+2\right)+3x(b+2cx)\right)+3bd^2x^3\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{6x^3\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-1 + d^2*x^2)*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)) + 3*b*d^2*x^3*Sqrt[-1
 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(6*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]
)
```

Maple [C] time = 0.019, size = 123, normalized size = 1.1

$$-\frac{(\mathrm{csgn}(d))^2}{6x^3}\sqrt{dx-1}\sqrt{dx+1}\left(3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)x^3bd^2-4\sqrt{d^2x^2-1}x^2ad^2-6\sqrt{d^2x^2-1}x^2c-3\sqrt{d^2x^2-1}xb-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(3*arctan(1/(d^2*x^2-1)^(1/2))*x
^3*b*d^2-4*(d^2*x^2-1)^(1/2)*x^2*a*d^2-6*(d^2*x^2-1)^(1/2)*x^2*c-3*(d^2*x^2
-1)^(1/2)*x*b-2*(d^2*x^2-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^3
```

Maxima [A] time = 1.79371, size = 119, normalized size = 1.03

$$-\frac{1}{2}bd^2\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)+\frac{2\sqrt{d^2x^2-1}ad^2}{3x}+\frac{\sqrt{d^2x^2-1}c}{x}+\frac{\sqrt{d^2x^2-1}b}{2x^2}+\frac{\sqrt{d^2x^2-1}a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(sqrt(d^2)*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + s
qrt(d^2*x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*
a/x^3
```

Fricas [A] time = 1.03175, size = 216, normalized size = 1.86

$$\frac{6bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+2\left(2ad^3+3cd\right)x^3+\left(2\left(2ad^2+3c\right)x^2+3bx+2a\right)\sqrt{dx+1}\sqrt{dx-1}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

Sympy [C] time = 56.6455, size = 219, normalized size = 1.89

$$-\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iad^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bd^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)) - b*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2,
9/4, 5/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*d**2*meijerg(((1, 5/4,
 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d
**2*x**2))/(4*pi**(3/2)) - c*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1,
 5/4, 3/2, 7/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*c*d*meijerg(((1
/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*
pi)/(d**2*x**2))/(4*pi**(3/2))
```

Giac [B] time = 1.68982, size = 266, normalized size = 2.29

$$-\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{8}-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}-96\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-\cdots}\right.\right.}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
 - 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```